САМ СЕБЕ РЕПЕТИТОР ®

ОТВЕТЫ И РЕШЕНИЯ

+ РЕШЕНИЯ ЗАДАЧ ПОВЫШЕННОЙ ТРУДНОСТИ И НА ПОСТРОЕНИЕ

10

К задачнику

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АЛГЕБРА

и начала анализа

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ПОДРОБНЫЙ РАЗБОР ЗАДАНИЙ ИЗ ЗАДАЧНИКА ПО АЛГЕБРЕ И НАЧАЛАМ АНАЛИЗА

10 класс

авторов: А.Г. Мордкович
Л.О. Денищева
Т.А.Корешкова и др.
(М.: Мнемозина, 2002-2004)

+ ЗАДАЧИ ПОВЫШЕННОЙ СЛОЖНОСТИ

+ ЗАДАЧИ НА ПОСТРОЕНИЕ

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Издание содержит алгоритмы решения типовых задач, подробный разбор абсолютно всех заданий, включая задачи на построение графиков и задачи повышенной сложности из учебника по алгебре для 10–11 класса авторов: А.Г. Мордкович, Л.О. Денищева, Т.А. Корешкова и др. (М.: Мнемозина, 2002–2004).

Пособие будет незаменимым помощником школьникам при приготовлении домашних работ, подготовке к экзамену, а также будет способствовать обретению прочных навыков самопроверки.

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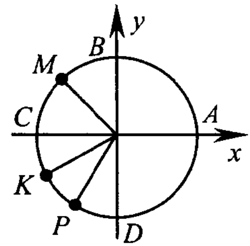
Глава 1. Тригонометрические функции

§1. Введение

В задачах 1 – 8 требуется найти длину дуги. Она находится по формуле: $l = \alpha \cdot r$, где α – радианная мера дуги, r – радиус окружности. Так как рассматривается единичная окружность, то $r = 1$, т.е. $l = \alpha$. Переход от градусной к радианной мере: $\alpha = \frac{\beta}{180} \cdot \pi$, где β – мера угла в градусах.

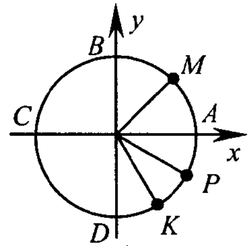


1. $AM = 90° + 45° = 135°$, $BK = 90° + 30° = 120°$.

$MP = 45° + 60° = 105°$, $DC = 270°$,

$KA = 150°$, $BP = 150°$,

$CB = 270°$, $BC = 90°$.



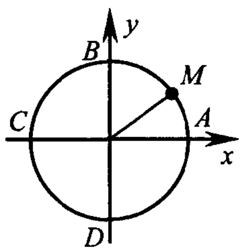
2. $AM = 45°$, $BD = 180°$,

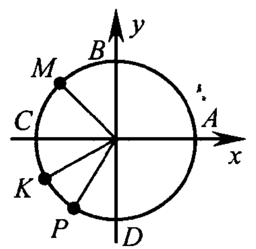
$CK = 120°$, $MP = 285°$,

$DM = 135°$, $MK = 360° - 105° = 255°$,

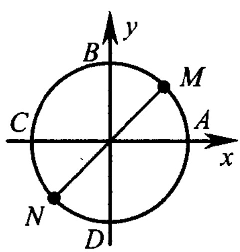
$CP = 150°$, $PC = 210°$.

3. $AM = 36°$, $MB = 54°$, $DM = 126°$, $MC = 144°$. **4.** $CP = 15°$, $PD = 75°$, $AP = 105°$.

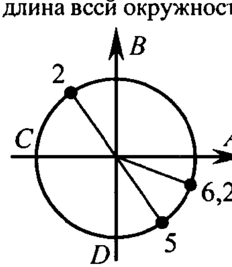




5. $AN = 225°$.



6. а) Да; **б)** да; **в)** да; **г)** нет, т.к. длина всей окружности $l = 2\pi r = 2 \cdot 3{,}1415\ldots < 6{,}3$.



7. $\angle AOM = 60°$; $MN = 2 \cdot \sin \angle AOM = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$;

$AM = 60°$, $MB = 30°$, $AN = 300°$, $NA = 60°$.

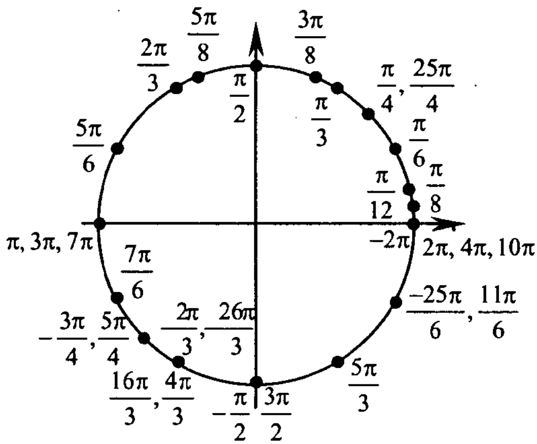
8. Воспользуемся результатами задачи № 7.

$\overset{\smile}{AM} = \overset{\smile}{NA} = 60°$, $\overset{\smile}{MB} = \overset{\smile}{DN} = 30°$. Аналогично $\overset{\smile}{BP} = \overset{\smile}{QD} = 30°$, $\overset{\smile}{PC} = \overset{\smile}{CQ} = 60°$.

Окончательно получаем: $\overset{\smile}{AM} = \overset{\smile}{MP} = \overset{\smile}{PC} = \overset{\smile}{CQ} = \overset{\smile}{QN} = \overset{\smile}{NA} = 60°$, ч.т.д.

§2. Числовая окружность

С № 9 по № 16 см. рисунок.



9. а) $\frac{\pi}{2};(0;1)$; **б)** $\pi;(-1;0)$; **в)** $\frac{3\pi}{2};(0;-1)$; **г)** $2\pi;(1;0)$.

10. а) $7\pi;(-1;0)$; **б)** $4\pi;(1;0)$; **в)** $10\pi;(1;0)$; **г)** $3\pi;(-1;0)$.

11. а) $\frac{\pi}{3};\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$; **б)** $\frac{\pi}{4};\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **в)** $\frac{\pi}{6};\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$.

г) $\frac{\pi}{8}$; $\cos^2\frac{\pi}{8} = \frac{1+\cos\frac{\pi}{4}}{2}$; $\cos\frac{\pi}{8} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$;

$\sin^2\frac{\pi}{8} = \frac{1-\sin^2\frac{\pi}{4}}{2}$; $\sin\frac{\pi}{8} = \sqrt{\frac{2-\sqrt{2}}{2}}$; $\left(\sqrt{\frac{2+\sqrt{2}}{2}};\sqrt{\frac{2-\sqrt{2}}{2}}\right)$.

12. а) $\frac{2\pi}{3};\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$; **б)** $\frac{3\pi}{4};\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **в)** $\frac{5\pi}{6};\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$; **г)** $\frac{5\pi}{4};\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$.

13. а) $\frac{4\pi}{3};\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$; **б)** $\frac{5\pi}{3};\left(\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$; **в)** $\frac{7\pi}{6};\left(-\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$; **г)** $\frac{11\pi}{6};\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$.

14. а) $\frac{\pi}{12}$; $\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2}$; $\cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$;

$\sin^2\frac{\pi}{12}=\dfrac{1-\sin^2\frac{\pi}{6}}{2}$; $\sin\frac{\pi}{12}=\sqrt{\frac{2-\sqrt{3}}{2}}$; $\left(\sqrt{\frac{2+\sqrt{3}}{2}};\sqrt{\frac{2-\sqrt{3}}{2}}\right)$;

б) $\frac{5\pi}{12}$; $\frac{5}{12}\pi=\frac{\pi}{4}+\frac{\pi}{6}$; $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$;

$\sin\frac{5\pi}{12}=\frac{\sqrt{2}\left(\sqrt{3}+1\right)}{4}$; $\left(\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4};\frac{\sqrt{2}\left(\sqrt{3}+1\right)}{4}\right)$;

в) $\frac{3\pi}{8}$; $\cos\frac{3\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$; $\sin\frac{3\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}$; $\left(\frac{\sqrt{2-\sqrt{2}}}{2};\frac{\sqrt{2+\sqrt{2}}}{2}\right)$;

г) $\frac{5\pi}{8}$; $\cos\frac{5\pi}{8}=-\frac{\sqrt{2-\sqrt{2}}}{2}$; $\sin\frac{5\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}$; $\left(-\frac{\sqrt{2-\sqrt{2}}}{2};\frac{\sqrt{2+\sqrt{2}}}{2}\right)$.

15. а) $-\frac{\pi}{2};(0;-1)$; **б)** $\frac{2\pi}{3};\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$; **в)** $-2\pi;(1;0)$; **г)** $-\frac{3\pi}{4};\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$.

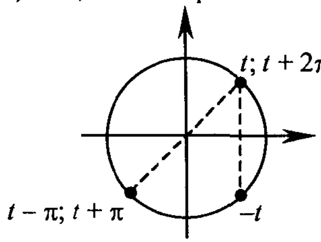
16. а) $\frac{25\pi}{4};\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **б)** $-\frac{25\pi}{3};\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$; **в)** $-\frac{25\pi}{6};\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$; **г)** $\frac{16\pi}{3};\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$.

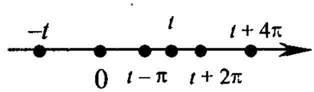
17. а) t; $-t$. На прямой: симметрично относительно нуля; на окружности: симметрично относительно оси x.

б) t; $t+2\pi k$. На прямой: стоят с периодом $2\pi k$; на окружности: совпадают.

в) t; $t+\pi$. На прямой: стоят на расстоянии в π; на окружности: диаметрально противоположны.

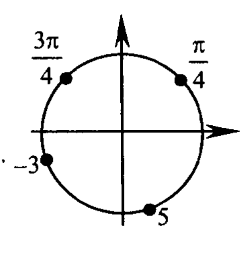
г) $t+\pi$, $t-\pi$. На прямой: стоят на расстоянии в 2π; на окружности: совпадают.





18. а) $M\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **б)** $M(5)=(0{,}284;-0{,}959)$;

в) $M\left(\frac{3\pi}{4}\right)=\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **г)** $M(-3)=(-0{,}990;-0{,}141)$.



19. а) A: $2\pi k$; **б)** C: $\pi+2\pi k$; **в)** A и C: πn.

20. См. рисунок к № 19.

а) B: $\frac{\pi}{2}+2\pi k$; **б)** D: $-\frac{\pi}{2}+2\pi k$; **в)** B и D: $\frac{\pi}{2}+\pi n$.

21. См. рисунок к № 19.

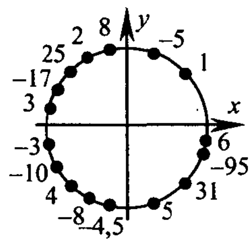
а) A: min положительное $= 2\pi$; max отрицательное $= -2\pi$;

б) B: min положительное $= \dfrac{\pi}{2}$; max отрицательное $= -\dfrac{3\pi}{2}$;

с) C: min положительное $= \pi$; max отрицательное $= -\pi$;

г) D: min положительное $= \dfrac{3\pi}{2}$; max отрицательное $= -\dfrac{\pi}{2}$.

С № 22 по № 25 см. рисунок.



22. **а)** 1: (0,540; 0,841); **б)** −5: (0,284; 0,950);
в) 4,5: (−0,211; −0,978); **г)** (−3): (−0,990; −0,141).

23. **а)** 6: IV; **б)** 2: II; **в)** 3: II; **г)** 4: III.

24. **а)** 5: IV; **б)** −5: I; **в)** 8: II; **г)** −8: III.

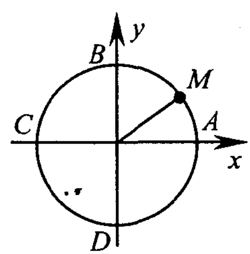
25. **а)** 10: III; **б)** −17: II; **в)** 31: IV; **г)** −95: IV.

26. **а)** AM: $t \in \left(2\pi k; \dfrac{\pi}{4} + 2\pi k\right)$;

б) CM: $t \in \left(-\pi + 2\pi k; \dfrac{\pi}{4} + 2\pi k\right)$;

в) MA: $t \in \left(\dfrac{\pi}{4} + 2\pi k; 2\pi + 2\pi k\right)$;

г) MC: $t \in \left(\dfrac{\pi}{4} + 2\pi k; \pi + 2\pi k\right)$.



27. См. рисунок к № 26

а) DM: $t \in \left(-\dfrac{\pi}{2} + 2\pi k; \dfrac{3\pi}{4} + 2\pi k\right)$; **б)** BD: $t \in \left(\dfrac{\pi}{2} + 2\pi k; \dfrac{3\pi}{2} + 2\pi k\right)$;

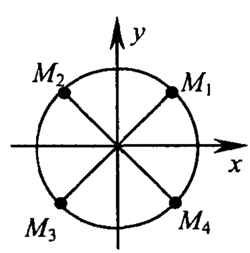
в) MD: $t \in \left(\dfrac{3\pi}{4} + 2\pi k; \dfrac{3\pi}{2} + 2\pi k\right)$; **г)** DB: $t \in \left(-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right)$.

28. **а)** M_1M_2: $t \in \left(\dfrac{\pi}{4} + 2\pi k; \dfrac{3\pi}{4} + 2\pi k\right)$;

б) M_4M_1: $t \in \left(-\dfrac{\pi}{4} + 2\pi k; \dfrac{\pi}{4} + 2\pi k\right)$;

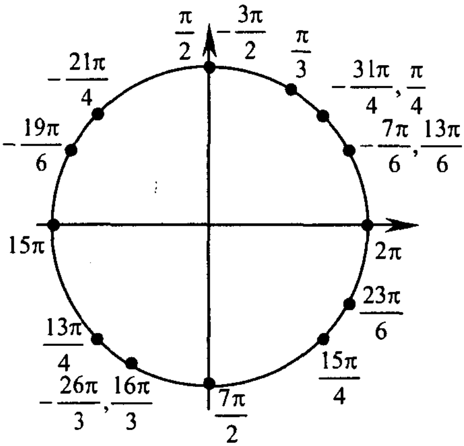
в) M_3M_2: $t \in \left(-\dfrac{3\pi}{4} + 2\pi k; \dfrac{3\pi}{4} + 2\pi k\right)$;

г) M_1M_3: $t \in \left(\dfrac{\pi}{4} + 2\pi k; \dfrac{5\pi}{4} + 2\pi k\right)$.



§3. Числовая окружность на координатной плоскости

С № 29 по № 32 см. рисунок.



29. а) $M\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **б)** $M\left(\frac{\pi}{6}\right)=\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$;

в) $M\left(\frac{\pi}{3}\right)=\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$; **г)** $M\left(\frac{\pi}{2}\right)=(0;1)$.

30. а) $M(2\pi)=(1;0)$; **б)** $M\left(-\frac{3\pi}{2}\right)=(0;1)$;

в) $M\left(\frac{7\pi}{2}\right)=(0;-1)$; **г)** $M(15\pi)=(-1;0)$.

31. а) $M\left(\frac{15\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$; **б)** $M\left(\frac{23\pi}{6}\right)=\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$;

в) $M\left(\frac{13\pi}{4}\right)=\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$; **г)** $M\left(\frac{16\pi}{3}\right)=\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$.

32. а) $M\left(-\frac{19\pi}{6}\right)=\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$; **б)** $M\left(-\frac{31\pi}{4}\right)=\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$;

в) $M\left(-\frac{21\pi}{4}\right)=\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; **г)** $M\left(-\frac{26\pi}{3}\right)=\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$.

33. а) $M=\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$: min положит. $=\frac{\pi}{6}$; max отрицат. $=-\frac{11\pi}{6}$;

б) $M=\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$: min положит. $=\frac{5\pi}{6}$; max отрицат. $=-\frac{7\pi}{6}$;

в) $M=\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$: min положит. $=\frac{11\pi}{6}$; max отрицат. $=-\frac{\pi}{6}$;

г) $M=\left(-\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$: min положит. $=\frac{7\pi}{6}$; max отрицат. $=-\frac{5\pi}{6}$.

34. а) $M=\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$: min положит. $=\frac{\pi}{3}$; max отрицат. $=-\frac{5\pi}{3}$;

б) $M=\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$: min положит. $=\frac{2\pi}{3}$; max отрицат. $=-\frac{4\pi}{3}$;

в) $M=\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$: min положит. $=\frac{4\pi}{3}$; max отрицат. $=-\frac{2\pi}{3}$;

г) $M=\left(\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$: min положит. $=\frac{5\pi}{3}$; max отрицат. $=-\frac{\pi}{3}$.

35. а) $y=\frac{\sqrt{2}}{2}$; $t=\frac{\pi}{4}+2\pi n;\ \frac{3\pi}{4}+2\pi n$; **б)** $y=\frac{1}{2}$; $t=\frac{\pi}{6}+2\pi n;\ \frac{5\pi}{6}+2\pi n$;

в) $y=0$; $t=\pi n$; **г)** $y=\frac{\sqrt{3}}{2}$; $t=\frac{\pi}{3}+2\pi n;\ \frac{2\pi}{3}+2\pi n$.

36. а) $y=-\frac{\sqrt{3}}{2}$; $x=(-1)^{k+1}\frac{\pi}{3}+\pi k$; **б)** $y=1$; $x=\frac{\pi}{2}+2\pi k$;

в) $y=-\frac{\sqrt{2}}{2}$; $x=(-1)^{k+1}\frac{\pi}{4}+\pi k$; **г)** $y=-1$; $x=-\frac{\pi}{2}+2\pi k$.

37. а) $x=\frac{\sqrt{3}}{2}$; $y=\pm\frac{\pi}{6}+2\pi k$; **б)** $x=\frac{1}{2}$; $y=\pm\frac{\pi}{3}+2\pi k$;

в) $x=1$; $y=2\pi k$; **г)** $x=\frac{\sqrt{2}}{2}$; $y=\pm\frac{\pi}{4}+2\pi k$.

38. а) $x=0$; $y=\frac{\pi}{2}+\pi k$; **б)** $x=-\frac{1}{2}$; $y=\pm\frac{2\pi}{3}+2\pi k$;

в) $x=-\frac{\sqrt{3}}{2}$; $y=\pm\frac{5\pi}{6}+2\pi k$; **г)** $x=-1$; $y=\pi+2\pi k$.

39. а) да; **б)** нет; **в)** да; **г)** нет.

40. а) $E\,(2)$ – +; **б)** $K\,(4)$ – –; **в)** $F\,(1)$ + +; **г)** $L\,(6)$ + –.

41. а) $M\,(12)$ + –; **б)** $N\,(15)$ – +; **в)** $P\,(49)$ + –; **г)** $Q\,(100)$ + –.

42. а) $y=-\frac{\sqrt{3}}{2}$, $x<0$ $\frac{4\pi}{3}+2\pi n$; $M\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$;

б) $y=\frac{1}{2}$, $x<0$ $\frac{5\pi}{6}+2\pi n$; $M\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$;

в) $y=\frac{1}{2}$, $x>0$ $\frac{\pi}{6}+2\pi n$; $M\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$;

г) $y=-\frac{\sqrt{3}}{2}$, $x>0$ $-\frac{\pi}{3}+2\pi n$; $M\left(\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$.

43. а) $x=\frac{\sqrt{3}}{2}$, $y>0$ $\frac{\pi}{6}+2\pi n$; $M\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$;

б) $x=-\frac{1}{2}$, $y<0$ $-\frac{2\pi}{3}+2\pi n$; $M\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$;.

в) $x=\frac{\sqrt{3}}{2}$, $y<0$ $-\frac{\pi}{6}+2\pi n$; $M\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$;

г) $x=-\frac{1}{2}$, $y>0$ $\frac{2\pi}{3}+2\pi n$; $M\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$.

44. а) $x>0$, $t\in\left(-\frac{\pi}{2}+2\pi n;\ \frac{\pi}{2}+2\pi n\right)$; б) $x<\frac{1}{2}$, $t\in\left(\frac{\pi}{3}+2\pi n;\ \frac{5\pi}{3}+2\pi n\right)$;

в) $x>\frac{1}{2}$, $t\in\left(-\frac{\pi}{3}+2\pi n;\ \frac{\pi}{3}+2\pi n\right)$; г) $x<0$, $t\in\left(\frac{\pi}{2}+2\pi n;\ \frac{3\pi}{2}+2\pi n\right)$.

45. а) $x<\frac{\sqrt{2}}{2}$, $t\in\left(\frac{\pi}{4}+2\pi k;\ \frac{7\pi}{4}+2\pi k\right)$; б) $x>-\frac{\sqrt{2}}{2}$, $t\in\left(-\frac{3\pi}{4}+2\pi k;\ \frac{3\pi}{4}+2\pi k\right)$;

в) $x>\frac{\sqrt{2}}{2}$, $t\in\left(-\frac{\pi}{4}+2\pi k;\ \frac{\pi}{4}+2\pi k\right)$; г) $x<-\frac{\sqrt{2}}{2}$, $t\in\left(\frac{3\pi}{4}+2\pi k;\ \frac{5\pi}{4}+2\pi k\right)$.

46. а) $x\le\frac{\sqrt{3}}{2}$, $t\in\left(\frac{\pi}{6}+2\pi k;\ \frac{11\pi}{6}+2\pi k\right)$; б) $x\le-\frac{\sqrt{3}}{2}$, $t\in\left(\frac{5\pi}{6}+2\pi k;\ \frac{7\pi}{6}+2\pi k\right)$;

в) $x\ge\frac{\sqrt{3}}{2}$, $t\in\left(-\frac{\pi}{6}+2\pi k;\ \frac{\pi}{6}+2\pi k\right)$; г) $x\ge-\frac{\sqrt{3}}{2}$, $t\in\left(-\frac{5\pi}{6}+2\pi k;\ \frac{5\pi}{6}+2\pi k\right)$.

47. а) $y>0$, $t\in(2\pi k;\ \pi+2\pi k)$; б) $y<\frac{1}{2}$, $t\in\left(\frac{7\pi}{6}+2\pi k;\ 6\frac{\pi}{3}+2\pi k\right)$;

в) $y>\frac{1}{2}$, $t\in\left(\frac{\pi}{6}+2\pi k;\ \frac{5\pi}{6}+2\pi k\right)$; г) $y<0$, $t\in(-\pi+2\pi k;\ 2\pi k)$.

48. а) $y<\frac{\sqrt{2}}{2}, -\frac{5\pi}{4}+2\pi k<t<\frac{\pi}{4}+2\pi k$; б) $y>-\frac{\sqrt{2}}{2}, -\frac{\pi}{4}+2\pi k<t<\frac{5\pi}{4}+2\pi k$;

в) $y > \frac{\sqrt{2}}{2}, \frac{\pi}{4} + 2\pi k < t < \frac{3\pi}{4} + 2\pi k$; **г)** $y < -\frac{\sqrt{2}}{2}, \frac{5\pi}{4} + 2\pi k < t < \frac{7\pi}{4} + 2\pi k$.

49. а) $y \le \frac{\sqrt{3}}{2}, -\frac{4\pi}{3} + 2\pi k < t < \frac{\pi}{3} + 2\pi k$; **б)** $y \le -\frac{\sqrt{3}}{2}, \frac{4\pi}{3} + 2\pi k < t < \frac{5\pi}{3} + 2\pi k$;

в) $y \ge \frac{\sqrt{3}}{2}, \frac{\pi}{3} + 2\pi k < t < \frac{2\pi}{3} + 2\pi k$; **г)** $y \ge -\frac{\sqrt{3}}{2}, -\frac{\pi}{3} + 2\pi k < t < \frac{4\pi}{3} + 2\pi k$.

§4. Синус и косинус

50. а) $\sin 0 = 0, \cos 0 = 1$; **б)** $\sin\frac{\pi}{2} = 1, \cos\frac{\pi}{2} = 0$;

в) $\sin\frac{3\pi}{2} = -1, \cos\frac{3\pi}{2} = 0$; **г)** $\sin\pi = 0, \cos\pi = -1$.

51. а) $\sin(-2\pi) = 0, \cos(-2\pi) = 1$; **б)** $\sin\left(-\frac{\pi}{2}\right) = -1, \cos\left(-\frac{\pi}{2}\right) = 0$;

в) $\sin\left(-\frac{3\pi}{2}\right) = 1, \cos\left(-\frac{3\pi}{2}\right) = 0$; **г)** $\sin(-\pi) = 0, \cos(-\pi) = -1$.

52. а) $\sin\frac{5\pi}{6} = \frac{1}{2}, \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; **б)** $\sin\frac{5\pi}{4} = \frac{\sqrt{2}}{2}, \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$;

в) $\sin\frac{7\pi}{6} = -\frac{1}{2}, \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$; **г)** $\sin\frac{9\pi}{4} = -\frac{\sqrt{2}}{2}, \cos\frac{9\pi}{4} = \frac{\sqrt{2}}{2}$.

53. а) $\sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$; **б)** $\sin\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(-\frac{4\pi}{3}\right) = -\frac{1}{2}$;

в) $\sin\left(-\frac{13\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(-\frac{13\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; **г)** $\sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}$.

54. а) $\sin\frac{13\pi}{6} = \frac{1}{2}, \cos\frac{13\pi}{6} = -\frac{\sqrt{3}}{2}$; **б)** $\sin\left(-\frac{8\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \cos\left(-\frac{8\pi}{3}\right) = -\frac{1}{2}$;

в) $\sin\frac{23\pi}{6} = -\frac{1}{2}, \cos\frac{23\pi}{6} = \frac{\sqrt{3}}{2}$; **г)** $\sin\left(-\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \cos\left(-\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

55. а) $\sin\left(-\frac{\pi}{4}\right) + \cos\frac{\pi}{3} + \cos\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} + 1 - \sqrt{2}}{2}$;

б) $\cos\frac{\pi}{6}\cos\frac{\pi}{4}\cos\frac{\pi}{3}\cos\frac{\pi}{2} = 0$; **в)** $\sin\left(-\frac{\pi}{2}\right)\cos(-\pi) + \sin\left(-\frac{3\pi}{2}\right) = -1 + 1 + 1 = 1$;

г) $\sin\frac{\pi}{6}\sin\frac{\pi}{4}\sin\frac{\pi}{3}\sin\frac{\pi}{2} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\cdot 1 = \frac{\sqrt{6}}{6}$.

56. а) $\sin\left(-\frac{3\pi}{4}\right)+\cos\left(-\frac{\pi}{4}\right)+\sin\frac{\pi}{4}\cos\frac{\pi}{2}+\cos 0\sin\frac{\pi}{2}=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+1=1$;

б) $\cos\frac{5\pi}{3}+\cos\frac{4\pi}{3}+\sin\frac{3\pi}{2}\sin\frac{5\pi}{8}\cos\frac{3\pi}{2}=\frac{1}{2}-\frac{1}{2}=0$.

57. а) $\cos 2t,\ t=\frac{\pi}{2},\ \cos\pi=-1$; б) $\sin\frac{t}{2},\ t=-\frac{\pi}{3},\ \sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$;

в) $\sin 2t,\ t=-\frac{\pi}{6},\ \sin\left(-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$; г) $\cos\frac{t}{2},\ t=-\frac{\pi}{3},\ \cos\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

58. а) $\sin^2 t-\cos^2 t,\ t=\frac{\pi}{3},\ \sin^2\frac{\pi}{3}-\cos^2\frac{\pi}{3}=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$;

б) $\sin^2 t+\cos^2 t,\ t=\frac{\pi}{4},\ \sin^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}=\frac{1}{2}+\frac{1}{2}=1$;

в) $\sin^2 t-\cos^2 t,\ t=\frac{\pi}{4},\ \sin^2\frac{\pi}{4}-\cos^2\frac{\pi}{4}=0$;

г) $\sin^2 t+\cos^2 t,\ t=\frac{\pi}{6},\ \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{6}=\frac{1}{4}+\frac{3}{4}=1$.

59. а) $f(t)=2\sin t$ $f_{max}=2$ $f_{min}=-2$. б) $f(t)=3+4\cos t$ $f_{max}=7$ $f_{min}=-1$.
в) $f(t)=-3\cos t$ $f_{max}=3$ $f_{min}=-3$. г) $f(t)=3-5\sin t$ $f_{max}=8$ $f_{min}=-2$.

60. а) $1-\sin^2 t=\cos^2 t$; б) $1-\cos^2 t=\sin^2 t$;
в) $1+\sin^2 t+\cos^2 t=2$; г) $\sin t-\sin t\cos^2 t=\sin t\left(1-\cos^2 t\right)=\sin^3 t$.

61. а) $(\sin t-\cos t)^2+2\sin t\cos t=1$;
б) $(\sin t+\cos t)^2-2\sin t\cos t=\sin^2 t+\cos^2 t+2\sin t\cos t-2\sin t\cos t=1$.

62. а) $\sin^2 t(1{,}5+2\pi k)+\cos^2 1{,}5+\cos\left(-\frac{\pi}{4}\right)+\sin\left(-\frac{\pi}{6}\right)=1+\frac{\sqrt{2}}{2}-\frac{1}{2}=\frac{\sqrt{2}+1}{2}$;

б) $\cos^2\left(\frac{\pi}{8}+4\pi\right)+\sin^2\left(\frac{\pi}{8}-44\pi\right)=1$.

63. а) $\cos t=\frac{\sqrt{2}}{2},\ t=\pm\frac{\pi}{4}+2\pi n$; б) $\sin t=-\frac{1}{2},\ t=(-1)^{n+1}\frac{\pi}{6}+\pi n$;

в) $\cos t=-\frac{1}{2},\ t=\pm\frac{2\pi}{3}+2\pi n$; г) $\sin t=\frac{\sqrt{2}}{2},\ t=(-1)^{n+1}\frac{\pi}{4}+\pi n$.

64. а) $\sin t=\frac{\sqrt{3}}{2},\ -t=(-1)^{k+1}\frac{\pi}{3}+\pi k$; б) $\cos=\sqrt{3}$, решения нет $|\cos t|\le 1$;

в) $\cos t=-\frac{\sqrt{3}}{2},\ t=\pm\frac{5\pi}{6}+2\pi n$; г) $\cos=-\frac{\pi}{3}$, решения нет $|\cos t|\le 1$.

65. а) 0; б) $\frac{\pi}{2}$; в) $-\frac{\pi}{6}$; г) $\frac{\pi}{3}$.

66. а) $\frac{\pi}{2}$; б) 0; в) $\frac{2\pi}{3}$; г) $\frac{5\pi}{6}$.

67. а) $\sin t+1=0$, $\sin t=-1$, $t=-\frac{\pi}{2}+2\pi n$;

б) $\cos t-1=0$, $\cos t=1$, $t=2\pi n$;

в) $1-2\sin t=0$, $\sin t=\frac{1}{2}$, $t=(-1)^n\frac{\pi}{6}+\pi n$;

г) $2\cos t+1=0$, $\cos t=-\frac{1}{2}$, $t=\pm\frac{2\pi}{3}+2\pi n$.

68. а) $\frac{\sin t-1}{\cos t}$, $\cos t\neq 0$, $t\neq -\frac{\pi}{2}+\pi n$; б) $\frac{\cos t+5}{2\sin t-1}$, $\sin t\neq\frac{1}{2}$, $t\neq(-1)^n\frac{\pi}{6}+\pi n$;

в) $\frac{\cos t}{1-\sin t}$, $\sin t\neq 1$, $t\neq\frac{\pi}{2}+2\pi n$; г) $\frac{\sin t}{1+\cos t}$, $\cos t\neq 0$, $t\neq\pm\pi+2\pi n$.

69. а) $\sin\frac{4\pi}{7}>0$; б) $\cos\left(-\frac{5\pi}{7}\right)<0$; в) $\sin\frac{9\pi}{8}<0$; г) $\sin\left(-\frac{3\pi}{8}\right)<0$.

70. а) $\sin(-2)<0$; б) $\cos 3<0$; в) $\sin 5<0$; г) $\cos(-6)>0$.

71. а) $\sin 10<0$; б) $\cos(-12)>0$; в) $\sin(-15)<0$; г) $\cos 8<0$.

72. а) $\sin 1\cos 2<0$; б) $\sin\frac{\pi}{7}\cos\left(-\frac{7\pi}{5}\right)<0$; в) $\cos 2\sin(-3)>0$; г) $\cos\left(-\frac{14\pi}{9}\right)\sin\frac{4\pi}{9}>0$.

73. а) $\frac{\sin^2 t}{1+\cos t}=\frac{1-\cos^2 t}{1+\cos t}=\frac{(1-\cos t)(1+\cos t)}{1+\cos t}=1-\cos t$;

б) $\sin^4 t+\cos^4 t+2\sin^2 t\cos^2 t=(\sin^2 t+\cos^2 t)^2=1$;

в) $\frac{\cos^2 t}{1+\sin t}+\sin t=1-\sin t+\sin t=1$;

г) $\cos^4 t+\cos^2 t\sin^2 t-\cos^2 t+1=\cos^4 t(\sin^2 t-1)+1=\cos^4 t-\cos^4 t+1=1$.

74. а) $10\sin t=\sqrt{75}$, $\sin t=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$, $t=(-1)^n\frac{\pi}{3}+\pi n$;

б) $\sqrt{8}\sin t+2=0$, $\sin t=-\frac{\sqrt{2}}{2}$, $t=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

в) $8\cos t-\sqrt{32}=0$, $\cos t=\frac{\sqrt{2}}{2}$, $t=\pm\frac{\pi}{4}+2\pi k$;

г) $8\cos t=-\sqrt{48}$, $\cos t=\frac{\sqrt{2}}{2}$, $t=\pm\frac{5\pi}{6}+2\pi k$.

75. а) $\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}=\sin\sqrt{2}$, $\sin t=\frac{\sqrt{2}}{2}$, $t=(-1)^k\frac{\pi}{4}+\pi k$;

б) $\sqrt{\frac{4}{3}}\cos t=\cos^2 1+\sin^2 1$, $\cos t=\frac{\sqrt{3}}{2}$, $t=\pm\frac{\pi}{6}+2\pi n$.

76. а) $|\sin t| = 1$, $\sin t = \pm 1$, $t = \frac{\pi}{2} + \pi n$;

б) $\sqrt{1 - \sin^2 t} = \frac{1}{2}$, $|\cos t| = \frac{1}{2}$, $\cos t = \pm\frac{1}{2}$, $t = \pm\frac{\pi}{3} + 2\pi k$;

в) $|\cos t| = 1$, $\cos t = \pm 1$, $t = \pi n$;

г) $\sqrt{1 - \cos^2 t} = \frac{\sqrt{2}}{2}$, $|\sin t| = \frac{\sqrt{2}}{2}$, $\sin t = \pm\frac{\sqrt{2}}{2}$, $t = \frac{\pi}{4} + \frac{\pi n}{2}$.

77. а) $\cos 1 + \cos(1 + \pi) + \sin t\left(-\frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{6}\right) = \cos 1 - \cos 1 - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$;

б) $\sin 2 + \sin(2 + \pi) + \cos^2\left(-\frac{\pi}{12}\right) + \sin^2\frac{\pi}{12} = \sin 2 - \sin 2 + 1 = 1$.

78. а) $\sqrt{\sin 10{,}2\pi}$ – Да; **б)** $\sqrt{\cos 1{,}3\pi}$ – Нет;

в) $\sqrt{\sin(-3{,}4)\pi}$ – Да; **г)** $\sqrt{\cos(-6{,}9\pi)}$ – Нет.

79. а) $\cos 2(2x - 1) < 0$, $x = \frac{1}{2}$

б) $\cos 3 \cos 5(x^2 - 4) < 0$, $-x^2 + 4 < 0$; $x \in (-\infty; -2)(2; +\infty)$.

80. а) $(\cos t - 5)(3x - 1) \geq 0$, $3x - 1 \leq 0$, $x \leq \frac{1}{3}$;

б) $(2 + \sin t)(9 - x^2) \geq 0$, $9 - x^2 \geq 0$, $x \in [-3; 3]$.

81. а) $a = \sin\frac{7\pi}{10}$, $b = \sin\frac{5\pi}{6}$, $a > b$; **б)** $a = \cos 2$, $b = \sin 2$, $a < b$;

в) $a = \cos\frac{\pi}{8}$, $b = \cos\frac{\pi}{3}$, $a > b$; **г)** $a = \sin 1$, $b = \cos 1$, $a > b$.

82. а) $\sin\frac{2\pi}{9} - \sin\frac{10\pi}{9} > 0$; **б)** $\sin 1 - \sin 1{,}1 < 0$;

в) $\sin\frac{15\pi}{8} - \cos\frac{\pi}{4} < 0$; **г)** $\cos 1 - \cos 0{,}9 < 0$.

83. а) $\sin\frac{4\pi}{3}, \sin\frac{7\pi}{7}, \sin\frac{\pi}{7}, \sin\frac{\pi}{5}, \sin\frac{2\pi}{3}$; **б)** $\cos\frac{5\pi}{6}, \cos\frac{5\pi}{4}, \cos\frac{\pi}{3}, \cos\frac{7\pi}{4}, \cos\frac{\pi}{8}$.

84. а) cos 4, sin 3, cos5, sin2; **б)** cos 3, cos 4, cos 7, cos 6;
в) sin 4, sin 6, sin 3, sin 7; **г)** cos 3, sin 5, sin 4, cos 2.

85. а) $\sqrt{\sin^2 1 + \sin^2 2 - 2\sin 1 \sin 2} + \sqrt{\frac{1}{4} - \sin 1 + \sin^2 1} + \sqrt{1 + \sin^2 2 - 2\sin 2} =$

$= |\sin 1 - \sin 2| + \left|\sin 1 - \frac{1}{2}\right| + |\sin 2 - 1| = \sin 2 - \sin 1 - \frac{1}{2} - \sin 2 + 1 = \frac{1}{2}$;

б) $\sqrt{\cos^2 6+\cos^2 7-2\cos 6\cos 7}+\sqrt{\frac{1}{4}-\cos 7+\cos^2 7}+\sqrt{1+\cos^2 6-2\cos 6}=$

$=\left|\cos 6-\cos 7\right|+\left|\cos 7-\frac{1}{2}\right|+\left|\cos 6-1\right|=1-\cos 6+\cos 6-\cos 7+\cos 7-\frac{1}{2}=\frac{1}{2}$.

86. а) $\sin(\pi-t)=\sin t$; $\sin(\pi-t)=-\sin(-t)=\sin t$;
б) $\sin(2\pi-t)=-\sin t$; $\sin(2\pi-t)=\sin(-t)=-\sin t$;
в) $\cos(\pi-t)=-\cos t$; $\cos(\pi-t)=-\cos(-t)=-\cos t$;
г) $\cos(2\pi-t)=\cos t$; $\cos(2\pi-t)=\cos(-t)=\cos t$.

87. а) $\sin t>0,\ t\in(2\pi k;\pi+2\pi k)$; б) $\sin t<\frac{\sqrt{3}}{2},\ t\in\left(-\frac{4\pi}{3}+2\pi k;\frac{\pi}{3}+2\pi k\right)$;

в) $\sin t<0,\ t\in(-\pi+2\pi k;2\pi k)$; г) $\sin t>\frac{\sqrt{3}}{2},\ t\in\left(\frac{\pi}{3}+2\pi k;\frac{2\pi}{3}+2\pi k\right)$.

88. а) $\cos t>0,\ t\in\left(-\frac{\pi}{3}+2\pi k;\frac{\pi}{2}+2\pi k\right)$; б) $\cos t<\frac{\sqrt{2}}{2},\ t\in\left(\frac{\pi}{4}+2\pi k;\frac{7\pi}{4}+2\pi k\right)$;

в) $\cos t<0,\ t\in\left(\frac{\pi}{2}+2\pi k;\frac{3\pi}{2}+2\pi k\right)$; г) $\cos t>\frac{\sqrt{2}}{2},\ t\in\left(-\frac{\pi}{4}+2\pi k;\frac{\pi}{4}+2\pi k\right)$.

89. а) $\sin t<-\frac{1}{2},\ t\in\left(\frac{7\pi}{6}+2\pi k;\frac{11\pi}{6}+2\pi k\right)$;

б) $\sin t>-\frac{\sqrt{2}}{2},\ t\in\left(-\frac{\pi}{4}+2\pi k;\frac{5\pi}{4}+2\pi k\right)$;

в) $\sin t>-\frac{1}{2},\ t\in\left(-\frac{\pi}{6}+2\pi k;\frac{7\pi}{6}+2\pi k\right)$;

г) $\sin t<-\frac{\sqrt{2}}{2},\ t\in\left(-\frac{5\pi}{4}+2\pi k;\frac{7\pi}{4}+2\pi k\right)$.

90. а) $\cos t>-\frac{\sqrt{3}}{2},\ t\in\left(-\frac{5\pi}{6}+2\pi k;\frac{5\pi}{6}+2\pi k\right)$;

б) $\cos t<-\frac{1}{2},\ t\in\left(\frac{2\pi}{3}+2\pi k;\frac{4\pi}{3}+2\pi k\right)$;

в) $\cos t<-\frac{\sqrt{3}}{2},\ t\in\left(\frac{5\pi}{6}+2\pi k;\frac{7\pi}{6}+2\pi k\right)$;

г) $\cos t>-\frac{1}{2},\ t\in\left(-\frac{2\pi}{3}+2\pi k;\frac{2\pi}{3}+2\pi k\right)$.

91. а) $\sin t\le\frac{1}{2},\ t\in\left[-\frac{7\pi}{6}+2\pi k;\frac{\pi}{6}+2\pi k\right]$; б) $\cos t\ge-\frac{\sqrt{2}}{2},\ t\in\left[-\frac{3\pi}{4}+2\pi k;\frac{3\pi}{4}+2\pi k\right]$;

в) $\sin t\ge-\frac{1}{2},\ t\in\left[-\frac{\pi}{6}+2\pi k;\frac{7\pi}{6}+2\pi k\right]$; г) $\cos t\le\frac{\sqrt{2}}{2},\ t\in\left[\frac{\pi}{4}+2\pi k;\frac{7\pi}{4}+2\pi k\right]$.

§5. Тангенс и котангенс

92. а) $\mathrm{tg}\frac{5\pi}{4}=1$; **б)** $\mathrm{ctg}\frac{4\pi}{3}=\frac{\sqrt{3}}{3}$; **в)** $\mathrm{tg}\frac{5\pi}{6}=-\frac{\sqrt{3}}{3}$; **г)** $\mathrm{ctg}\frac{7\pi}{4}=-1$.

93. а) $\mathrm{tg}\left(-\frac{5\pi}{4}\right)=-1$; **б)** $\mathrm{ctg}\left(-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{3}$; **в)** $\mathrm{tg}\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{3}$; **г)** $\mathrm{ctg}\left(-\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{3}$

94. а) $\mathrm{tg}\frac{\pi}{4}+\mathrm{ctg}\frac{5\pi}{4}=1+1=2$; **б)** $\mathrm{ctg}\frac{\pi}{3}-\mathrm{tg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}-\frac{\sqrt{3}}{3}=0$;

в) $\mathrm{tg}\frac{\pi}{6}\cdot\mathrm{ctg}\frac{\pi}{6}=1$; **г)** $\mathrm{tg}\frac{9\pi}{4}+\mathrm{ctg}\frac{\pi}{4}=1+1=2$.

95. а) $\mathrm{tg}\frac{\pi}{4}\cdot\sin\frac{\pi}{3}\cdot\mathrm{ctg}\frac{\pi}{6}=1\cdot\frac{\sqrt{3}}{2}\cdot\sqrt{3}=\frac{3}{2}$;

б) $2\sin\frac{\pi}{3}\cos\frac{\pi}{6}-\frac{1}{2}\mathrm{tg}\frac{\pi}{3}=2\cdot\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}-\frac{1}{2}\sqrt{3}=\frac{3-\sqrt{3}}{2}$;

в) $2\sin\pi+3\cos\pi+\mathrm{ctg}\frac{\pi}{2}=0-3+0=-3$;

г) $\mathrm{tg}\,0+8\cos\frac{3\pi}{2}-6\sin\frac{\pi}{3}=0+0-6\frac{\sqrt{3}}{2}=3\sqrt{3}$.

96. а) $\mathrm{tg}\frac{\pi}{5}\cdot\mathrm{ctg}\frac{\pi}{5}=1$; **б)** $\mathrm{tg}\,2{,}3\cdot\mathrm{ctg}\,2{,}3=3$; **в)** $\mathrm{tg}\frac{\pi}{7}\cdot\mathrm{ctg}\frac{\pi}{7}=1$; **г)** $7\,\mathrm{ctg}\frac{\pi}{12}\cdot\mathrm{ctg}\frac{\pi}{12}=7$.

97. а) $\mathrm{tg}\,2{,}5\cdot\mathrm{ctg}\,2{,}5+\cos^2\pi-\sin^2\frac{\pi}{8}-\cos^2\frac{\pi}{8}=1+1-1=1$;

б) $\sin^2\frac{3\pi}{7}-2\,\mathrm{tg}\,1\cdot\mathrm{ctg}\,1+\cos^2\left(-\frac{3\pi}{7}\right)+\sin^2\frac{5\pi}{2}=1-2+1=0$.

98. а) $\mathrm{tg}\frac{6\pi}{7}<0$; **б)** $\mathrm{ctg}\frac{10\pi}{7}>0$; **в)** $\mathrm{tg}\frac{8\pi}{7}>0$; **г)** $\mathrm{ctg}\frac{11\pi}{7}<0$.

99. а) $\sin t\cdot\mathrm{ctg}\,t=\cos t,\ \sin t\cdot\frac{\cos t}{\sin t}=\cos t$; **б)** $\frac{\sin t}{\mathrm{tg}\,t}=\cos t,\ \sin t\cdot\frac{\cos t}{\sin t}=\cos t$;

в) $\cos t\cdot\mathrm{tg}\,t=\sin t$; **г)** $\frac{\cos t}{\mathrm{ctg}\,t}=\sin t,\ \cos t\cdot\frac{\sin t}{\cos t}=\sin t$.

100. а) $\sin t\cdot\cos t\cdot\mathrm{tg}\,t=\sin t\cdot\cos t\cdot\frac{\sin t}{\cos t}=\sin^2 t$;

б) $\sin t\cdot\cos t\cdot\mathrm{ctg}\,t-1=\sin t\cdot\cos t\cdot\frac{\cos t}{\sin t}-1=\cos^2 t-1=-\sin^2 t$;

в) $\sin^2 t-\mathrm{tg}\,t\cdot\mathrm{ctg}\,t=\sin^2 t-1=-\cos^2 t$; **г)** $\frac{1-\cos^2 t}{1-\sin^2 t}=\frac{\sin^2 t}{\cos^2 t}=tg^2 t$.

101. $\dfrac{\sin\frac{\pi}{4}-\cos\pi-\mathrm{tg}\frac{\pi}{4}}{2\sin\frac{\pi}{6}-\sin\frac{3\pi}{2}}=\frac{\sqrt{2}}{4}$; $\dfrac{\frac{\sqrt{2}}{2}+1-1}{1+1}=\frac{\sqrt{2}}{4}$.

102. а) $\cos\frac{5\pi}{9}-\text{tg}\frac{25\pi}{18}<0$; б) $\text{tg}1-\cos2>0$;

в) $\sin\frac{7\pi}{10}-\text{ctg}\frac{3\pi}{5}>0$; г) $\sin2-\text{ctg}5{,}5>0$.

103. а) $\sin1\cdot\cos2\cdot\text{tg}3\cdot\text{ctg}4>0$; б) $\sin(-5)\cos(-6)\text{tg}(-7)\text{ctg}(-8)<0$.

104. а) $1+\text{tg}^2t=\cos^{-2}t,\ 1+\frac{\sin^2t}{\cos^2t}=\frac{1}{\cos^2t}=\cos^{-2}t$;

б) $1+\text{ctg}^2t=\sin^{-2}t,\ 1+\frac{\cos^2t}{\sin^2t}=\frac{1}{\sin^2t}=\sin^{-2}t$;

в) $\sin^2t\left(1+\text{ctg}^2t\right)=1,\ \sin^2t\cdot\sin^{-2}t=1$; г) $\cos^2t\left(1+\text{tg}^2t\right)=1,\ \cos^2t\cdot\cos^{-2}t=1$

105. а) $\text{tg}(\pi-t)=-\text{tg}t$, $\text{tg}(\pi-t)=\text{tg}(-t)=-\text{tg}t$;
б) $\text{tg}(2\pi+t)=\text{tg}t$, $\text{tg}(2\pi+t)=\text{tg}(\pi+t)=\text{tg}t$;
в) $\text{ctg}(\pi-t)=-\text{ctg}t$, $\text{ctg}(\pi-t)=\text{ctg}(-t)=-\text{ctg}t$;
г) $\text{ctg}(2\pi+t)=\text{ctg}t$, $\text{ctg}(2\pi+t)=\text{ctg}(\pi+t)=\text{ctg}t$.

106. а) $\cos^2t\cdot\text{tg}^2t-\sin^2t\cdot\cos^2t=\sin^2t\left(1-\cos^2t\right)=\sin^4t$;

б) $1-\cos^2t+tg^2t\cdot\cos^2t=\sin^2t+\sin^2t=2\sin^2t$;

в) $\left(1-\sin^2t\right)\left(\text{tg}^2t+1\right)=\cos^2t\frac{1}{\cos^2t}=1$; г) $\left(1-\cos^2t\right)\left(\text{ctg}^2t+1\right)=\sin^2t\frac{1}{\sin^2t}=1$.

107. а) $$\frac{\cos^2t-\sin^2t}{\cos t\sin t}=\frac{\dfrac{\cos^2t}{\cos^2t}-\dfrac{\sin^2t}{\cos^2t}}{\dfrac{\cos t\sin t}{\cos^2t}}=\frac{1-\text{tg}^2t}{\text{tg}t};$$

б) $$\frac{\cos^2t-\sin^2t}{\sin t\cos t}=\frac{\dfrac{\cos^2t}{\sin^2t}-\dfrac{\sin^2t}{\sin^2t}}{\dfrac{\cos t\sin t}{\sin^2t}}=\frac{\text{ctg}^2t-1}{\text{ctg}t}.$$

108. а) cos 1, sin 1, 1, tg 1. б) ctg 2, cos 2, sin 2, 2.

109. а) $\text{ctg}5(x-1)\geq0,\ -x+1\geq0,\ x\leq1$;

б) $\frac{\text{tg}7\cdot\cos1}{\sin1}\left(2x^2-72\right)>0,\ 2x^2-72<0,\ x^2>36,\ x\in(-6;6)$;

в) $(\text{tg}2\cdot\sin5)(7-5x)\leq0,\ 7-5x\leq0,\ x\geq\frac{7}{5}$;

г) $\text{tg}1\cdot\text{ctg}2\cdot\text{tg}3\cdot\text{ctg}4\cdot\left(x^2+2\right)>0,\ x^2+2>0,\ x\in R$.

§6. Тригонометрические функции числового аргумента

110. а) $1-\sin^2t=\cos^2t$; б) $\cos^3t-1=-\sin^2t$;
в) $1-\cos^2t=\sin^2t$; г) $\sin^2t-1=-\cos^2t$.

111. а) $(1-\sin t)(1+\sin t)=1-\sin^2 t=\cos^2 t$; **б)** $\cos^2 t+1-\sin^2 t=\cos^2 t+\cos^2 t=2\cos^2 t$;

в) $(1-\cos t)(1+\cos t)=1-\cos^2 t=\sin^2 t$; **г)** $\sin^2 t+2\cos^2 t-1=1+\cos^2 t-1=\cos^2 t$.

112. а) $\dfrac{1}{\cos^2 t}-1=\dfrac{1-\cos^2 t}{\cos^2 t}=\operatorname{tg}^2 t$; **б)** $\dfrac{1-\sin^2 t}{\cos^2 t}=1$;

в) $1-\dfrac{1}{\sin^2 t}=\dfrac{\sin^2 t-1}{\sin^2 t}=-\operatorname{ctg}^2 t$; **г)** $\dfrac{1-\cos^2 t}{1-\sin^2 t}=\dfrac{\sin^2 t}{\cos^2 t}=\operatorname{tg}^2 t$.

113. а) $\dfrac{(\sin t+\cos t)^2}{1+2\sin t\cos t}=\dfrac{1+2\sin t\cos t}{1+2\sin t\cos t}=1$; **б)** $\dfrac{1-2\sin t\cos t}{(\cos t-\sin t)^2}=\dfrac{1-2\sin t\cos t}{1-2\sin t\cos t}=1$.

114. а) $\dfrac{\cos^2 t}{1-\sin t}-\sin t=1,\ \dfrac{1-\sin^2 t}{1-\sin t}-\sin t=\dfrac{1-\sin^2 t-\sin t+\sin^2 t}{1-\sin t}=1$;

б) $\dfrac{\sin^2 t}{1+\cos t}+\cos t=1,\ \dfrac{1-\cos^2 t}{1+\cos^2 t}+\cos t=\dfrac{\sin^2 t+\cos t+\cos^2 t}{1+\cos t}=1$.

115. а) $(\sin t+\cos t)^2-2\sin t\cos t=1+2\sin t\cos t-2\sin t\cos t=1$;

б) $\dfrac{2-\sin^2 t-\cos^2 t}{3\sin^2 t+3\cos^2 t}=\dfrac{2-1}{3}=\dfrac{1}{3}$; **в)** $\sin^4 t+\cos^4 t+2\sin^2 t\cos^2 t=(\sin^2 t+\cos^2 t)^2=1$;

г) $\dfrac{\sin^4 t-\cos^4 t}{\sin^2 t-\cos^2 t}=\dfrac{(\sin^2 t-\cos^2 t)(\sin^2 t+\cos^2 t)}{\sin^2 t-\cos^2 t}=\sin^2 t+\cos^2 t=1$.

116. а) $\sin t=\dfrac{4}{5},\ \dfrac{\pi}{2}<t<\pi,\ \cos t=-\dfrac{3}{5},\ \operatorname{tg} t=-\dfrac{4}{3},\ \operatorname{ctg} t=-\dfrac{3}{4}$;

б) $\sin t=\dfrac{5}{13},\ 0<t<\dfrac{\pi}{2},\ \sqrt{1-\dfrac{25}{169}}=\dfrac{12}{13},\ \operatorname{tg} t=\dfrac{5}{12},\ \operatorname{ctg} t=\dfrac{12}{5}$;

в) $\sin t=-0{,}6,\ -\dfrac{\pi}{2}<t<0,\ \cos t=\dfrac{4}{5},\ \operatorname{tg} t=-\dfrac{3}{4},\ \operatorname{ctg} t=-\dfrac{4}{3}$;

г) $\sin t=-0{,}28,\ \pi<t<\dfrac{3\pi}{2},\ \cos t=-\sqrt{1-\dfrac{43}{625}}=-\dfrac{24}{25},\ \operatorname{tg} t=\dfrac{7}{24},\ \operatorname{ctg} t=\dfrac{24}{7}$.

117. а) $\cos t=0{,}8,\ 0<t<\dfrac{\pi}{2},\ \sin t=\dfrac{3}{5},\ \operatorname{tg} t=\dfrac{3}{4},\ \operatorname{ctg} t=\dfrac{4}{3}$;

б) $\cos t=-\dfrac{5}{13},\ \dfrac{\pi}{2}<t<\pi,\ \sin t=\dfrac{12}{13},\ \operatorname{tg} t=-\dfrac{12}{5},\ \operatorname{ctg} t=-\dfrac{5}{12}$;

в) $\cos t=0{,}6,\ \dfrac{3\pi}{2}<t<2\pi,\ \sin t=-\dfrac{4}{5},\ \operatorname{tg} t=-\dfrac{4}{3},\ \operatorname{ctg} t=-\dfrac{3}{4}$;

г) $\cos t=-\dfrac{24}{25},\ \pi<t<\dfrac{3\pi}{2},\ \sin t=-\dfrac{7}{25},\ \operatorname{tg} t=\dfrac{7}{24},\ \operatorname{ctg} t=\dfrac{24}{7}$.

118. а) $\operatorname{tg} t=\dfrac{3}{4},\ 0<t<\dfrac{\pi}{2},\ \operatorname{ctg} t=\dfrac{4}{3},\ \sin t=\dfrac{3}{5},\ \cos t=\dfrac{4}{5}$;

б) $\operatorname{tg} t=2{,}4,\ \pi<t<\dfrac{3\pi}{2},\ \operatorname{ctg} t=\dfrac{5}{12},\ \cos t=-\dfrac{5}{13},\ \sin t=-\dfrac{12}{13}$;

в) $\operatorname{tg} t = -\frac{3}{4},\ \frac{\pi}{2} < t < \pi,\ \operatorname{ctg} t = -\frac{4}{3},\ \sin t = \frac{3}{5},\ \cos t = -\frac{4}{5}$;

г) $\operatorname{tg} t = -\frac{1}{3},\ \frac{3\pi}{2} < t < 2\pi,\ \operatorname{ctg} t = -3,\ \frac{\cos t}{\sqrt{1-\cos^2 t}} = -3,\ \cos^2 t = 9 - 9\cos^2 t$,

$\cos t = \frac{3}{\sqrt{10}},\ \sin t = -\sqrt{1-\frac{9}{10}} = -\frac{1}{\sqrt{10}}$.

119. а) $\operatorname{ctg} t = \frac{12}{5},\ \pi < t < \frac{3\pi}{2},\ \operatorname{tg} t = \frac{5}{12},\ \sin t = -\frac{5}{13},\ \cos t = -\frac{12}{13}$;

б) $\operatorname{ctg} t = \frac{7}{24},\ 0 < t < \frac{\pi}{2},\ \operatorname{tg} t = \frac{24}{7},\ \sin t = \frac{24}{25},\ \cos t = \frac{7}{25}$;

в) $\operatorname{ctg} t = -\frac{5}{12},\ \frac{3\pi}{2} < t < 2\pi,\ \operatorname{tg} t = -\frac{12}{5},\ \cos t = \frac{5}{13},\ \sin t = -\frac{12}{13}$;

г) $\operatorname{ctg} t = -\frac{8}{15},\ \frac{\pi}{2} < t < \pi,\ \operatorname{tg} t = -\frac{15}{8},\ \frac{\cos t}{\sqrt{1-\cos^2 t}} = -\frac{8}{15},\ \cos^2 t = \frac{64}{225} - \frac{64}{225}\cos^2$

$\cos t = -\frac{8}{17},\ \sin t = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$.

120. а) $f(x) = 1 - (\cos^2 t - \sin^2 t) = 2\sin^2 t,\ f_{\max} = 2,\ f_{\min} = 0$;

б) $f(t) = 1 - \sin t \cdot \cos t \cdot tg\, t = 1 - \sin t \cdot \cos t \cdot \frac{\sin t}{\cos t} = \cos^2 t$, $f_{\max} = 1,\ f_{\min} = 0$.

в) $f(t) = \cos^2 t \cdot \operatorname{tg}^2 t + 5\cos^2 t - 1 = \cos^2 t \cdot \frac{\sin^2 t}{\cos^2 t} + 5\cos^2 t - 1 = \sin^2 t + 5\cos^2 t - 1$

$= 4\cos^2 t$, $f_{\max} = 4,\ f_{\min} = 0$;

г) $f(t) = \sin t + 3\sin^2 t + 3\cos^2 t = \sin t + 3,\ f_{\max} = 4,\ f_{\min} = 2$.

121. а) $\operatorname{ctg} t - \frac{\cos t - 1}{\sin t} = \frac{\cos t - \cos t + 1}{\sin t} = \frac{1}{\sin t}$;

б) $\operatorname{ctg}^2 t - \left(\frac{1}{\sin^2 t} - 1\right) = \frac{\cos^2 t - 1 + \sin^2 t}{\sin^2 t} = 0$;

в) $\cos^2 t - (\operatorname{ctg}^2 t + 1)\sin^2 t = \cos^2 t - \cos^2 t - \sin^2 t = -\sin^2 t$;

г) $\frac{\sin^2 t - 1}{\cos^2 t - 1} + \operatorname{tg} t \cdot \operatorname{ctg} t = \operatorname{ctg}^2 t + 1 = \frac{\sin^2 t + \cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$.

122. а) $\frac{\sin t}{1+\cos t} + \frac{\sin t}{1-\cos t} = \frac{\sin t - \sin t\cos t + \sin t + \cos t \sin t}{1-\cos^2 t} = \frac{2\sin t}{\sin^2 t} = \frac{2}{\sin t}$;

б) $\operatorname{ctg}^2 t(\cos^2 t - 1) + 1 = -\cos^2 t + 1 = \sin^2 t$;

в) $\frac{\cos t}{1+\sin t} + \frac{\cos t}{1-\sin t} = \frac{\cos t - \sin t\cos t + \cos t + \sin t\cos t}{1-\sin^2 t} = \frac{2\cos t}{\cos^2 t} = \frac{2}{\cos t}$;

г) $\frac{\operatorname{tg} t + 1}{1 + \operatorname{ctg} t} = \frac{\frac{\sin t + \cos t}{\cos t}}{\frac{\sin t + \cos t}{\sin t}} = \operatorname{tg} t$.

123. а) $(3\sin t+4\cos t)^2+(4\sin t-3\cos t)^2=9\sin^2 t+16\cos^2 t+$
$+24\sin t\cos t+16\sin^2 t+9\cos^2 t-24\sin t\cos t=25$;

б) $(\operatorname{tg}t+\operatorname{ctg}t)^2-(\operatorname{tg}t-\operatorname{ctg}t)^2=\operatorname{tg}^2 t+\operatorname{ctg}^2 t+2-\operatorname{tg}^2 t-\operatorname{ctg}^2 t+2=4$;

в) $\sin t\cdot\cos t(\operatorname{tg}t+\operatorname{ctg}t)=\sin t\cdot\cos t\dfrac{\sin^2 t+\cos^2 t}{\sin t\cos t}=1$;

г) $\sin^2 t\cdot\cos^2 t\left(\operatorname{tg}^2 t+\operatorname{ctg}^2 t+2\right)=\sin^2 t\cdot\cos^2 t(\operatorname{tg}t+\operatorname{ctg}t)^2=$

$=\sin^2 t\cdot\cos^2 t\dfrac{1}{\sin^2 t\cos^2 t}=1$.

124. а) $\dfrac{1-\sin^2 t}{1-\cos^2 t}+\operatorname{tg}t\cdot\operatorname{ctg}t=\dfrac{\cos^2 t}{\sin^2 t}+1=\operatorname{ctg}^2 t+1=\dfrac{1}{\sin^2 t}$;

б) $$\frac{\cos^2 t-\operatorname{ctg}^2 t}{\sin^2 t-\operatorname{tg}^2 t}=\frac{\dfrac{\cos^2 t\cdot\sin^2 t-\cos^2 t}{\sin^2 t}}{\dfrac{\sin^2 t\cdot\cos^2 t-\sin^2 t}{\cos^2 t}}=\frac{\cos^2 t\left(\sin^2 t-1\right)\cos^2 t}{\sin^2 t\sin^2 t\left(\cos^2 t-1\right)}=\frac{-\cos^6 t}{-\sin^6 t}=\operatorname{ctg}^6 t.$$

125. а) $\dfrac{\operatorname{tg}t}{\operatorname{tg}t+\operatorname{ctg}t}=\sin^2 t$, $\dfrac{\operatorname{tg}t}{\dfrac{1}{\sin t\cos t}}=\dfrac{\sin t}{\cos t}=\sin t\cdot\cos t=\sin^2 t$;

б) $\dfrac{1+\operatorname{tg}t}{1+\operatorname{ctg}t}=\operatorname{tg}t$, $\dfrac{\dfrac{\sin t+\cos t}{\cos t}}{\dfrac{\sin t+\cos t}{\sin t}}=\dfrac{\sin t}{\cos t}=\operatorname{tg}t$;

в) $\dfrac{\operatorname{ctg}t}{\operatorname{tg}t+\operatorname{ctg}t}=\cos^2 t$, $\dfrac{\operatorname{ctg}t}{\dfrac{1}{\sin t\cos t}}=\operatorname{ctg}t\cdot\sin t\cdot\cos t=\cos^2 t$;

г) $\dfrac{1-\operatorname{ctg}t}{1-\operatorname{tg}t}=-\operatorname{ctg}t$, $\dfrac{\dfrac{\sin t-\cos t}{\sin t}}{\dfrac{-\sin t+\cos t}{\cos t}}=-\dfrac{\cos t}{\sin t}=-\operatorname{ctg}t$.

126. а) $1+\sin t=\dfrac{\cos t+\operatorname{ctg}t}{\operatorname{ctg}t}$, $\dfrac{\dfrac{\cos t\sin t+\cos t}{\sin t}}{\dfrac{\cos t}{\sin t}}=\dfrac{\cos t\sin t+\cos t}{\cos t}=\sin t+1$;

б) $\dfrac{\sin t+\operatorname{tg}t}{\operatorname{tg}t}=1+\cos t$, $\sin t\dfrac{\cos t}{\sin t}+1=\cos t+1$;

в) $\dfrac{1-\sin t}{\cos t}=\dfrac{\cos t}{1+\sin t}$, $\dfrac{1-\sin^2 t}{\cos t(1+\sin t)}=\dfrac{\cos t}{1+\sin t}$;

г) $\dfrac{\sin t}{1-\cos t}=\dfrac{1+\cos t}{\sin t}$, $\dfrac{\sin t(1+\cos t)}{1-\cos^2 t}=\dfrac{1+\cos t}{\sin t}$.

127. а) $\dfrac{(\sin t+\cos t)^2-1}{\operatorname{ctg}t-\sin t\cos t}=2\operatorname{tg}^2 t$, $\dfrac{2\sin t\cos t}{\cos t-\sin^2 t\cos t}\sin t=\dfrac{2\sin^2 t\cos t}{\cos t\left(1-\sin^2 t\right)}=2\operatorname{tg}^2 t$;

б) $\sin^3 t(1+\operatorname{ctg} t)+\cos^3 t(1+\operatorname{tg} t)=\sin t+\cos t$,

$$\sin^3 t\,\frac{\sin t+\cos t}{\sin t}+\cos^3 t\,\frac{\sin t+\cos t}{\cos t}=(\sin t+\cos t)(\sin^2 t+\cos^2 t)=\sin t+\cos t;$$

в) $\dfrac{(\sin t+\cos t)^2}{\operatorname{tg} t-\sin t\cos 2t}=2\operatorname{ctg}^2 t=\dfrac{2\sin t\cos t}{\sin t-\sin t\cos^2 t}\cos t=\dfrac{2\sin t\cos^2 t}{\sin^3 t}=2\operatorname{ctg}^2 t$;

г) $\dfrac{1-4\sin^2 t\cos^2 t}{(\sin t+\cos t)^2}+2\sin t\cdot\cos t=1$,

$$\frac{(1-2\sin t\cos t)(1+2\sin t\cos t)}{1+2\sin t\cos t}+2\sin t\cos t=1-2\sin t\cos t+2\sin t\cos t=1.$$

128. а) $\sin(4\pi+t)=\dfrac{3}{5}$, $0<t<\dfrac{\pi}{2}$, $\cos t=\dfrac{4}{5}$, $\operatorname{tg} t=\dfrac{3}{4}$, $\operatorname{tg}(-t)=-\dfrac{3}{4}$, $\operatorname{tg}(\pi-t)=-\dfrac{3}{4}$;

б) $\cos(2\pi+t)=\dfrac{12}{13}$, $\dfrac{3\pi}{2}<t<2\pi$, $\sin t=-\dfrac{5}{13}$, $\operatorname{ctg} t=-\dfrac{12}{5}$, $\operatorname{ctg}(-t)=\dfrac{12}{5}$, $\operatorname{ctg}(\pi-t)=\dfrac{12}{5}$.

129. а) $\cos t=-\dfrac{5}{13}$, $8{,}5\pi<t<9\pi$, $\sin t=\dfrac{12}{13}$, $\sin(-t)=-\dfrac{12}{13}$;

б) $\sin t=\dfrac{4}{5}$, $\dfrac{9\pi}{2}<t<5\pi$, $\cos t=-\dfrac{3}{5}$, $\cos(-t)=-\dfrac{3}{5}$,

$\sin(-t)=-\dfrac{4}{5}$, $\cos(-t)+\sin(-t)=-\dfrac{7}{5}$.

130. а) $\sin t+\cos t=0{,}8$, $(\sin t+\cos t)^2=\dfrac{16}{25}$, $2\cos t\cdot\sin t=-\dfrac{9}{25}$, $\cos t\cdot\sin t=-\dfrac{9}{50}$;

б) $\sin t-\cos t=\dfrac{1}{3}$, $(\cos t-\sin t)^2=\dfrac{1}{9}$, $-2\sin t\cdot\cos t=-\dfrac{8}{9}$, $9\sin t\cdot\cos t=4$.

131. $\operatorname{tg} t+\operatorname{ctg} t=2{,}3$, $(\operatorname{tg} t+\operatorname{ctg} t)^2=\operatorname{tg}^2 t+2\operatorname{tg} t\cdot\operatorname{ctg} t+\operatorname{ctg}^2 t=5{,}29$, $\operatorname{tg}^2 t+\operatorname{ctg}^2 t=3{,}29$.

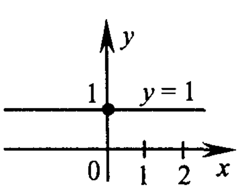
132. $\sin t\cos t=-\dfrac{1}{2}$, $\sin^4 t+\cos^4 t=1-2\sin^2 t\cdot\cos^2 t=1-\dfrac{1}{2}=\dfrac{1}{2}$.

133. $\operatorname{tg} t-\dfrac{1}{\operatorname{tg} t}=-\dfrac{7}{12}$, $0<t<\dfrac{\pi}{2}$, $12\operatorname{tg}^2 t+7\operatorname{tg} t-12=0$,

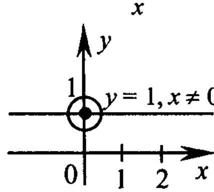
$\operatorname{tg} t=\dfrac{-7\pm\sqrt{49-4\cdot12(-12)}}{24}=\dfrac{-7\pm25}{24}$, $\operatorname{tg} t=-\dfrac{4}{3}$ не подходит, т.к. $0<t<\dfrac{\pi}{2}$,

$\operatorname{tg} t=\dfrac{3}{4}\Rightarrow\cos t=\dfrac{4}{5}$, $\quad\sin t=\dfrac{3}{5}\Rightarrow\sin t+\cos t=\dfrac{7}{5}$.

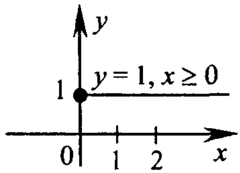
134. а) $y=\cos^2 t+\sin^2 t=1$;

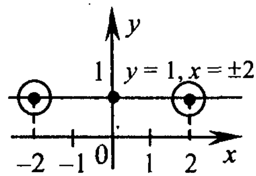


б) $y=\cos^2\dfrac{1}{x}+\sin^2\dfrac{1}{x}=1\ (x\neq0)$;



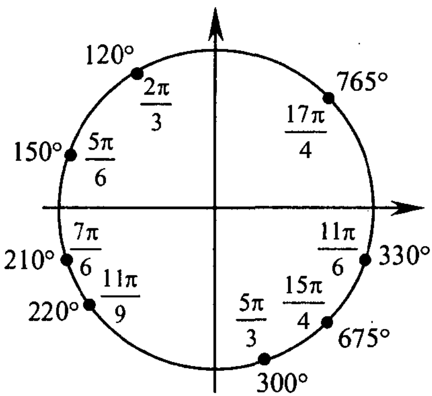
в) $y=\sin^2\sqrt{x}+\cos^2\sqrt{x}=1\ (x\ge 0)$; **г)** $y=\sin^2\dfrac{1}{x^2-4}+\cos^2\dfrac{1}{x^2-4}=1\ (x\ne\pm 2)$.





§7. Тригонометрические функции углового аргумента

135 – 138 см. рис.



135. а) $120^\circ=\dfrac{2\pi}{3}$; **б)** $220^\circ=\dfrac{11\pi}{9}$; **в)** $300^\circ=\dfrac{5\pi}{3}$; **г)** $765^\circ=\dfrac{17\pi}{4}$.

136. а) $210^\circ=\dfrac{7\pi}{6}$; **б)** $150^\circ=\dfrac{5\pi}{6}$; **в)** $330^\circ=\dfrac{11\pi}{6}$; **г)** $675^\circ=\dfrac{15\pi}{4}$.

137. а) $\dfrac{3\pi}{4}=135^\circ$; **б)** $\dfrac{11\pi}{3}=660^\circ$; **в)** $\dfrac{6\pi}{5}=216^\circ$; **г)** $\dfrac{46\pi}{9}=920^\circ$.

138. а) $\dfrac{5\pi}{8}=112{,}5^\circ$; **б)** $\dfrac{7\pi}{12}=105^\circ$; **в)** $\dfrac{11\pi}{12}=165^\circ$; **г)** $\dfrac{47\pi}{9}=940^\circ$.

139. а) $\sin 90^\circ=1$, $\cos 90^\circ=0$, $\operatorname{tg} 90^\circ=$ не сущ., $\operatorname{ctg} 90^\circ=0$;
б) $\sin 180^\circ=0$, $\cos 180^\circ=-1$, $\operatorname{tg} 180^\circ=0$, $\operatorname{ctg} 180^\circ=$ не сущ.;
в) $\sin 270^\circ=-1$, $\cos 270^\circ=0$, $\operatorname{tg} 270^\circ=$ не сущ., $\operatorname{ctg} 270^\circ=0$;
г) $\sin 360^\circ=0$, $\cos 360^\circ=1$, $\operatorname{tg} 360^\circ=0$, $\operatorname{ctg} 360^\circ=$ не сущ..

140. а) $\sin 30^\circ=\dfrac{1}{2}$, $\cos 30^\circ=\dfrac{\sqrt{3}}{2}$, $\operatorname{tg} 30^\circ=\dfrac{\sqrt{3}}{3}$, $\operatorname{ctg} 30^\circ=\sqrt{3}$;

б) $\sin 150^\circ=\dfrac{1}{2}$, $\cos 150^\circ=-\dfrac{\sqrt{3}}{2}$, $\operatorname{tg} 150^\circ=-\dfrac{\sqrt{3}}{3}$, $\operatorname{ctg} 150^\circ=-\sqrt{3}$;

в) $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\mathrm{tg}\, 210° = \frac{\sqrt{3}}{3}$, $\mathrm{ctg}\, 210° = \sqrt{3}$;

г) $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\mathrm{tg}\, 240° = \sqrt{3}$, $\mathrm{ctg}\, 240° = \frac{\sqrt{3}}{3}$.

141. sin 160°, sin 40°, sin 120°, sin 80°.

142. cos 160°, cos 120°, cos 80°, cos 40°.

143. sin 210°, sin 20°, sin 400°, sin 110°.

144. а) $\mathrm{tg}\,\alpha = \frac{x}{2}$, $x = 2\,\mathrm{tg}\,\alpha$; **б)** $\cos\alpha = \frac{x}{4}$, $x = 4\cos\alpha$;

в) $\cos\alpha = \frac{3}{x}$, $x = \frac{3}{\cos\alpha}$; **г)** $\mathrm{ctg}\,\alpha = x$.

145. а) $\sin 30° = \frac{2}{x}$, $x = \frac{2}{\frac{1}{2}} = 4$; **б)** $x = \frac{\sqrt{2}}{2}$;

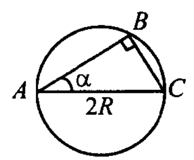
в) $\frac{\sqrt{3}}{2} = \frac{2}{x}$, $x = \frac{4}{\sqrt{3}}$; **г)** $\frac{x}{2} = \cos 60° = \frac{1}{2}$, $x = 1$.

146. а) $c = 12$, $\alpha = 60°$, $a = 6$, $b = 6\sqrt{3}$, $S = 18\sqrt{3}$, $R = 6$;

б) $c = 6$, $\alpha = 45°$, $a = b = 3\sqrt{2}$, $S = 9$, $R = 3$;

в) $c = 4$, $\alpha = 30°$, $a = 2$, $b = 2\sqrt{3}$, $S = 2\sqrt{3}$, $R = 2$;

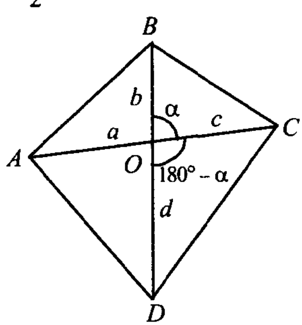
г) $c = 60$, $\alpha = 60°$, $a = 30$, $b = 30\sqrt{3}$, $S = 450\sqrt{3}$, $R = 30$.



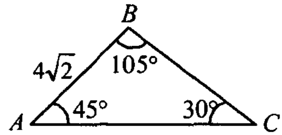
147. $\cos\alpha = \frac{AB}{AC}$, $AB = AC \cdot \cos\alpha = 2R\cos\alpha$.

148. $S_{ABCD} = \frac{1}{2}ab\sin\alpha + \frac{1}{2}bc\sin\alpha + \frac{1}{2}cd\sin\alpha + \frac{1}{2}da\sin\alpha =$

$= \frac{1}{2}\sin\alpha(ab + bc + cd + da) = \frac{1}{2}\sin\alpha(b(a+c) + d(c+a)) = \frac{1}{2}\sin\alpha(b+d)(a+c)$.



149. $\frac{AB}{\sin C}=\frac{BC}{\sin A}$, $\frac{4\sqrt{2}}{1/2}=\frac{BC}{\sqrt{2}/2}\Rightarrow BC=8$,



$\angle B=180°-45°-30°=105°$,

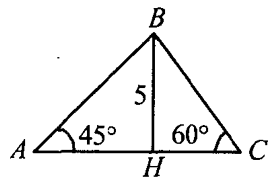
$\frac{AC}{\sin 105°}=\frac{16}{\sqrt{2}}$, $AC=\frac{16}{\sqrt{2}}\sin 105°$; $\sin 105°=\sin 75°=\sin(45°+30°)$,

$$AC=\frac{16}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\frac{1}{2}\right)=\frac{16}{\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}=4\sqrt{3}+4;$$

$$S=\frac{1}{2}\cdot AC\cdot AB\cdot\sin\angle C;\ S=\frac{1}{2}\cdot\frac{1}{2}\left(4\sqrt{3}+4\right)8=8\left(\sqrt{3}+1\right);$$

<u>**Ответ:**</u> $BC=8,\ AC=4\sqrt{3}+4,\ S=8\left(\sqrt{3}+1\right)\left(\text{см}^2\right)$.

150. $AH=BH=5$ $\left(\text{т.к.}\,\angle A=\angle ABH=45°\right)$



$\operatorname{tg}60°=\frac{5}{HC}$, $HC=\frac{5}{\operatorname{tg}60°}=\frac{5}{\sqrt{3}}$;

$$S=\frac{1}{2}\cdot 5\cdot\left(5+\frac{5}{\sqrt{3}}\right)=\frac{25\left(3+\sqrt{3}\right)}{6}.$$

§8. Формулы приведения

151. а) $\sin\left(\frac{\pi}{2}-t\right)=\cos t$; б) $\cos(2\pi-t)=\cos t$;

в) $\cos\left(\frac{3\pi}{2}+t\right)=-\sin t$; г) $\sin(\pi+t)=-\sin t$.

152. а) $\sin(\pi-t)=\sin t$; б) $\cos\left(\frac{\pi}{2}+t\right)=-\sin t$;

в) $\cos(2\pi+t)=\cos t$; г) $\sin\left(\frac{3\pi}{2}-t\right)=-\cos t$.

153. а) $\cos(90°-\alpha)=\sin\alpha$; б) $\sin(360°-\alpha)=-\sin\alpha$;

в) $\sin(270°+\alpha)=-\cos\alpha$; г) $\cos(180°+\alpha)=-\cos\alpha$.

154. а) $\operatorname{tg}(90°-\alpha)=\operatorname{ctg}\alpha$; б) $\operatorname{ctg}(180°-\alpha)=-\operatorname{ctg}\alpha$;

в) $\operatorname{tg}(180°+\alpha)=-\operatorname{ctg}\alpha$; г) $\operatorname{ctg}(360°+\alpha)=\operatorname{ctg}\alpha$.

155. а) $\sin 240°=-\sin 60°=-\frac{\sqrt{3}}{2}$; б) $\operatorname{tg}300°=\operatorname{tg}120°=-\operatorname{tg}60°=-\sqrt{3}$;

в) $\cos 330°=\cos(-30°)=\frac{\sqrt{3}}{2}$; г) $\operatorname{ctg}315°=-\operatorname{ctg}45°=-1$.

156. а) $\cos\frac{5\pi}{3}=\frac{1}{2}$; б) $\sin\left(-\frac{11\pi}{6}\right)=\frac{1}{2}$; в) $\sin\frac{7\pi}{6}=-\frac{1}{2}$; г) $\cos\left(-\frac{7\pi}{3}\right)=\frac{1}{2}$.

157. **а)** $\cos 630^\circ - \sin 1470^\circ - \operatorname{ctg} 1125^\circ = -\frac{1}{2} - 1 = -\frac{3}{2}$;

б) $\sin(-7\pi) + 2\cos\frac{31\pi}{3} - \operatorname{tg}\frac{7\pi}{4} = 1 + 1 = 2$;

в) $\operatorname{tg} 1800^\circ - \sin 495^\circ + \cos 945^\circ = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$;

г) $\cos(-9\pi) + 2\sin\left(-\frac{49\pi}{6}\right) - \operatorname{ctg}\left(-\frac{21\pi}{4}\right) = -1 - 1 + 1 = -1$.

158. **а)** $\sin(90^\circ - \alpha) + \cos(180^\circ + \alpha) + \operatorname{tg}(270^\circ + \alpha) + \operatorname{ctg}(360^\circ + \alpha) =$
$= \cos\alpha - \cos\alpha - \operatorname{ctg}\alpha + \operatorname{ctg}\alpha = 0$;

б) $\sin\left(\frac{\pi}{2} + t\right) - \cos(\pi - t) + \operatorname{tg}(\pi - t) + \operatorname{ctg}\left(\frac{5\pi}{2} - t\right) = \cos t + \cos t - \operatorname{tg} t + \operatorname{tg} t = 2\cos t$;

159. **а)** $\frac{\cos(180 + \alpha)\cdot\sin(-\alpha)}{\sin(-\alpha)\cdot\sin(90 + \alpha)} = \frac{-\cos\alpha\cos\alpha}{-\sin\alpha\cos\alpha} = \operatorname{ctg}\alpha$;

б) $\frac{\sin(\pi - t)\cos(2\pi - t)}{\operatorname{tg}(\pi - t)\cos(\pi - t)} = \frac{\sin t\cos t}{-\operatorname{tg} t\cdot(-\cos t)} = \cos t$;

в) $\frac{\sin(-\alpha)\operatorname{ctg}(-\alpha)}{\cos(360 - \alpha)\operatorname{tg}(180 + \alpha)} = \frac{\cos\alpha}{\cos\alpha\operatorname{tg}\alpha} = \operatorname{ctg}\alpha$;

г) $\frac{\sin(\pi + t)\sin(2\pi + t)}{\operatorname{tg}(\pi + t)\cos\left(\frac{3\pi}{2} + t\right)} = \frac{-\sin t\sin t}{\frac{\sin t}{\cos t}\sin t} = -\cos t$.

160. **а)** $\frac{\cos(\pi - t) + \cos\left(\frac{\pi}{2} - t\right)}{\sin(2\pi - t) - \sin\left(\frac{3\pi}{2} + t\right)} = \frac{-\cos t + \sin t}{-\sin t + \cos t} = -1$; **б)** $\frac{\sin^2(\pi - t) + \sin^2\left(\frac{\pi}{2} - t\right)}{\sin(\pi - t)}$.

161. **а)** $\frac{\operatorname{tg}(\pi - t)}{\cos(\pi + t)}\cdot\frac{\sin\left(\frac{3\pi}{2} + t\right)}{\operatorname{tg}\left(\frac{3\pi}{2} + t\right)} = \operatorname{tg}^2 t$, $\quad \frac{-\operatorname{tg} t}{-\cos t}\cos\left(\frac{3\pi}{2} + t\right) = \operatorname{tg}^2 t$;

б) $\frac{\sin(\pi - t)}{\operatorname{tg}(\pi + t)}\cdot\frac{\operatorname{ctg}\left(\frac{\pi}{2} - t\right)}{\operatorname{tg}\left(\frac{\pi}{2} + t\right)}\cdot\frac{\cos(2\pi - t)}{\sin(-t)} = \sin t$, $\quad \frac{\sin t}{\operatorname{tg} t}\cdot\frac{\operatorname{tg} t}{-\operatorname{ctg} t}\cdot\frac{\cos t}{-\sin t} = \operatorname{tg} t\cdot\cos t = \sin t$

162. **а)** $\frac{\cos^2(\pi - t) + \sin^2\left(\frac{\pi}{2} - t\right) + \cos(\pi + t)\cos(2\pi - t)}{\operatorname{tg}^2\left(t - \frac{\pi}{2}\right)\operatorname{ctg}^2\left(\frac{3\pi}{2} + t\right)} = \cos^2 t$,

$$\frac{\cos^2 t+\cos^2 t-\cos^2 t}{c\,\mathrm{tg}^2 t\,\mathrm{tg}^2 t}=\cos^2 t\,;$$

б) $\dfrac{\sin^2\left(t-\frac{3\pi}{2}\right)\cos(2\pi-t)}{\mathrm{tg}^2\left(t-\frac{\pi}{2}\right)\cos\left(t-\frac{3\pi}{2}\right)}=\cos t\,,\qquad \dfrac{\cos^2 t\cos t}{\mathrm{ctg}^2 t\sin^2 t}=\cos t\,.$

163. а) $\dfrac{11\cos 287^\circ-25\sin 557^\circ}{\sin 17^\circ}=\dfrac{-11\cos 167^\circ+25\sin 17^\circ}{\sin 17^\circ}=\dfrac{11\sin 17^\circ+25\sin 17^\circ}{\sin 17^\circ}=36\,;$

б) $\dfrac{13\sin 469^\circ-8\cos 341^\circ}{\cos 19^\circ}=\dfrac{13\sin 109^\circ-8\cos 19^\circ}{\cos 19^\circ}=\dfrac{13\cos 109^\circ-8\cos 19^\circ}{\cos 19^\circ}=5\,.$

164. а) $\dfrac{2\cos\frac{11\pi}{5}+8\sin\frac{13\pi}{10}}{\cos\frac{\pi}{5}}=\dfrac{2\cos\frac{\pi}{5}+8\sin\frac{3\pi}{10}}{\cos\frac{\pi}{5}}=\dfrac{2\cos\frac{\pi}{5}-8\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}}=-6\,;$

б) $\dfrac{5\sin\frac{5\pi}{7}+2\cos\frac{25\pi}{14}}{\sin\frac{2\pi}{7}}=\dfrac{5\sin\frac{2\pi}{7}-2\cos\frac{11\pi}{14}}{\sin\frac{2\pi}{7}}=\dfrac{5\sin\frac{2\pi}{7}+2\sin\frac{4\pi}{14}}{\sin\frac{2\pi}{7}}=7\,.$

165. а) $2\cos(2\pi+t)+\sin\left(\frac{\pi}{2}+t\right)=3,\ 2\cos t+\cos t=3,\ \cos t=1,\ t=2\pi n\,;$

б) $\sin(\pi+t)+2\cos\left(\frac{\pi}{2}+t\right)=3,\ -\sin t-2\sin t=3,\ \sin t=-1,\ t=-\frac{\pi}{2}+2\pi n\,;$

в) $2\sin(\pi+t)+\cos\left(\frac{\pi}{2}-t\right)=-\frac{1}{2},\ -2\sin t+\sin t=-\frac{1}{2},\ \sin t=\frac{1}{2},\ t=(-1)^k\frac{\pi}{6}+2\pi k\,;$

г) $3\sin\left(\frac{\pi}{2}+t\right)-\cos(2\pi+t)=1,\ 3\cos t-\cos t=1,\ \cos t=\frac{1}{2},\ t=\pm\frac{\pi}{3}+2\pi n\,.$

166. а) $5\sin\left(\frac{\pi}{2}+t\right)-\sin\left(\frac{3\pi}{2}+t\right)-8\cos(2\pi-t)=1\,,$

$5\cos t+\cos t-8\cos t=1,\ \cos t=-\frac{1}{2},\ t=\pm\frac{2\pi}{3}+2\pi n\,;$

б) $\sin(2\pi+t)-\cos\left(\frac{\pi}{2}-t\right)+\sin(\pi-t)=1\,,$

$\sin t-\sin t+\sin t=1,\ \sin t=1,\ t=\frac{\pi}{2}+2\pi n\,.$

167. а) $\sin^2(\pi+t)+\cos^2(2\pi-t)=0,\ \sin^2 t+\cos^2 t=0$ корней нет;

б) $\sin^2(\pi+t)+\cos^2(2\pi-t)=1,\ \sin^2 t+\cos^2(2\pi-t)=1,\ \sin^2 t+\cos^2 t=1,\ t\in R\,.$

§9. Функция y = sin x, ее свойства и график

168. а) $\sin\pi = 0$; **б)** $\sin\left(-\frac{\pi}{2}\right) = -1$; **в)** $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; **г)** $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

169. а) $f(x) = \sin x,\ f(-x) = -\sin x$; **б)** $f(x) = \sin x,\ f(2x) = \sin 2x$;
в) $f(x) = \sin x,\ f(x+1) = \sin(x+1)$; **г)** $f(x) = \sin x,\ f(x) - 5 = \sin x - 5$.

170. а) $y = 2\sin\left(x - \frac{\pi}{6}\right) + 1,\ x = \frac{4\pi}{3},\ y = 2\sin\frac{7\pi}{6} + 1 = -1 + 1 = 0$;

б) $y = -\sin\left(x + \frac{\pi}{4}\right),\ x = -\frac{\pi}{2},\ y = -\sin\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$;

в) $y = 2\sin\left(x - \frac{\pi}{6}\right) + 1,\ x = \frac{7\pi}{6},\ y = 2\sin\pi + 1 = 1$;

г) $y = -\sin\left(x + \frac{\pi}{4}\right),\ x = -\frac{15\pi}{4},\ y = -\sin\left(\frac{14\pi}{4}\right) = \sin\frac{7\pi}{4} = -1$.

171. а) $y = \sin x,\ \sin\left(-\frac{\pi}{2}\right) = -1,\ \left(-\frac{\pi}{2};-1\right)$ принадлежит;

б) $y = \sin x,\ \frac{1}{2} \neq \sin\frac{\pi}{2},\ \left(\frac{\pi}{2};\frac{1}{2}\right)$ не принадлежит;

в) $y = \sin x,\ 1 \neq \sin\pi,\ (\pi;1)$ не принадлежит;

г) $y = \sin x,\ -1 = \sin\frac{3\pi}{2},\ \left(\frac{3\pi}{2};-1\right)$ принадлежит.

172. а) $y = \sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin\frac{\pi}{6} + 2 = \frac{3}{2},\ \left(0;\frac{3}{2}\right)$ принадлежит;

б) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin\frac{\pi}{3} + 2 = -\frac{\sqrt{3}}{2} + 2,\ \left(\frac{\pi}{6};-\frac{\sqrt{3}}{2} + 2\right)$ принадлежит;

в) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2,\ \frac{3}{2} = -\sin\frac{5\pi}{6} + 2 = -\frac{1}{2} + 2,\ \left(\frac{2\pi}{3};\frac{3}{2}\right)$ принадлежит;

г) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2,\ -\sin\left(4\pi + \frac{\pi}{6}\right) + 2 = -\frac{1}{2} + 2 \neq 2{,}5,\ (4\pi;2{,}5)$ не принадлежит.

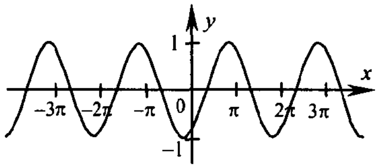
173. а) $y = \sin x,\ x \in \left[\frac{\pi}{4};\frac{2\pi}{3}\right],\ f_{\max} = 1,\ f_{\min} = \frac{\sqrt{2}}{2}$;

б) $y = \sin x,\ x \in \left[\frac{\pi}{4};\infty\right],\ f_{\max} = 1,\ f_{\min} = -1$;

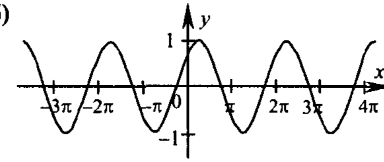
в) $y = \sin x,\ x \in \left[-\frac{3\pi}{2};\frac{3\pi}{4}\right],\ f_{\max} = 1,\ f_{\min} = -1$;

г) $y = \sin x,\ x \in \left[-\pi;\frac{\pi}{3}\right],\ f_{\max} = \frac{\sqrt{3}}{2},\ f_{\min} = -1$.

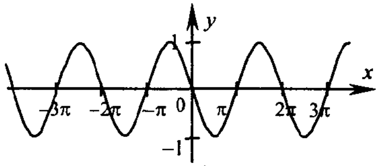
174. а)



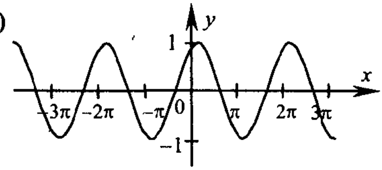
б)



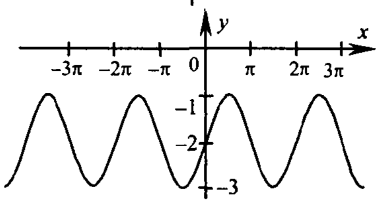
в)



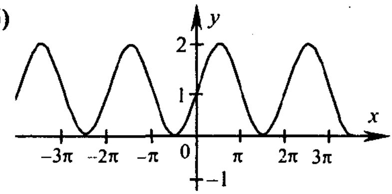
г)



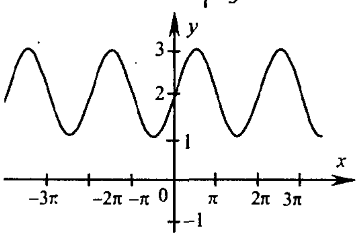
175. а)



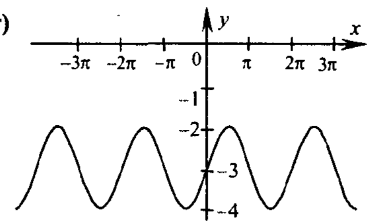
б)



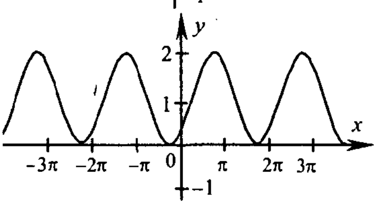
в)



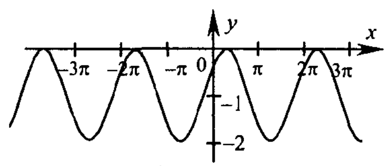
г)



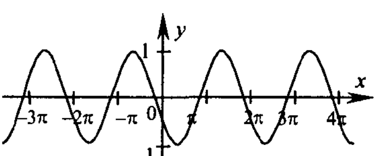
176. а)



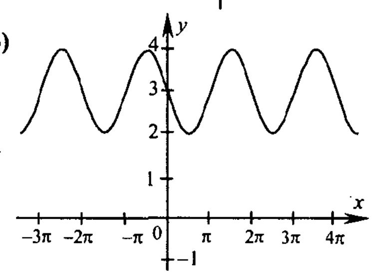
б)



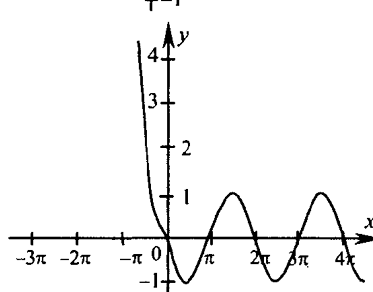
177. а)



б)



178. $f(x)=\begin{cases} x^2,\ x<0 \\ \sin x,\ x\geq 0 \end{cases}$



а) 1) Область определения $D(f)=R$

2) Область значений $E(f)=[-1;+\infty)$

3) При $x>0$ функция периодична, $T=2\pi$

4) Функция ни четная, ни нечетная

5) $f(x)=0$ при $x=\pi n$, $n\geq 0$, $x=0$ при $y=0$

6) Промежутки знакопостоянства:

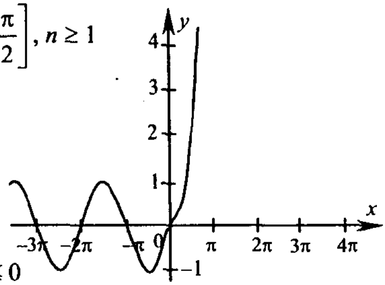
$f(x) > 0$ при $x < 0$, $x \in (2\pi n, \pi + 2\pi n)$, $n \geq 0$

$f(x) < 0$ при $x \in (2\pi n - \pi, 2\pi n)$, $n \geq 0$

7) $f_{\min} = -1$, $f_{\max} = +\infty$

8) Функция убывает при $x \leq 0$ и $x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right)$, $n \geq 0$

возрастает при $x \in \left[0; \frac{\pi}{2}\right] \cup \left[2\pi n - \frac{\pi}{2}; 2\pi n + \frac{\pi}{2}\right]$, $n \geq 1$



б) 1) Область определения $D(f) = R$

2) Область значений $E(f) = [-1; +\infty)$

3) При $x < 0$ функция периодична, $T = 2\pi$

4) Функция ни чстная, ни нечетная

5) $f(x) = 0$ при $x = \pi n$, $n \leq 0$, $x = 0$ при $y = 0$

6) Промежутки знакопостоянства:

$f(x) > 0$ при $x > 0$, $x \in (2\pi n, \pi + 2\pi n)$, $n \leq 0$

$f(x) < 0$ при $x \in (2\pi n - \pi, 2\pi n)$, $n \leq 0$

7) $f_{\min} = -1$, $f_{\max} = +\infty$

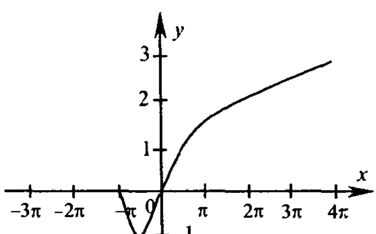
8) Функция убывает при $x \leq 0$ и $x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right)$, $n \leq 0$

возрастает при $x \in \left[2\pi n - \frac{\pi}{2}; 2\pi n + \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}; +\infty\right]$, $n < 0$

179. $f(x) = \begin{cases} \sin x, & -\pi \leq x \leq 0 \\ x\sqrt{x}, & x \geq 0 \end{cases}$

а) $f\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$, $f(0) = 0$, $f(1) = 1$, $f(\pi^2) = \pi$

б)



в) 1) Область определения $D(f) = [-\pi; +\infty)$

2) Область значений $E(f) = [-1; +\infty)$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = 0$

6) Промежутки знакопостоянства:

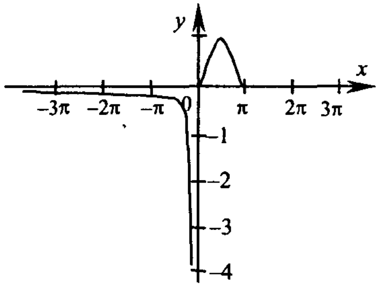
$f(x) > 0$ при $x > 0$; $f(x) < 0$ при $x \in [-\pi; 0)$

7) $f_{\min} = -1$, $f_{\max} = +\infty$

8) Функция убывает при $x \leq 0$ и $x \in \left[-\pi; -\frac{\pi}{2}\right]$, возрастает при $x \geq -\frac{\pi}{2}$.

180. $f(x)=\begin{cases}\dfrac{1}{x}, & x<0\\ \sin x, & 0\le x\le\pi\end{cases}$ **а)** $f(-2)=-\dfrac{1}{2}$, $f(0)=0$, $f(1)=\sin 1$;

б)



в) 1) Область определения $D(f)=(-\infty;\ \pi]$

2) Область значений $E(f)=(-\infty;\ 1]$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

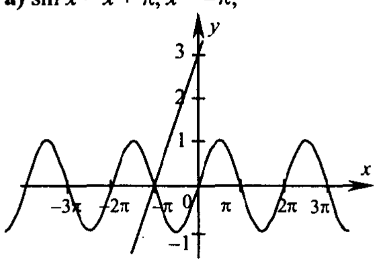
5) $f(x)=0$ при $x=0$, $x=\pi$

6) Промежутки знакопостоянства: $f(x)<0$ при $x<0$; $f(x)>0$ при $x\in(0;\ \pi)$

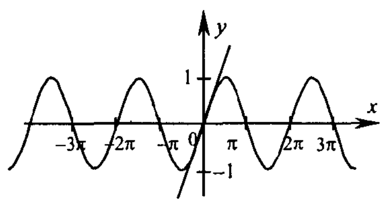
7) $f_{\min}=-\infty$, $f_{\max}=1$

8) Функция убывает при $x<0$ и $x\in\left[-\dfrac{\pi}{2};\pi\right]$, возрастает при $x\ge\left[0;\dfrac{\pi}{2}\right]$.

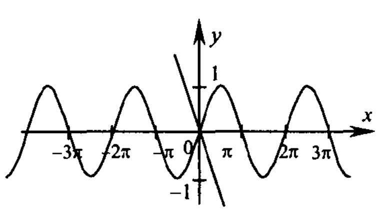
181. а) $\sin x=x+\pi$, $x=-\pi$;



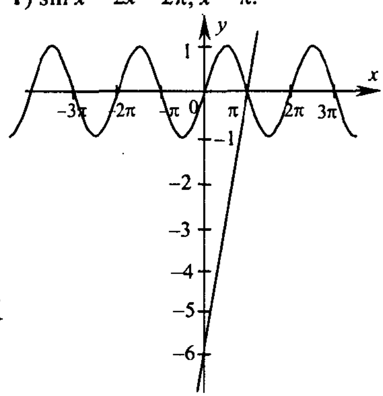
б) $\sin x=2x$, $x=0$;



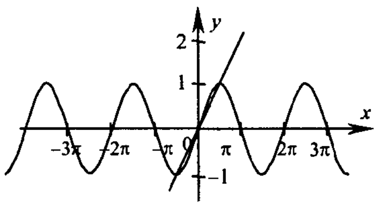
в) $\sin x=-x$, $x=0$;

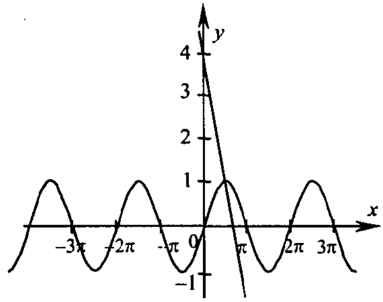


г) $\sin x=2x-2\pi$, $x=\pi$.

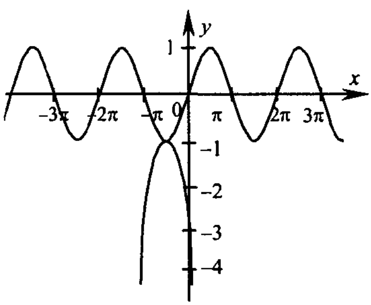


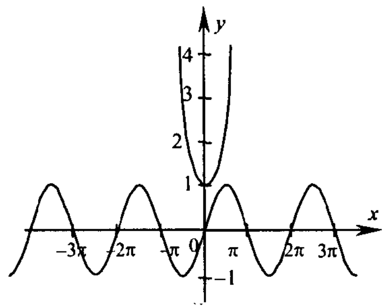
182. а) $\sin x = \frac{2}{\pi}x,\ x = 0,\ x = \pm\frac{\pi}{2}$; **б)** $\sin x = -\frac{4}{\pi}x + 3,\ x = \frac{\pi}{2}$.





183. а) $\sin x + 1 = -\left(x + \frac{\pi}{2}\right)^2,\ x = -\frac{\pi}{2}$; **б)** $\sin x = x^2 + 1$. решений нет.





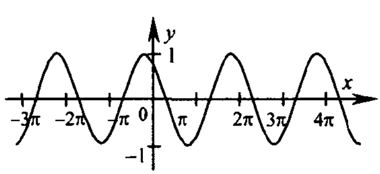
184. а) $\begin{cases} y = \sin x \\ y = x^2 + 4x - 1 \end{cases}$; $\begin{cases} y = \sin x \\ y = (x+2)^2 - 3 \end{cases}$ система имеет 2 решения;

б) $\begin{cases} y = \sin x \\ y = \dfrac{1}{x} \end{cases}$ система имеет бесконечное множество решений;

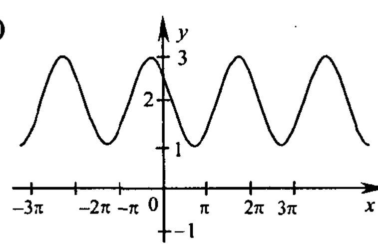
в) $\begin{cases} y = \sin x \\ y = -3x^2 - 2 \end{cases}$ система не имеет решений;

г) $\begin{cases} y = \sin x \\ |x| - y = 0 \end{cases}$ система имеет одно решение.

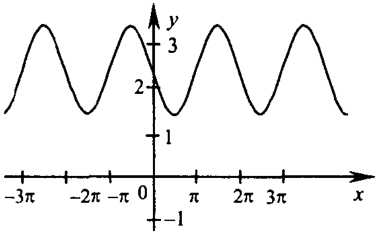
185. а)



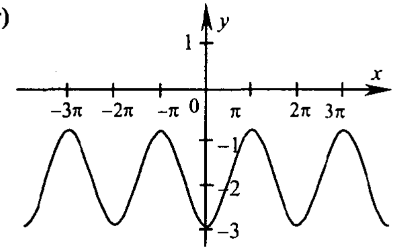
б)



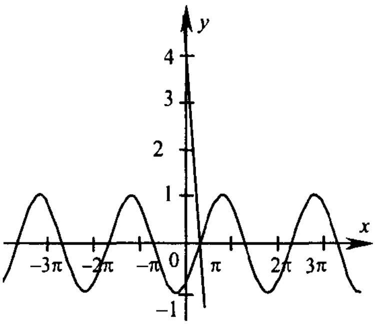
в)



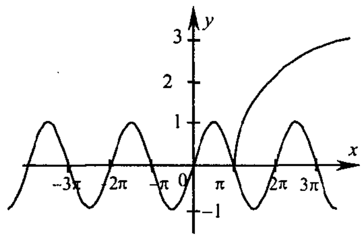
г)



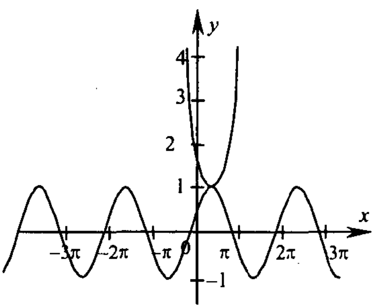
186. а) $\sin\left(x-\frac{\pi}{3}\right)=\pi-3x$, $x=\frac{\pi}{3}$;



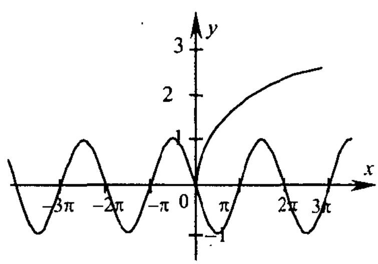
б) $\sin x=\sqrt{x-\pi}$, $x=\pi$;



в) $\sin\left(x+\frac{\pi}{6}\right)-1=\left(x-\frac{\pi}{3}\right)^2$, $x=\frac{\pi}{3}$;



г) $-\sin x=\sqrt{x}$, $x=0$.



187. $y=\sin\left(x-\frac{\pi}{4}\right)+\frac{1}{2}$

а) $x\in\left[\frac{\pi}{4};\frac{3\pi}{4}\right]$, $y_{\max}=\frac{3}{2}$, $y_{\min}=\frac{1}{2}$;

б) $\left(\frac{3\pi}{4};\frac{9\pi}{4}\right)$, $y_{\min}=-\frac{1}{2}$;

в) $[0;\pi)$, $y_{\max}=\frac{3}{2}$, $y_{\min}=\frac{1-\sqrt{2}}{2}$;

г) $\left[\frac{\pi}{4};\infty\right)$, $y_{\max}=\frac{3}{2}$, $y_{\min}=-\frac{1}{2}$.

188. а) $f(x)=x^5\sin\frac{x}{2}$, $f(-x)=-x^5(-1)\sin\frac{x}{2}=f(x)$;

б) $f(x)=\frac{\sin^2 x}{x^2-1}$, $f(-x)=\frac{\sin^2(-x)}{(-x^2)-1}=\frac{\sin^2 x}{x^2-1}=f(x)$;

в) $f(x)=\dfrac{2\sin\frac{x}{2}}{x^3}$, $f(-x)=\dfrac{-2\sin\frac{x}{2}}{-x^3}=\dfrac{2\sin\frac{x}{2}}{x^3}=f(x)$;

г) $f(x)=\sin^2 x-x^4$, $f(-x)=\sin^2(x)-\left(-x^4\right)=f(x)$.

189. а) $f(x)=-x-\sin x, f(-x)=-(-x)-\sin(-x)=-(-x-\sin x)=-f(x)$;

б) $f(x)=x^3\sin x^2, f(-x)=-x^3\sin(-x^2)=-x^3\sin x^2=-f(x)$;

в) $f(x)=\dfrac{x^2\sin x}{x^2-9}$, $f(-x)=-\dfrac{x^2\sin x}{x^2-9}=-f(x)$;

г) $f(x)=x^3-\sin x, f(-x)=-x^3+\sin x=-f(x)$.

190. $f(x)=2x^2-x+1$;

$f(\sin x)=2\sin^2 x-\sin x+1=2-2\cos^2 x-\sin x+1=3-2\cos^2 x-\sin x$.

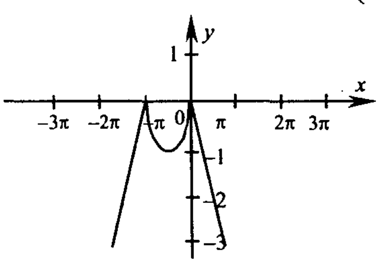
191. $f(x)=3x^2+2x-7$;

$f(\sin x)=3\sin^2 x+2\sin x-7=3-3\cos^2 x+2\sin x-7=-4-3\cos^2 x+2\sin x$.

192. $f(x)=\begin{cases}2x+2\pi, & x\le-\pi\\ \sin x, & -\pi<x\le 0\\ -2x, & x>0\end{cases}$

а) $f(-\pi-2)=-2\pi-4+2\pi=-4$, $f\left(-\dfrac{\pi}{6}\right)=\sin\left(-\dfrac{\pi}{6}\right)=-\dfrac{1}{2}$, $f(2)=-4$;

б)



в) 1) Область определения $D(f)=R$

2) Область значений $E(f)=(-\infty;0]$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

5) $f(x)=0$ при $x=-\pi, x=0$

6) Промежутки знакопостоянства:

$f(x)<0$ при $x<-\pi$, $x\in(-\pi;0)$, $x>0$

7) $f_{\min}=-\infty, f_{\max}=0$

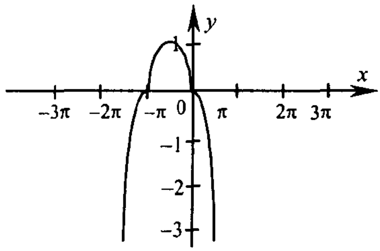
8) Функция возрастает при $x\in(-\infty;-\pi)\cup\left[-\dfrac{\pi}{2};0\right]$,

убывает при $x\in\left[-\pi;\dfrac{\pi}{2}\right]\cup[0;+\infty)$.

193. $f(x)=\begin{cases}-x^2, & x<0\\ \sin x, & 0\le x\le\pi\\ -(x-\pi)^2, & x>\pi\end{cases}$

а) $f(-3)=-9,\ f\left(\frac{\pi}{2}\right)=1,\ f(2\pi-3)=-(\pi-3)^2=-\pi^2+6\pi-9$

б)



в) 1) Область определения $D(f)=R$

2) Область значений $E(f)=(-\infty;\ 1]$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

5) $f(x)=0$ при $x=0,\ x=\pi$

6) Промежутки знакопостоянства:

$f(x)>0$ при $x\in(0;\ \pi),\ x>0$

$f(x)<0$ при $x<0;\ x>\pi$

7) $f_{\min}=-\infty, f_{\max}=1$

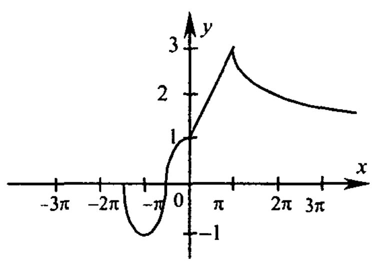
8) Функция возрастает при $x\in\left(-\infty;\frac{\pi}{2}\right]$,

убывает при $x\in\left[\frac{\pi}{2};+\infty\right]$.

194. $f(x)=\begin{cases}\sin\left(x+\frac{\pi}{2}\right), & -\frac{3\pi}{2}\le x\le 0\\ x+1, & 0<x<2\\ -\sqrt{x-2}+3, & x\ge 2\end{cases}$

а) $f(0)=1,\ f(6)=1,\ f(-\pi-2)=$ не определено, тт.к$(-\pi-2)<-\frac{3\pi}{2}$,

б)



в) 1) Область определения D $(f)=\left[-\frac{3\pi}{2};+\infty\right)$

2) Область значений E $(f)=[-1;\ 3]$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\frac{\pi}{2}$, $x = 11$

6) Промежутки знакопостоянства:

$f(x) > 0$ при $x \in \left(-\frac{\pi}{2};11\right)$; $f(x) < 0$ при $x \in \left(-\frac{3\pi}{2};-\frac{\pi}{2}\right)\cup(11;+\infty)$,

7) $f_{\min} = -\infty, f_{\max} = 3$

8) Функция возрастает при $x \in[-\pi; 2]$
убывает при $x \in[2; +\infty)$.

§10. Функция y = cos x, ее свойства и график

195. а) $\cos\frac{\pi}{2} = 0$; б) $\cos(-\pi) = -1$; в) $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; г) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$.

196. а) $f(x) = \cos x$, $f(-x) = \cos x$; б) $f(x) = \cos x$, $f(3x) = \cos 3x$;
в) $f(x) = \cos x$, $f(x + 2) = \cos(x + 2)$; г) $f(x) = \cos x$, $f(x) - 6 = \cos x - 6$.

197. а) $y = 2\sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = -2$; б) $y = 2\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2}$.

198. а) $y = \cos\left(-\frac{\pi}{3}\right) - \left(-\frac{\pi}{3}\right) = \frac{1}{2} - \frac{\pi^2}{9}$; б) $y = \cos\pi - \pi^2 = -1 - \pi^2$.

199. а) $\frac{1}{\cos\frac{2\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$; б) $\frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$.

200. а) $y = 2\cos\left(-\frac{\pi}{2} - \frac{\pi}{4}\right) - 1 = -\sqrt{2} - 1$; б) $y = 2\cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) - 1 = 1$.

201. а) $y = \cos x$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\left(\frac{\pi}{3};\frac{1}{2}\right)$ принадлежит;

б) $y = \cos x$, $\frac{1}{2} \neq \cos\frac{\pi}{6}$, $\left(\frac{\pi}{6};\frac{1}{2}\right)$ не принадлежит;

в) $y = \cos x$, $-\frac{1}{2} = \cos\frac{2\pi}{3}$, $\left(\frac{2\pi}{3};-\frac{1}{2}\right)$ принадлежит;

г) $y = \cos x$, $-\frac{\sqrt{3}}{2} = \cos\frac{5\pi}{6}$, $\left(\frac{5\pi}{6};-\frac{\sqrt{3}}{2}\right)$ принадлежит.

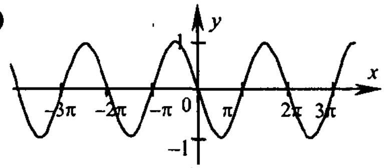
202. а) $\sqrt{3} + 1 = 2\cos\left(-\frac{\pi}{6}\right) + 1$, $(0;\sqrt{3} + 1)$ – принадлежит;

б) $1 \neq 2\cos\left(\frac{\pi}{6} - \frac{\pi}{6}\right) + 1$, $\left(\frac{\pi}{6};1\right)$ – не принадлежит;

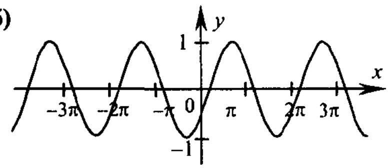
в) $2 = 2\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + 1$, $\left(\frac{\pi}{2};2\right)$ – принадлежит;

г) $3 = 2\cos\left(\dfrac{\pi}{6} - \dfrac{\pi}{6}\right) + 1$, $\left(\dfrac{\pi}{6};3\right)$ – принадлежит.

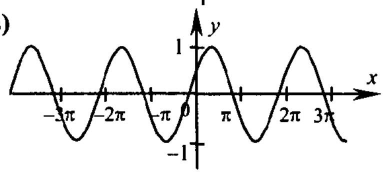
203. а)



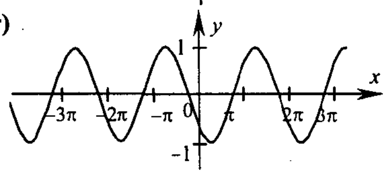
б)



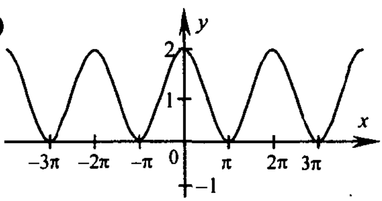
в)



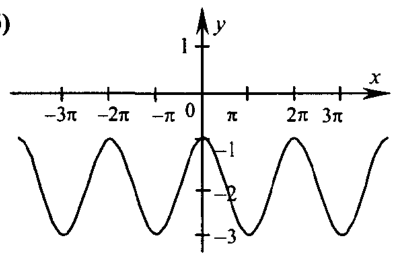
г)



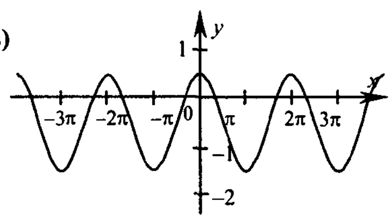
204. а)



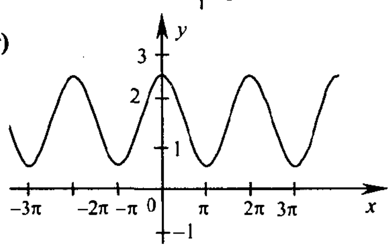
б)



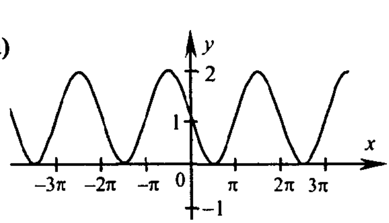
в)



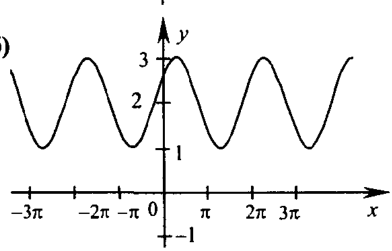
г)



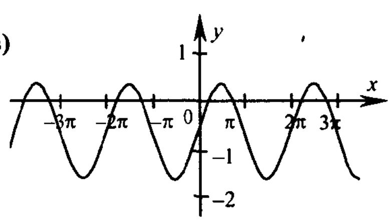
205. а)



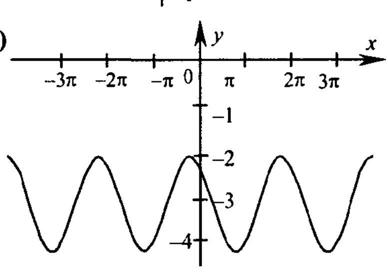
б)



в)



г)



206. $y = \cos x$.

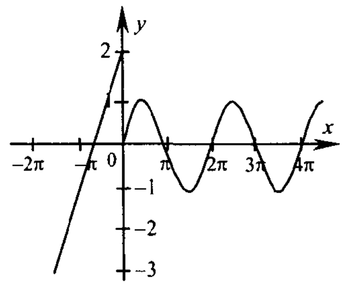
а) $x \in \left[\dfrac{\pi}{6}; \dfrac{2\pi}{3}\right]$, $y_{\min} = -\dfrac{1}{2}$, $y_{\max} = \dfrac{\sqrt{3}}{2}$;

б) $x \in \left(-\pi; \dfrac{\pi}{4}\right)$, $y_{\min}$ = не существует, $y_{\max} = 1$;

в) $x \in \left[\dfrac{\pi}{4}; +\infty\right)$, $y_{\min} = -1$, $y_{\max} = 1$;

г) $x \in \left[-\dfrac{\pi}{3}; \dfrac{3\pi}{2}\right)$, $y_{\min} = -1$, $y_{\max} = 1$.

207. а) $f(x) = \begin{cases} x + 2, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$



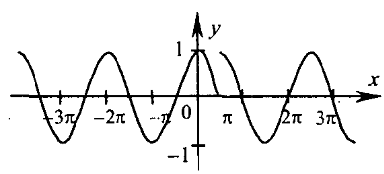
1) Область определения $D(f) = \mathrm{R}$
2) Область значений $E(f) = (-\infty; 2]$
3) При $x \geq 0$ функция периодична, $T = 2\pi$
4) Функция ни четная, ни нечетная
5) $f(x) = 0$ при $x = -2$, $x = \dfrac{\pi}{2} + \pi n$, $n \geq 0$
6) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in (-\infty; -2) \cup \left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3\pi}{2} + 2\pi n\right)$, $n \geq 0$

$f(x) > 0$ при $x \in \left(-2; \dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n\right)$, $n \geq 1$

7) $f_{\min} = -\infty$, $f_{\max} = 2$
8) Функция возрастает при $x \in (-\infty; 0) \cup (-\pi + 2\pi n; 2\pi n)$, $n \geq 0$
убывает при $x \in (2\pi n; 2\pi n + \pi)$, $n \geq 0$.

б) $f(x) = \begin{cases} \cos x, & x \leq \dfrac{\pi}{2} \\ \sin x, & x > \dfrac{\pi}{2} \end{cases}$



1) Область определения $D(f) = R$
2) Область значений $E(f) = [-1; 1]$
3) Функция периодична на промежутках $\left(-\infty; \dfrac{\pi}{2}\right]$ и $\left(\dfrac{\pi}{2}; +\infty\right)$, $\mathrm{T} = 2\pi$
4) Функция ни четная, ни нечетная
5) $f(x) = 0$ при $x = \dfrac{\pi}{2} - \pi n$, $n \geq 0$, $x = \pi(1 + k)$, $k \geq 0$
6) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in \left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3\pi}{2} + 2\pi n\right)$, $n \leq -1$, $x \in (-\pi + 2\pi k; 2\pi k)$, $k \geq 1$,

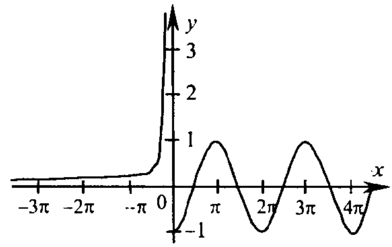
7) $f_{\min} = -1$, $f_{\max} = 1$

8) Функция возрастает при $x \in [-\pi + 2\pi n; 2\pi n)$, $n \le 0$

$x \in \left[-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right]$, $k \ge 1$

убывает при $x \in [2\pi n; \pi + 2\pi n)$, $n \le -1$, $x \in \left[0; \dfrac{\pi}{2}\right] \cup \left[\dfrac{\pi}{2} + 2\pi k; \dfrac{3\pi}{2} + 2\pi k\right]$, $k \ge 0$.

в) $f(x) = \begin{cases} -\dfrac{2}{x}, & x < 0 \\ -\cos x, & x \ge 0 \end{cases}$



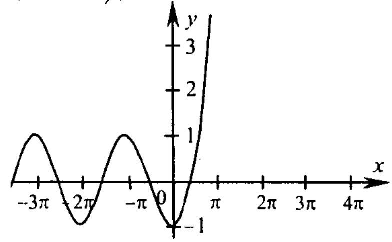
1) Область определения $D(f) = R$
2) Область значений $E(f) = [-1; +\infty)$
3) При $x \ge 0$ функция периодична, $T = 2\pi$
4) Функция ни четная, ни нечетная
5) $f(x) = 0$ при $x = \dfrac{\pi}{2} + \pi n$, $n \ge -1$
6) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in \left[0; \dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n\right)$, $n \ge 1$

$f(x) > 0$ при $x \in (-\infty; 0) \cup \left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3\pi}{2} + 2\pi n\right)$, $n \ge 0$

7) $f_{\min} = -1$, $f_{\max} = +\infty$
8) Функция возрастает при $x \in (-\infty; 0) \cup [2\pi n; \pi + 2\pi n)$, $n \ge 0$
убывает при $x \in [2\pi k - \pi; 2\pi k)$, $k \ge 1$.

г) $f(x) = \begin{cases} -\cos x, & x < 0 \\ 2x^2 - 1, & x \ge 0 \end{cases}$



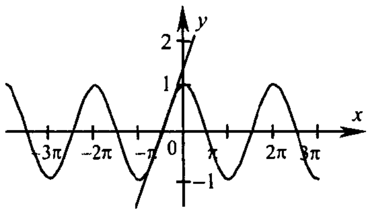
1) Область определения $D(f) = R$
2) Область значений $E(f) = [-1; +\infty)$
3) При $x < 0$ функция периодична, $T = 2\pi$
4) Функция ни четная, ни нечетная
5) $f(x) = 0$ при $x = \dfrac{\pi}{2} - \pi n$, $n \ge 1$, $x = \dfrac{\sqrt{2}}{2}$
6) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in \left(-\dfrac{\pi}{2} - 2\pi n; \dfrac{\pi}{2} - 2\pi n\right) \cup \left(-\dfrac{\pi}{2}; \dfrac{\sqrt{2}}{2}\right)$, $n \ge 1$

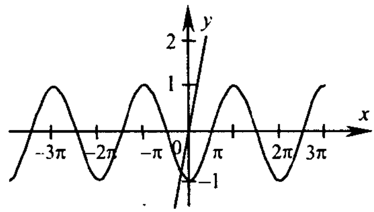
$f(x) > 0$ при $x \in \left(\dfrac{\pi}{2} - 2\pi k; \dfrac{3\pi}{2} - 2\pi k\right) \cup \left(\dfrac{\sqrt{2}}{2}; +\infty\right)$, $k \ge 1$

7) $f_{\min} = -1$, $f_{\max} = +\infty$
8) Функция возрастает при $x \in [-2\pi n; -2\pi n + \pi) \cup [0; +\infty)$, $n \ge 1$
убывает при $x \in [-2\pi n - \pi; -2\pi n)$, $n \ge 0$.

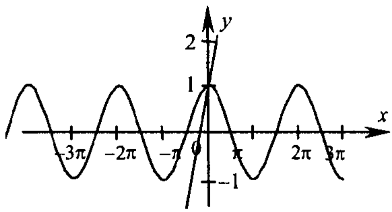
208. а) $\cos x = x + \frac{\pi}{2}$; $x = -\frac{\pi}{2}$;



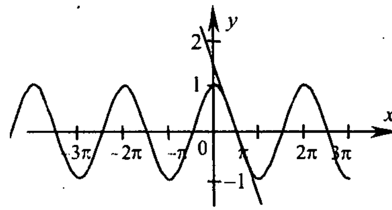
б) $-\cos x = 3x - 1$; $x = 0$;



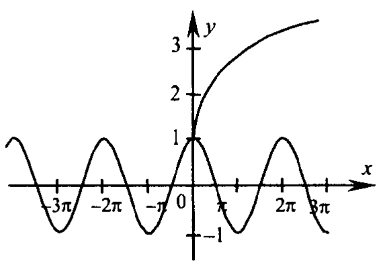
в) $\cos x = 2x + 1$; $x = 0$;



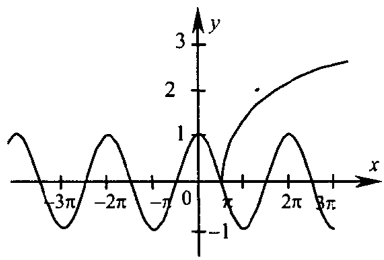
г) $\cos x = -x + \frac{\pi}{2}$; $x = \frac{\pi}{2}$.



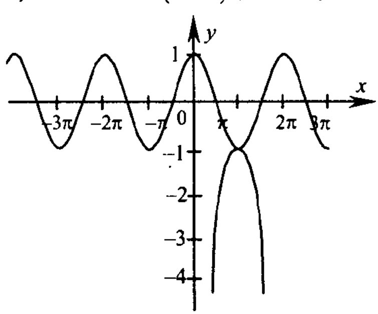
209. а) $\cos x = \sqrt{x} + 1$, $x = 0$;



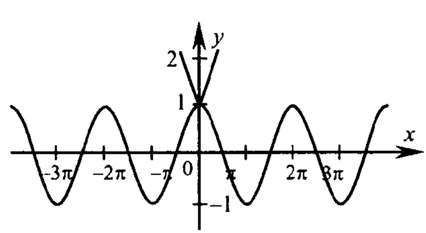
б) $\cos x = \sqrt{x - \frac{\pi}{2}}$, $x = \frac{\pi}{2}$;



в) $\cos x + 1 = -(x - \pi)^2$; $x = \pi$;



г) $\cos x = |x| + 1$, $x = 0$.



210. а) $\begin{cases} y = \cos x \\ y = -x^2 + 2x - 3 \end{cases}$; $\begin{cases} y = \cos x \\ y = -(x-1)^2 - 2 \end{cases}$ решений нет;

б) $\begin{cases} y = \cos x \\ y = \frac{2}{} \end{cases}$ бесконечное множество решений;

в) $\begin{cases} y = \cos x \\ y = x^2 - 3 \end{cases}$ 2 решения

г) $\begin{cases} y = \cos x \\ |x| - y = 0 \end{cases}$; $\begin{cases} y = \cos x \\ y = |x| \end{cases}$ 2 решения.

211. а) $f(x) = x^2 \cos x,\ f(-x) = (-x)^2 \cos(-x) = x^2 \cos x = f(x)$;

б) $f(x) = \dfrac{\cos x^3}{4 - x^2},\ f(-x) = \dfrac{\cos(-x)^3}{4 - (-x)^2} = \dfrac{\cos x^3}{4 - x^2} = f(x)$;

в) $f(x) = \dfrac{\cos 5x + 1}{|x|},\ f(-x) = \dfrac{\cos(-5x) + 1}{|-x|} = \dfrac{\cos 5x + 1}{|x|} = f(x)$;

г) $f(x) = (4 + \cos x)(\sin^6 x - 1)$,

$f(x) = (4 + \cos(-x))(\sin^6(-x) - 1) = (4 + \cos x)(\sin^6 x - 1) = f(x)$.

212. а) $f(x) = \sin x \cos x,\ f(-x) = -\sin(-x)\cos(-x) = -\sin x \cos x = -f(x)$;

б) $f(x) = x^5 \cos 3x,\ f(-x) = (-x)^5 \cos(-3x) = -x^5 \cos 3x = -f(x)$;

в) $f(x) = \dfrac{\cos x^3}{x(25 - x^2)},\ f(-x) = \dfrac{\cos(-x)^3}{-x(25 - (-x)^2)} = -\dfrac{\cos x^3}{x(25 - x^2)} = -f(x)$;

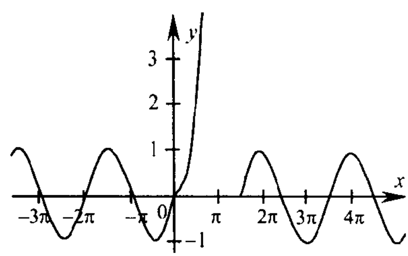
г) $f(x) = x^{11} \cdot \cos x + \sin x$,

$f(-x) = (-x)^{11} \cdot \cos(-x) + \sin(-x) = (-x)^{11} \cdot \cos x - \sin x = f(-x)$.

213. $-f(\cos x) = -2\cos^2 x + 3\cos x + 2 = 2(1 - \cos^2 x) + 3\cos x = 2\sin^2 x + 3\cos x$

214. $f(\cos x) = 5\cos^2 x + \cos x + 4 = 5 - 5\sin^2 x + \cos x - 4 = -5\sin^2 x + \cos x + 9$

215. $f(x) = \begin{cases} \sin x, & x \le 0 \\ x^2, & 0 < x < \dfrac{\pi}{2} \\ \cos x, & x \ge \dfrac{\pi}{2} \end{cases}$



1) Область определения $D(f) = R$
2) Область значений $E(f) = [-1; +\infty)$
3) Функция ни четная, ни нечетная
4) При $x \le 0$ и $x \ge \dfrac{\pi}{2}$ функция периодична, $T = 2\pi$
5) $f(x) = 0$ при $x = -\pi n$, $n \ge 0$, $x = \dfrac{\pi}{2} + \pi k$, $k \ge 0$
6) Промежутки знакопостоянства:

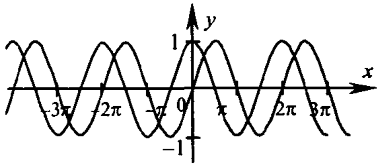
$f(x) > 0$ при $x \in (-2\pi n; -2\pi n + \pi) \cup \left(0; \dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right)$, $n \ge 1$, $k \ge 1$

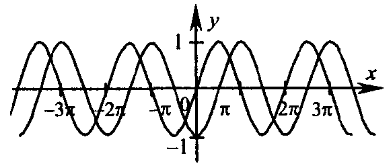
7) $f_{min} = -1, f_{max} = +\infty$

8) Функция возрастает при $x \in \left[-\frac{\pi}{2}-2\pi n; \frac{\pi}{2}-2\pi n\right]$, $n \geq 1$

$x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \cup \left[2\pi; \frac{5\pi}{2}\right] \cup \left[\pi+2\pi n; 2\pi+2\pi n\right]$, $n \geq 1$.

216. а) $\sin x = \cos x$, $x = \frac{\pi}{4}+\pi k$; б) $\sin x = -\cos x$, $x = -\frac{\pi}{4}+\pi k$.





§11. Периодичность функции y = sin x, y = cos x

217. См. рис. 73. **218.** См. рис. 74. **219.** См. рис. 75. **220.** См. рис. 76.

221. 32π является периодом функции $y = \sin x$, $y = \cos x$, но не основным.

222. а) $\sin 50,\ 5\pi = \sin\frac{\pi}{2} = 1$; б) $\cos 51,\ 75\pi = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$;

в) $\sin 25,\ 25\pi = -\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$; г) $\sin 30,\ 5\pi = \sin\frac{\pi}{2} = 1$.

223. а) $\sin 390° = \sin 30° = \frac{1}{2}$; б) $\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$;

в) $\sin 540° = \sin 180° = 0$; г) $\cos 930° = \cos 150° = -\frac{\sqrt{3}}{2}$.

224. а) $\sin^2(x-8\pi) = 1-\cos^2(16\pi - x)$,

$\sin^2(x-8\pi) = \sin^2 x$; $1-\cos^2(16\pi-x) = 1-\cos^2 x = \sin^2 x$;

б) $\cos^2(4\pi+x) = 1-\sin^2 x(22\pi - x)$; $\cos^2(4\pi+x) = \cos^2 x$,

$10\sin^2 x(22\pi - x) = 1-\sin^2 x = \cos^2 x$.

225. а) $y = \sin 2x$, $T = \pi$, $y(x+T) = \sin(2x+2\pi) = \sin 2x = y(x)$;

б) $y = \cos 3x$, $T = \frac{2\pi}{3}$, $y(x+T) = \sin(3x+2\pi) = \sin 3x = y(x)$;

в) $y = \sin\frac{x}{2}$, $T = 4\pi$, $y(x+T) = \sin\left(\frac{x}{2}+2\pi\right) = \sin\frac{x}{2} = y(x)$;

г) $y = \cos\frac{3x}{4}$, $T = \frac{8\pi}{3}$, $y(x+T) = \cos\left(\frac{3x}{4}+2\pi\right) = \cos\frac{3x}{4} = y(x)$.

226. а) $\sin 8 = \sin(8-2\pi)$; б) $\cos(-10) = \cos(-10+4\pi)$;

в) $\sin(-25) = \sin(-25+8\pi)$; г) $\cos 35 = \cos(35-10\pi)$.

227. а) $\cos(t+4\pi) = ?$ $\cos(2\pi - t) = -\frac{3}{5}$, $\cos t = -\frac{3}{5}$, $\cos(t+4\pi) = \cos t = -\frac{3}{5}$;

б) $\sin(32\pi - t) = ?$ $\sin(2\pi - t) = \frac{5}{13}$, $\sin(32\pi - t) = \sin(2\pi - t) = \frac{5}{13}$.

228. а) $\sin(t+2\pi)+\sin(t-4\pi)=1,\ \sin t+\sin t=1,\ \sin t=;\ t=(-1)^k\dfrac{\pi}{6}+\pi k$;

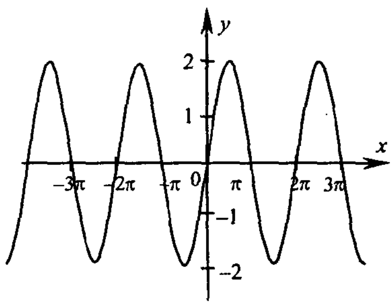
б) $3\cos(2\pi+t)+\cos(t-2\pi)+2=0,\ 4\cos t=-2,\ \cos t=-\dfrac{1}{2},\ t=\pm\dfrac{2\pi}{3}+2\pi n$;

в) $\sin(t+4\pi)+\sin(t-6\pi)=\sqrt{3},\ 2\sin t=\sqrt{3},\ \sin t=\dfrac{\sqrt{3}}{2},\ t=(-1)^k\dfrac{\pi}{3}+\pi k$;

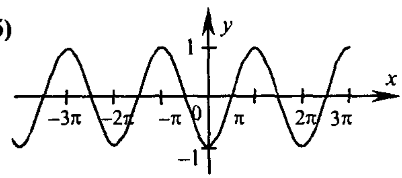
г) $\cos(t+2\pi)+\cos(t-8\pi)=\sqrt{2},\ 2\cos t=\sqrt{2},\ \cos t=\dfrac{\sqrt{2}}{2},\ t=\pm\dfrac{\pi}{4}+2\pi k$.

§12. Как построить график функции y = mf (x), если известен график функции y = f (x)

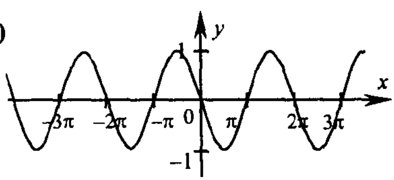
229. а)



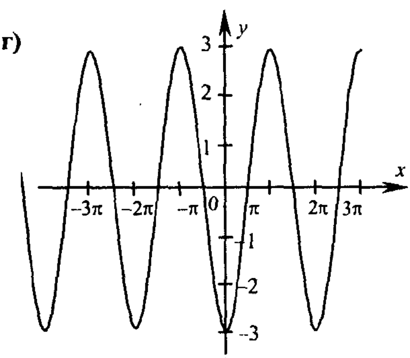
б)



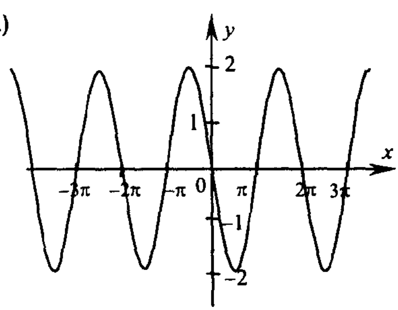
в)



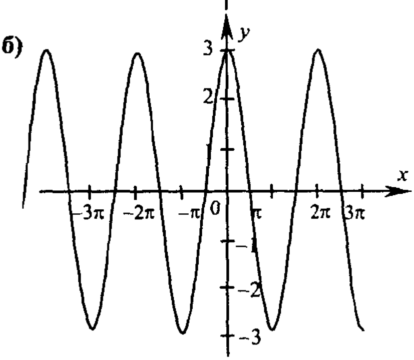
г)



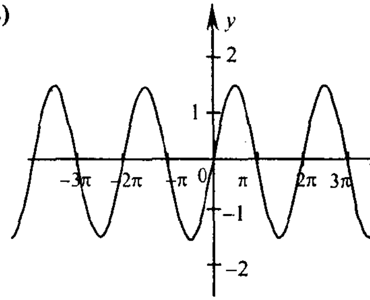
230. а)



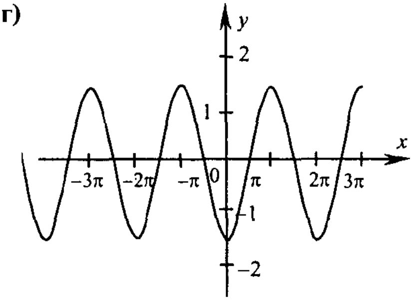
б)



в)



г)



231. а) $x \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$, $y_{\max}=2$, $y_{\min}=0$; **б)** $x \in \left(0;\frac{3\pi}{2}\right)$, $y_{\max}=0$, $y_{\min}=-2$;

в) $x \in \left[\frac{\pi}{3};\frac{3\pi}{2}\right]$, $y_{\max}=1$, $y_{\min}=-2$; **г)** $x \in \left[-\frac{3\pi}{2};-\frac{\pi}{4}\right]$, $y_{\max}=\sqrt{2}$, $y_{\min}=-2$.

232. а) $x \in [0;+\infty)$, $y_{\max}=3$, $y_{\min}=-3$; **б)** $x \in \left(-\infty;\frac{\pi}{2}\right)$, $y_{\max}=3$, $y_{\min}=-3$;

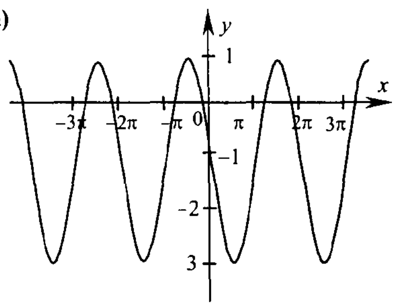
в) $x \in \left[\frac{\pi}{4};+\infty\right)$, $y_{\max}=3$, $y_{\min}=-3$; **г)** $x \in (-\infty;0)$, $y_{\max}=3$, $y_{\min}=-3$.

233. а) $f(-x) = -3\sin x$; **б)** $2f(x) = 6\sin x$;
в) $2f(x) + 1 = 6\sin x + 1$; **г)** $f(-x) + f(x) = -3\sin x + 3\sin x = 0$.

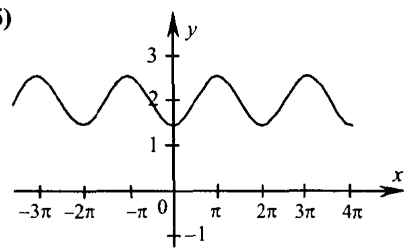
234. $f(x) = -\frac{1}{2}\cos x$; **а)** $f(-x) = -\frac{1}{2}\cos x$; **б)** $2f(x) = -\cos x$;

в) $f(x+2\pi) = -\frac{1}{2}\cos x$; **г)** $f(-x) - f(x) = -\frac{1}{2}\cos x + \frac{1}{2}\cos x = 0$.

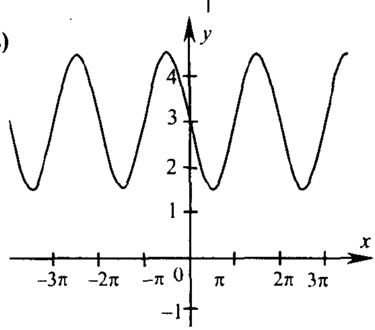
235. а)



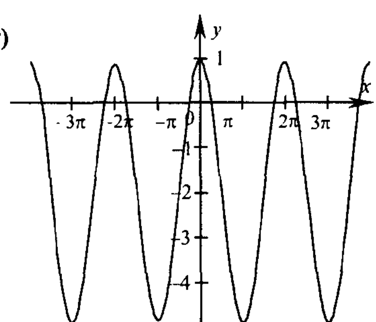
б)



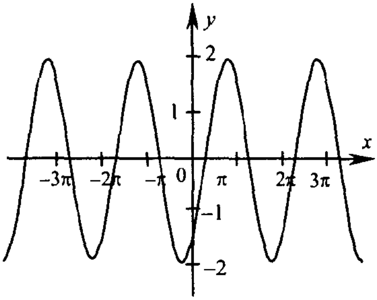
в)



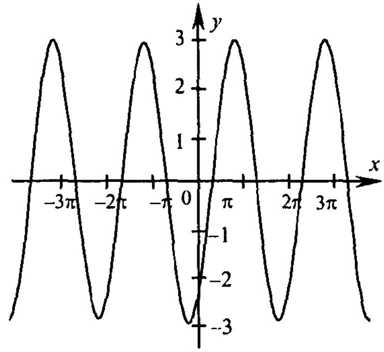
г)



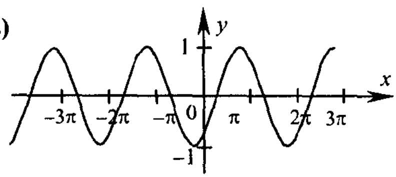
236. а)



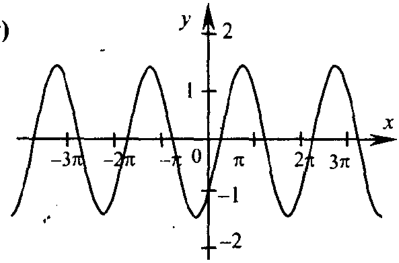
б)



в)



г)



237. а) $\begin{cases} x^2, & x<0 \\ \dfrac{1}{2}\sin x, & 0\le x\le\pi \end{cases}$

б) $\begin{cases} 1{,}5\cos x, & x\in\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right] \\ x-\dfrac{\pi}{2}, & x>\dfrac{\pi}{2} \end{cases}$

238. а) $f(x)=\begin{cases} 3\sin x, & x<\dfrac{\pi}{2} \\ 3x^3, & x\ge\dfrac{\pi}{2} \end{cases}$

1) Область определения $D(f) = R$

2) Область значений $E(f) = [-3;3]\cup\left[\dfrac{3\pi^3}{8};+\infty\right)$

3) Функция периодична при и $x<\dfrac{\pi}{2}$, $T = 2\pi$

4) Функция ни четная, ни нечетная

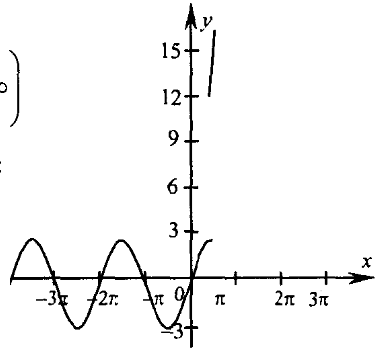
5) $f(x) = 0$ при $x = -\pi n$, $n \ge 0$

6) $f_{\min} = -3$, $f_{\max} = +\infty$

7) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in (-2\pi n - \pi; -2\pi n)$, $n \ge 0$,

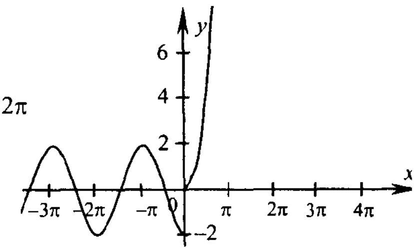
$f(x) > 0$ при $x \in (-2\pi n; -2\pi n + \pi)$, $n \ge 0$,



8) Функция возрастает при $x \in \left[-2\pi n-\dfrac{\pi}{2};-2\pi n+\dfrac{\pi}{2}\right]$, $n\ge 0$

убывает при $x\in\left[-2\pi n+\dfrac{\pi}{2};-2\pi n+\dfrac{3\pi}{2}\right]$, $n\ge 0$.

б) $f(x)=\begin{cases}-2\cos x, & x<0\\ \frac{1}{2}x^4, & x\geq 0\end{cases}$



1) Область определения $D(f) = R$
2) Область значений $E(f) = [-2;+\infty)$
3) Функция периодична при и $x<0$, $T=2\pi$
4) Функция ни четная, ни нечетная
5) $f(x)=0$ при $x=-\frac{\pi}{2}-\pi n$, $n\geq 1$, $x=0$
6) $f_{\min}=-3$, $f_{\max}=+\infty$
7) Промежутки знакопостоянства:

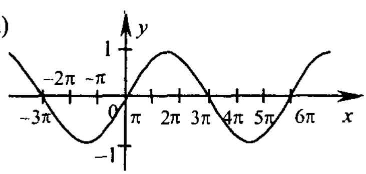
$f(x)<0$ при $x\in\left(-\frac{\pi}{2}-2\pi n;\frac{\pi}{2}-2\pi n\right)\cup\left(-\frac{\pi}{2};0\right)$, $n\geq 1$,

$f(x)>0$ при $x\in\left(\frac{\pi}{2}-2\pi n;\frac{3\pi}{2}-2\pi n\right)$, $n\geq 1$,

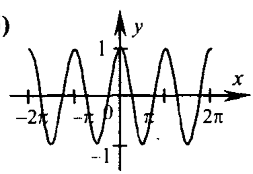
8) Функция возрастает при $x\in[-2\pi n;-2\pi n+\pi]$, $n\geq 1$, $x\geq 0$,
убывает при $x\in[-2\pi n-\pi;-2\pi n]$, $n\geq 0$

§13. Как построить график функции y = f (kx), если известен график функции y = f (x)

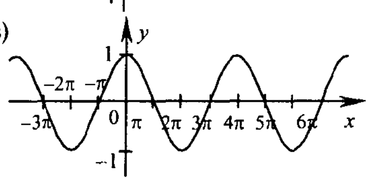
239. а)



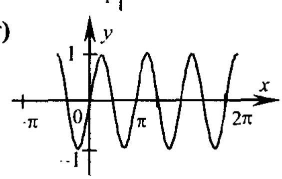
б)



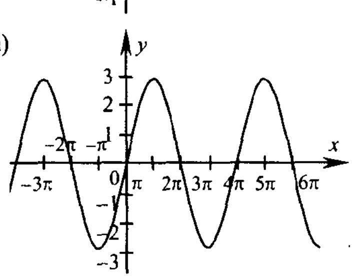
в)



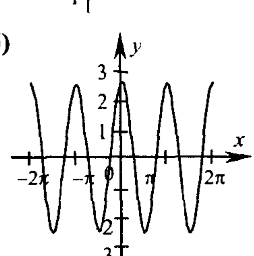
г)



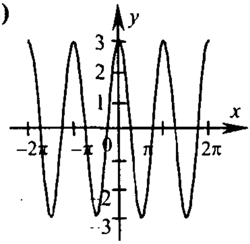
240. а)



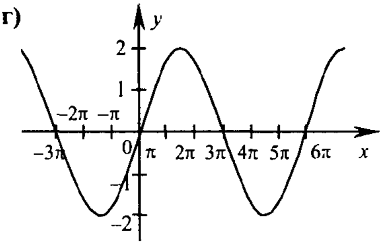
б)



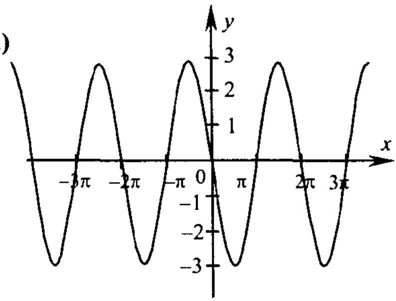
в)



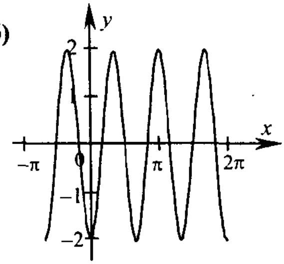
г)



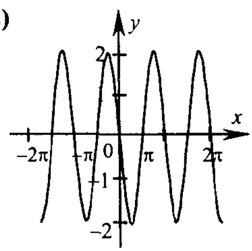
241. а)



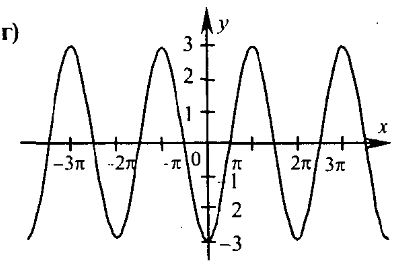
б)



в)



г)



242. а) $x \in \left[-\frac{\pi}{2};0\right]$, $y_{max} = 0$, $y_{min} = -1$; **б)** $x \in \left(-\frac{\pi}{4};\frac{\pi}{2}\right)$, $y_{max} = 1$, $y_{min} = -1$;

в) $x \in \left[-\frac{\pi}{4};\frac{\pi}{4}\right]$, $y_{max} = 1$, $y_{min} = -1$; **г)** $x \in (0;\pi]$, $y_{max} = 1$, $y_{min} = -1$.

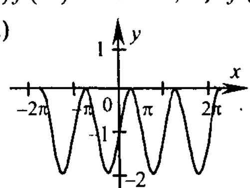
243. а) $x \in [0;+\infty)$, $y_{max} = 1$, $y_{min} = -1$; **б)** $x \in (-\infty;\pi)$, $y_{max} = 1$, $y_{min} = -1$;

в) $x \in \left(-\infty;\frac{\pi}{2}\right]$, $y_{max} = 1$, $y_{min} = -1$; **г)** $x \in \left(\frac{\pi}{3};+\infty\right)$, $y_{max} = 1$, $y_{min} = -1$.

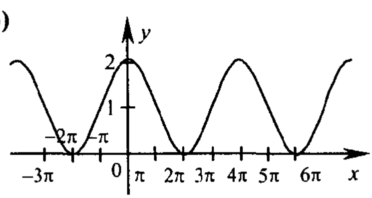
244. а) $f(-x) = \cos\frac{x}{3}$; **б)** $3f(x) = 3\cos\frac{x}{3}$; **в)** $f(-3x) = \cos x$; **г)** $f(-x) - f(x) = 0$.

245. а) $f(-x) = -\sin 2x$; **б)** $2f(x) = 2\sin 2x$; **в)** $f(-3x) = -\sin 6x$; **г)** $f(-x) + f(x) = 0$.

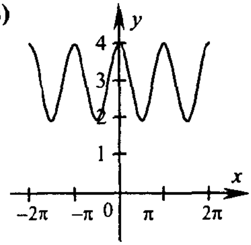
246. а)



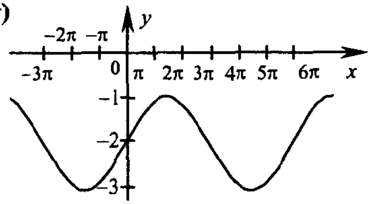
б)



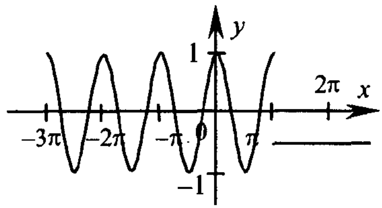
в)



г)



247. а) $f(x)=\begin{cases}\cos 2x, & x\le\pi\\ -\dfrac{1}{2}, & x>\pi\end{cases}$



1) Область определения $D(f)=R$

2) Область значений $E(f)=[-1;1]$

3) Функция периодична при и $x\le\pi$, $T=\pi$

4) Функция ни четная, ни нечетная

5) $f(x)=0$ при $x=-\dfrac{\pi}{4}-\dfrac{\pi n}{2}$, $n\ge 0$, $x=\dfrac{3\pi}{4}$;

6) $f_{\min}=-1, f_{\max}=1$

7) Промежутки знакопостоянства:

$f(x)<0$ при $x\in\left(\dfrac{\pi}{4}-\pi n;\dfrac{2\pi}{4}-\pi n\right)\cup(\pi;+\infty)$, $n\ge 0$,

$f(x)>0$ при $x\in\left(\dfrac{\pi}{4}-\pi n;\dfrac{\pi}{4}-\pi n\right)\cup\left(\dfrac{3\pi}{4};\pi\right]$, $n\ge 0$,

8) Функция возрастаст при $x\in\left[\dfrac{\pi}{2}-\pi n;\pi-\pi n\right]$, $n\ge 0$

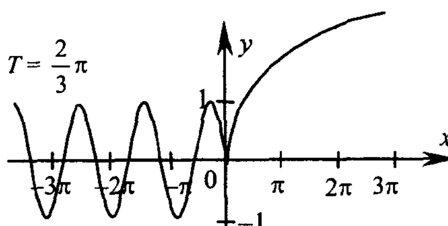
убывает при $x\in\left[-\pi n;\dfrac{\pi}{2}-\pi n\right]$, $n\ge 0$.

б) $f(x)=\begin{cases}-\sin 3x, & x<0\\ \sqrt{x}, & x\ge 0\end{cases}$

1) Область определения $D(f)=R$

2) Область значений $E(f)=[-1;+\infty)$

3) Функция периодична при и $x\le 0$, $T=\dfrac{2}{3}\pi$



4) Функция ни четная, ни нечетная

5) $f(x)=0$ при $x=-\dfrac{\pi n}{3}$, $n\ge 0$,

6) $f_{\min}=-1, f_{\max}=+\infty$

7) Промежутки знакопостоянства:

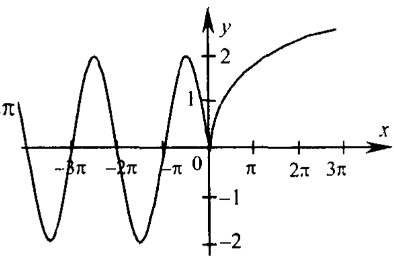
$f(x)<0$ при $x\in\left(-\dfrac{2\pi}{3};\dfrac{\pi}{3}-\dfrac{2\pi n}{3}\right)$, $n\ge 1$,

$f(x)>0$ при $x\in\left(-\dfrac{\pi}{3}-\dfrac{2\pi n}{\cdot 3};-\dfrac{2\pi n}{3}\right)$, $n\ge 0, x\ge 0$

8) Функция возрастает при $x \in \left[\frac{\pi}{6}-\frac{2\pi n}{3};\frac{\pi}{3}-\frac{2\pi n}{3}\right]$, $n \ge 1$,

убывает при $x \in \left[-\frac{\pi}{6}-\frac{3\pi n}{2};\frac{\pi}{6}-\frac{3\pi n}{2}\right]$, $n \ge 1$, $x \in \left[-\frac{\pi}{3};0\right)$.

248. а) $f(x)=\begin{cases}-2\sin x, & x<0\\ \sqrt{2x}, & x\ge 0\end{cases}$

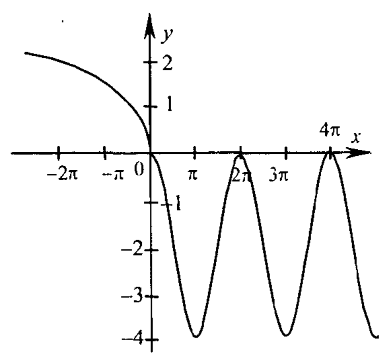


1) Область определения $D(f)=R$
2) Область значений $E(f)=[-2;+\infty)$
3) Функция периодична при и $x \le 0$, $T=2\pi$
4) Функция ни четная, ни нечетная
5) $f(x)=0$ при $x=-\pi n$, $n \ge 0$;
6) $f_{\min}=-2$, $f_{\max}=+\infty$
7) Промежутки знакопостоянства:
$f(x)<0$ при $x \in (-2\pi n; -2\pi n+\pi)$, $n \ge 1$,
$f(x)>0$ при $x \in (-2\pi n-\pi; -2\pi n)$, $n \ge 0$,

8) Функция возрастает при $x \in \left[-2\pi n+\frac{\pi}{2};-2\pi n+\frac{3\pi}{2}\right]\cup[0;+\infty)$, $n \ge 1$

убывает при $x \in \left[-2\pi n-\frac{\pi}{2};-2\pi n+\frac{\pi}{2}\right]\cup\left[-\frac{\pi}{2};0\right)$, $n \ge 1$.

б) $f(x)=\begin{cases}\sqrt{-x}, & x\le 0\\ 3\cos x-3, & x>0\end{cases}$



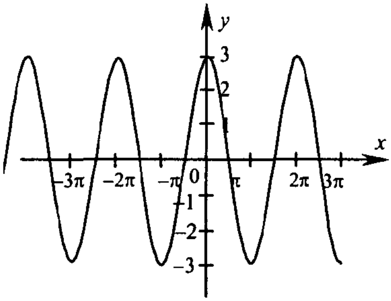
1) Область определения $D(f)=R$
2) Область значений $E(f)=[-6;+\infty)$
3) Функция периодична при и $x \ge 0$, $T=2\pi$
4) Функция ни четная, ни нечетная
5) $f(x)=0$ при $x=2\pi n$, $n \ge 0$,
6) $f_{\min}=-6$, $f_{\max}=+\infty$
7) Промежутки знакопостоянства:
$f(x)<0$ при $x \ne 2\pi n$, $n \ge 0$,
$f(x)>0$ при $x<0$;
8) Функция возрастает при $x \in (-2\pi n-\pi; 2\pi n)$, $n \ge 1$,
убывает при $x \in (2\pi n; 2\pi n+\pi)$, $n \ge 0$, $x \le 0$.

249. а) $y=\begin{cases}-x, & x<0\\ \sin 2x, & x\ge 0\end{cases}$; б) $y=\begin{cases}\cos 3x, & x\in\left[-\frac{\pi}{6};\frac{\pi}{3}\right]\\ -1, & x>\frac{\pi}{3}\end{cases}$;

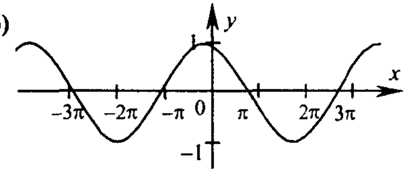
в) $y=\begin{cases}\sin 2x, & x<0\\ 2\cos x, & x>0\end{cases}$; г) $y=\begin{cases}-2\sin x, & x\in[-2\pi;0]\\ \cos\frac{x}{2}, & x\in(0;3\pi]\end{cases}$.

§14. График гармонического колебания

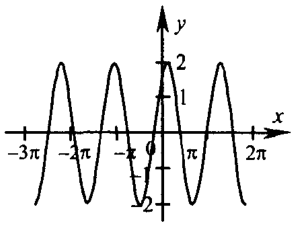
250. а)



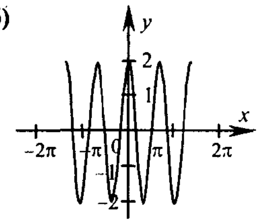
б)



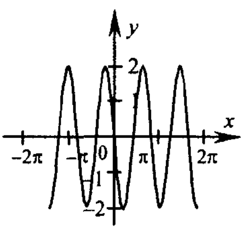
251. а)



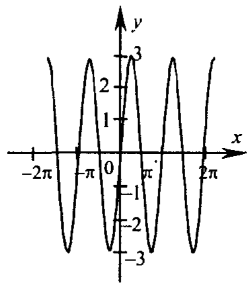
б)



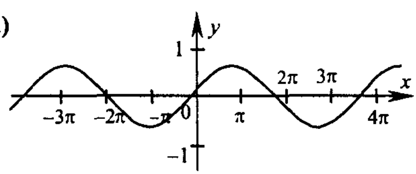
252. а)



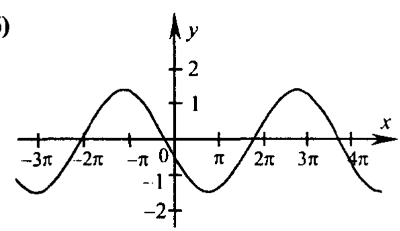
б)



253. а)



б)



§15. Функции y = tg x, y = ctg x, их свойства и графики

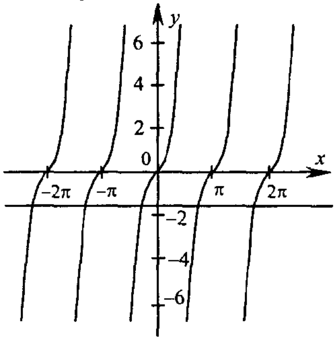
254. а) $\operatorname{tg}\dfrac{\pi}{4}=1$; **б)** $\operatorname{tg}\dfrac{2\pi}{3}=-\sqrt{3}$; **в)** $\operatorname{tg}\dfrac{3\pi}{4}=-1$; **г)** $\operatorname{tg}\pi=0$.

255. $t=\operatorname{tg}x$

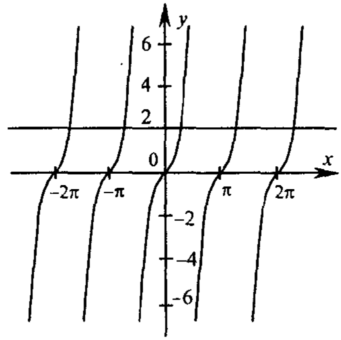
а) $x\in\left(\dfrac{\pi}{2};\dfrac{3\pi}{2}\right)$, $y_{\max}=-$, $y_{\min}=-$; **б)** $x\in\left(\dfrac{3\pi}{4};\pi\right]$, $y_{\max}=0$, $y_{\min}=-$;

в) $x\in\left[-\dfrac{\pi}{4};\dfrac{\pi}{6}\right]$, $y_{\max}=\dfrac{\sqrt{3}}{3}$, $y_{\min}=-1$; **г)** $x\in\left[\pi;-\dfrac{3\pi}{2}\right)$, $y_{\max}=-$, $y_{\min}=0$.

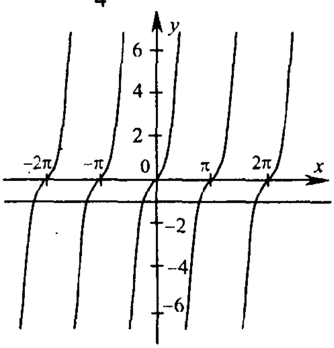
256. **а)** $x = \dfrac{\pi}{3} + \pi n,\ n \geq 0$;



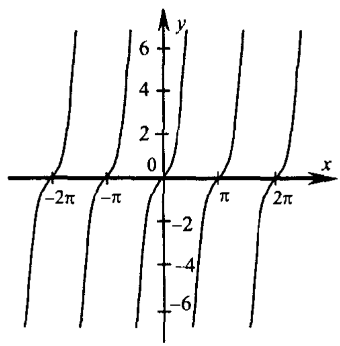
б) $x = \dfrac{\pi}{4} + \pi n$;



в) $x = -\dfrac{\pi}{4} - \pi n$;



г) $x = \pi n$.

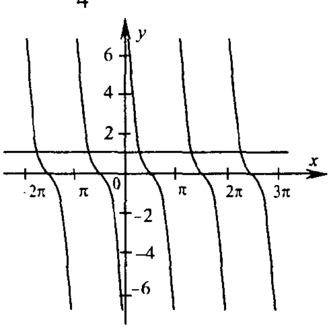


257. **а)** $\operatorname{ctg}\dfrac{\pi}{4} = 1$; **б)** $x = \operatorname{ctg}\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{3}$; **в)** $\operatorname{ctg} 2\pi = –$; **г)** $\operatorname{ctg}\dfrac{\pi}{2} = 0$.

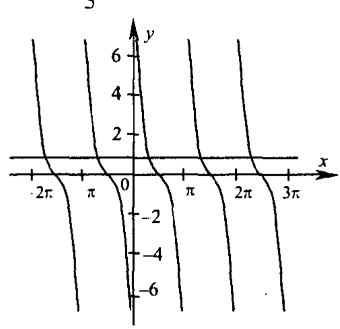
258. **а)** $x \in \left[\dfrac{\pi}{4}; \dfrac{\pi}{2}\right]$, $y_{\max} = 1$, $y_{\min} = 0$; **б)** $x \in \left[\dfrac{\pi}{2}; \pi\right]$, $y_{\max} = –$, $y_{\min} = 0$;

в) $x \in [-\pi; 0]$, $y_{\max} = –$, $y_{\min} = –$; **г)** $x \in \left[\dfrac{\pi}{6}; \dfrac{3\pi}{4}\right]$, $y_{\max} = \sqrt{3}$, $y_{\min} = -1$.

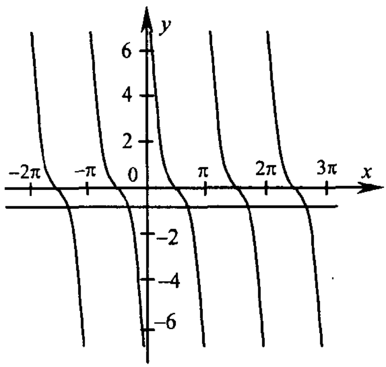
259. **а)** $x = \dfrac{\pi}{4} + \pi k$;



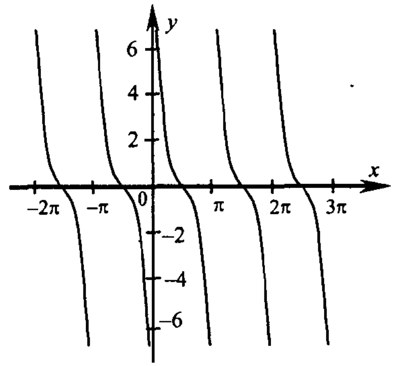
б) $x = \dfrac{\pi}{3} + \pi k$;



в) $x = -\dfrac{\pi}{3} + \pi k$;



г) $x = \dfrac{\pi}{2} + \pi k$.



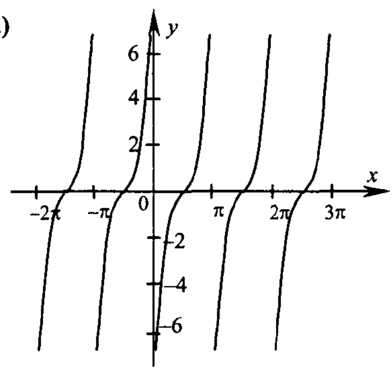
260. а) $f(x) = \operatorname{tg} x - \operatorname{ctg} x, f(-x) = -\operatorname{tg} x - \cos x$, ни четная, ни нечетная;

б) $f(x) = \operatorname{tg} x + x, f(-x) = -\operatorname{tg} x - x = -f(x)$, нечетная;

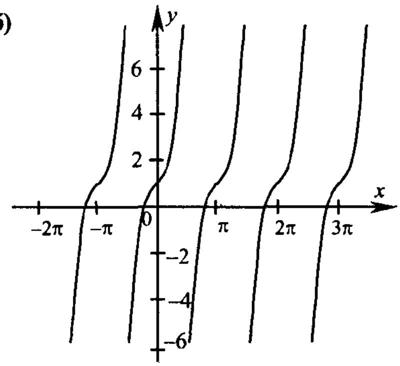
в) $f(x) = \operatorname{ctg}^2 x - x^4, f(-x) = \operatorname{ctg}^2 x - x^4$, четная;

г) $f(x) = x^3 - \operatorname{ctg} x, f(-x) = -x^3 + \operatorname{ctg} x = -f(x)$, нечетная.

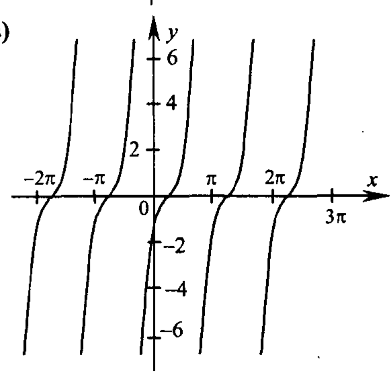
261. а)



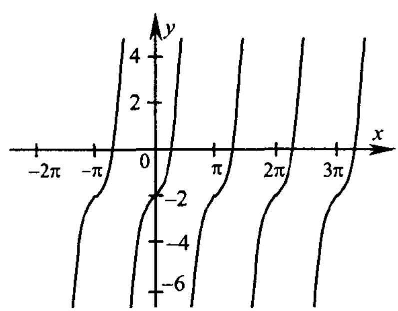
б)



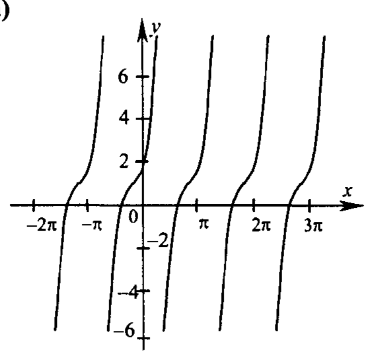
в)



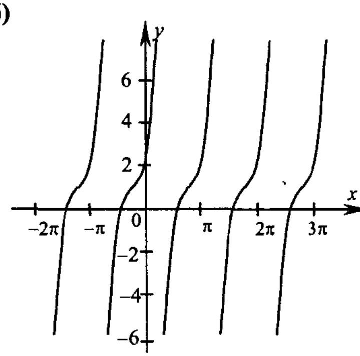
г)



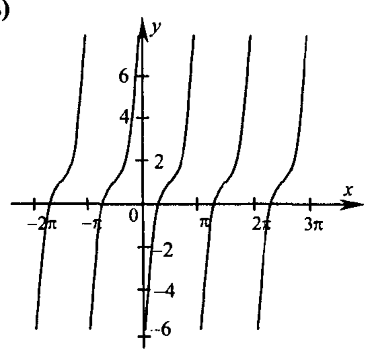
262. а)



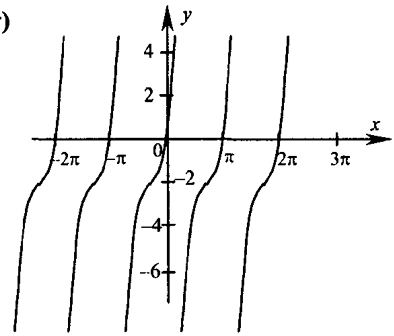
б)



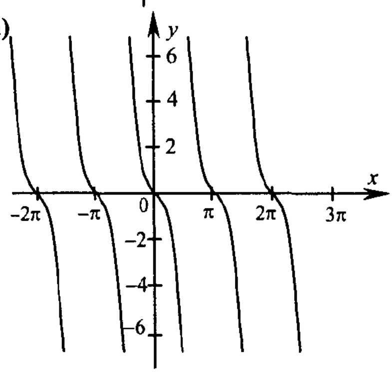
в)



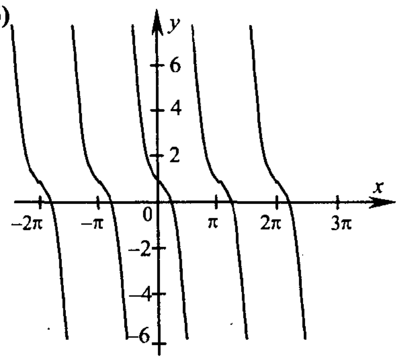
г)



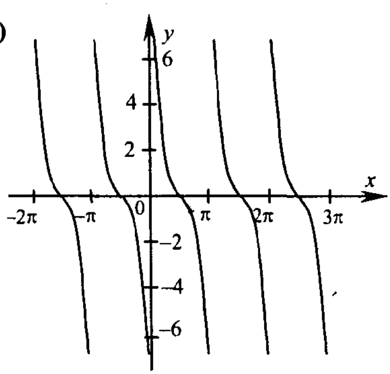
263. а)



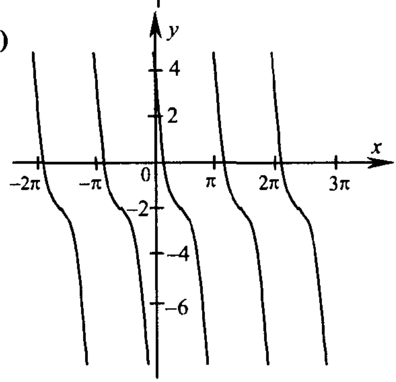
б)



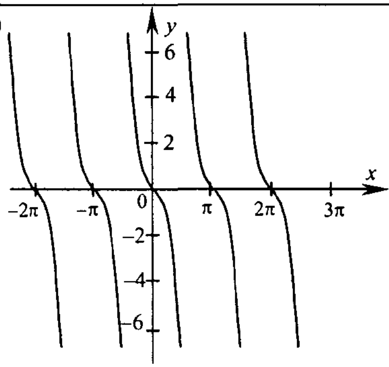
в)



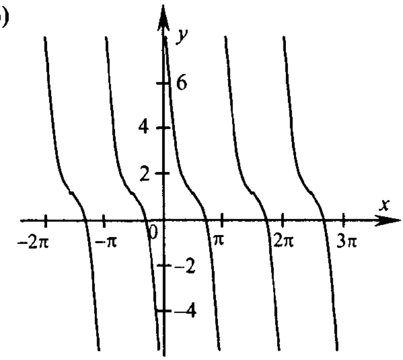
г)



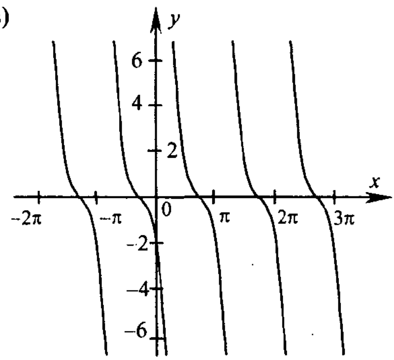
264. а)



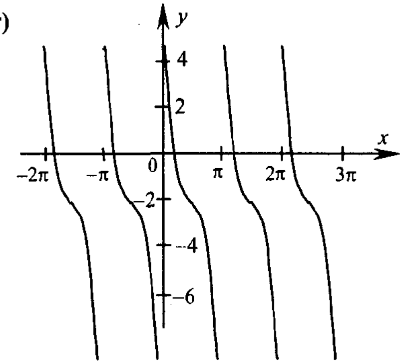
б)



в)



г)



265. а) $y=\operatorname{tg}2x$, $T=\frac{\pi}{2}$, $y(x+T)=\operatorname{tg}(2x+\pi)=\operatorname{tg}x$;

б) $y=\operatorname{tg}\frac{x}{3}$, $T=3\pi$, $y(x+T)=\operatorname{tg}\left(\frac{x}{3}+\pi\right)=\operatorname{tg}\frac{x}{3}$;

в) $y=\operatorname{tg}5x$, $T=\frac{\pi}{5}$, $y(x+T)=\operatorname{tg}(5x+\pi)=\operatorname{tg}5x$;

г) $y=\frac{2x}{5}$, $T=\frac{5\pi}{2}$, $y(x+T)=\operatorname{tg}\left(\frac{2x}{5}+\pi\right)=\operatorname{tg}\frac{2x}{5}$.

266. а) $y=\operatorname{tg}x+\sin 2x-\operatorname{tg}3x-\cos 4x$, $T=\pi$,
$y(x+\pi)=\operatorname{tg}(x+\pi)+\sin(2x+2\pi)-\operatorname{tg}(3x-3\pi)-\cos(4x+4\pi)=y(x)$;
б) $y=\sin 3x+\cos 5x+\operatorname{ctg}x-2\operatorname{tg}2x$, $T=\pi$,
$y(x+\pi)=\sin(3x+3\pi)+\cos(5x+5\pi)+\operatorname{ctg}(\pi+x)-2\operatorname{tg}(2x+\pi)=$
$=-\sin 3x-\cos 5x+\operatorname{ctg}x-2\operatorname{tg}2x\neq y(x)$, $\Rightarrow$ π не есть период.

267. $\operatorname{tg}(9\pi-x)=-\frac{3}{4}$; $\operatorname{tg}(9\pi-x)=-\operatorname{tg}x$; $\operatorname{tg}x=\frac{3}{4}$, $\operatorname{ctg}x=\frac{4}{3}$.

268. $\operatorname{ctg}(7\pi-x)=\frac{5}{7}$; $\operatorname{ctg}x=-\frac{5}{7}$; $\operatorname{tg}=-\frac{7}{5}$.

269. а) $\operatorname{tg}200°-\operatorname{tg}201°<0$; б) $\operatorname{tg}1-\operatorname{tg}1{,}01<0$;

в) $\operatorname{tg}2{,}2-\operatorname{tg}2{,}1>0$; г) $\operatorname{tg}\frac{3\pi}{5}-\operatorname{tg}\frac{6\pi}{5}<0$.

270. а) $f(x) = \operatorname{tg} x \sin^2 x$, $f(-x) = -\operatorname{tg} x \cdot \sin^2 x = -f(x)$, нечетная;

б) $f(x) = \dfrac{\operatorname{tg}^2 x}{x^2 - 1}$, $\quad f(-x) = \dfrac{\operatorname{tg}^2 x}{x^2 - 1} = \mathrm{f}(x)$, четная;

в) $f(x) = x^5 \operatorname{tg} x$, $\quad f(-x) = x^5 \operatorname{tg} x = f(x)$, четная;

г) $f(x) = x^2 + \sin x + \operatorname{tg} x$, $\quad f(-x) = x^2 - \sin x - \operatorname{tg} x$, ни четная, ни нечетная.

271. а) $f(x) = \sin x + \operatorname{ctg} x$, $\quad f(-x) = -\sin x - \operatorname{ctg} x = -f(x)$, нечетная;

б) $f(x) = \dfrac{2 \operatorname{ctg} x}{x^3}$, $\quad f(-x) = \dfrac{-2 \operatorname{ctg} x}{-x^3} = f(x)$, чстная;

в) $f(x) = \dfrac{x^4 \operatorname{ctg} x}{x^2 - 4}$, $\quad f(-x) = -\dfrac{x^4 \operatorname{ctg} x}{x^2 - 4} = -f(x)$, нечетная;

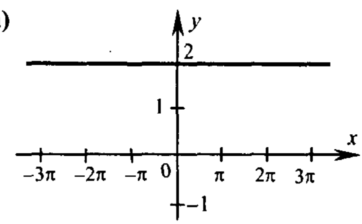
г) $f(x) = \operatorname{ctg} x - x\cos x$, $\quad f(-x) = -\operatorname{ctg} x + x\cos x = -f(x)$, нечетная.

272. $f(x) = \operatorname{tg} x$, $\quad f(2x + 2\pi) + f(7\pi - 2x) = \operatorname{tg}(2x + 2\pi) + \operatorname{tg}(7\pi - 2x) = \operatorname{tg} 2x - \operatorname{tg} 2x = 0.$

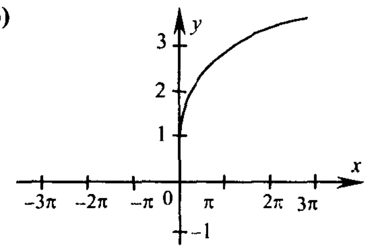
273. $f(x) = x^2 + 1$, $\quad f(\operatorname{tg} x) = \operatorname{tg}^2 x + 1 = \dfrac{1}{\cos^2 x}$.

274. $f(x) = x^2 + 1$, $\quad f(\operatorname{ctg} x) = \operatorname{ctg}^2 x + 1 = \dfrac{1}{\sin^2 x}$.

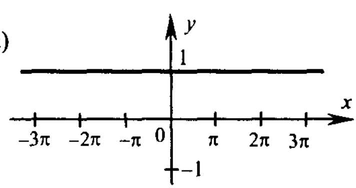
275. а)



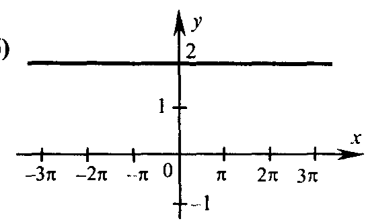
б)



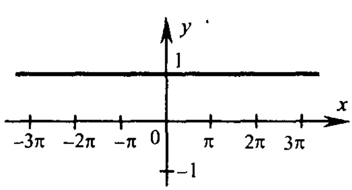
276. а)



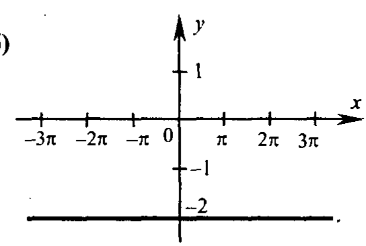
б)



277. а)



б)



Глава 2. Тригонометрические уравнения

§16. Первые представления о решении тригонометрических уравнений

278. а) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi n$; б) $\sin t = -\frac{1}{2}$, $t = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

в) $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$; г) $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k\frac{\pi}{4} + \pi k$.

279. а) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm\frac{\pi}{6} + 2\pi k$; б) $\sin t = -\frac{\sqrt{3}}{2}$, $t = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

б) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm\frac{5\pi}{6} + 2\pi k$; г) $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k\frac{\pi}{3} + \pi k$.

280. а) $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi k$; б) $\cos t = 2$, решений нет;

в) $\cos t = -1$, $t = \pi + 2\pi k$; г) $\sin t = -3$, решений нет.

281. а) $\operatorname{tg} t = \sqrt{3}$, $t = \frac{\pi}{3} + \pi n$; б) $\operatorname{ctg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{3} + 2\pi n$;

в) $\operatorname{tg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{6} + \pi n$; г) $\operatorname{ctg} t = \sqrt{3}$, $t = \frac{\pi}{6} + \pi n$.

282. а) $\sin t(2\cos t + 1) = 0$, $\left\{\begin{array}{l}\sin t = 0\\ \cos t = -\frac{1}{2}\end{array}\right.$, $t = \pi n$, $t = \pm\frac{2\pi}{3} + 2\pi n$;

б) $(\sin t - 1)(\cos t + 1) = 0$, $\left\{\begin{array}{l}\sin t = 1\\ \cos t = -1\end{array}\right.$, $t = \frac{\pi}{2} + 2\pi n$, $t = \pi + 2\pi n$;

в) $\cos t \cdot (2\sin t + 1) = 0$, $\left\{\begin{array}{l}\cos t = 0\\ \sin t = -\frac{1}{2}\end{array}\right.$, $t = \frac{\pi}{2} + \pi n$, $t = (-1)^{n+1}\frac{\pi}{6} + \pi n$;

г) $\left(2\sin t - \sqrt{2}\right)(2\cos t + 1) = 0$, $\left\{\begin{array}{l}\sin t = \frac{\sqrt{2}}{2}\\ \cos t = -\frac{1}{2}\end{array}\right.$, $t = (-1)^{n+1}\frac{\pi}{4} + \pi n$, $t = \pm\frac{2\pi}{3} + 2\pi n$.

283. а) $\cos\left(\frac{\pi}{2} - t\right) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$; б) $\cos(t - \pi) = 1$, $-\cos t = 1$, $t = \pi + 2\pi n$;

в) $\sin t(\pi - t) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$; г) $\sin\left(t - \frac{\pi}{2}\right) = 1$, $\cos t = -1$, $t = \pi + 2\pi n$.

284. а) $3 - 4\sin^2 t = 0$, $\sin t = \pm\frac{\sqrt{3}}{2}$, $t = (-1)^k\frac{\pi}{3} + \pi k$, $t = (-1)^{k-1}\frac{\pi}{3} + \pi k$;

б) $\sin^2 t - \sin t = 0$, $\left\{\begin{array}{l}\sin t = 0\\ \sin t = 1\end{array}\right.$, $t = \pi n$, $t = \frac{\pi}{2} + 2\pi n$;

в) $4\sin^2 t-1=0,\ \sin t=\pm\frac{1}{2},\ t=(-1)^k\frac{\pi}{6}+\pi k,\ t=(-1)^{k+1}\frac{\pi}{6}+\pi k$;

г) $2\sin^2 t+\sin t=0,\ \begin{cases}\sin t=0\\ \sin t=-\frac{1}{2}\end{cases},\ t=\pi n,\ t=(-1)^{k+1}\frac{\pi}{6}+\pi k$.

285. а) $3-4\cos^2 t=0\ \cos t=\pm\frac{\sqrt{3}}{2},\ t=\pm\frac{\pi}{6}+\pi n$;

б) $2\cos^2 t-\cos t=0,\ \begin{cases}\cos t=0\\ \cos t=\frac{1}{2}\end{cases},\ t=\frac{\pi}{2}+\pi n,\ t=\pm\frac{\pi}{3}+2\pi n$;

в) $4\cos^2 t-1=0,\ \cos t=\pm\frac{1}{2},\ t=\pm\frac{\pi}{3}+2\pi n,\ t=\pm\frac{2\pi}{3}+2\pi n$;

г) $2\cos^2 t+\cos t=0,\ \begin{cases}\cos t=0\\ \cos t=-\frac{1}{2}\end{cases},\ t=\frac{\pi}{2}+\pi n,\ t=\pm\frac{2\pi}{3}+2\pi n$.

286. а) $2\sin^2 t+3\sin t-2=0,\ \sin t=\frac{-3+\sqrt{9-4\cdot 2(-2)}}{4}=\frac{1}{2}$,

$t=(-1)^k\frac{\pi}{6}+\pi k,\ \sin t=-2$ не подходит;

б) $2\cos^2 t-5\cos t+2=0,\ \cos t=\frac{5+3}{4}=2$ – не подходит, $\cos t=\frac{1}{2},\ t=\pm\frac{\pi}{3}+2$

в) $2\sin^2 t+\sin t-1=0,\ \sin t=\frac{-1+3}{4}=\frac{1}{2},\ t=(-1)^k\frac{\pi}{6}+\pi k$,

$\sin t=-1,\ t=-\frac{\pi}{2}+2\pi n$;

г) $4\cos^2 t+9\cos t+5=0,\ \cos t=\frac{-9+1}{8}=-1,\ t=\pi+2\pi n$,

$\cos t=\frac{-9-1}{8}$ – не подходит.

287. а) $2\cos^2 t+\sin t+1=0,\ 2-2\sin^2 t+\sin t+1=0,\ 2\sin^2 t-\sin t-3=0$,

$\sin t=\frac{1+5}{4}$ не подходит, $\sin t=\frac{1-5}{4}=-1,\ t=-\frac{\pi}{2}+2\pi k$;

б) $\sin^2 t+\cos t-3=0,\ \cos^2 t-3\cos t+2=0,\ \cos t=2$ не подходит,

$\cos t=1,\ t=2\pi n$.

288. а) $\sin\left(\frac{\pi}{2}+t\right)-\cos(\pi+t)=1,\ \cos t+\cos t=1,\ \cos t=\frac{1}{2},\ t=\pm\frac{\pi}{3}+2\pi n$;

б) $\sin(\pi+t)+\sin(2\pi-t)-\cos\left(\frac{3\pi}{2}+t\right)+1{,}5=0$,

$-\sin t - \sin t - \sin t = -\frac{3}{2}$, $\sin t = \frac{1}{2}$, $t = (-1)^k \frac{\pi}{6} + \pi k$;

в) $\cos\left(\frac{\pi}{2} - t\right) - \sin(\pi + t) = \sqrt{2}$, $\sin t + \sin t = \sqrt{2}$, $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$;

г) $\sin(\pi + t) + \cos\left(\frac{\pi}{2} + t\right) = \sqrt{3}$, $-\sin t - \sin t = \sqrt{3}$, $\sin t = -\frac{\sqrt{3}}{3}$, $t = (-1)^{k+1} \frac{\pi}{3} + \pi k$.

§17. Арккосинус и решение уравнения cos t = a

289. а) $\arccos 0 = \frac{\pi}{2}$; б) $\arccos 1 = 0$; в) $\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$; г) $\arccos\frac{1}{2} = \frac{\pi}{3}$.

290. а) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$; б) $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$;

в) $\arccos(-1) = \pi$; г) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

291. а) $\arccos(-1) + \arccos 0 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$; б) $\arccos\frac{1}{2} - \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$;

в) $\arccos\left(-\frac{\sqrt{2}}{2}\right) + \arccos\frac{\sqrt{2}}{2} = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$; г) $\arccos\left(-\frac{1}{2}\right) - \arccos\frac{1}{2} = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$.

292. а) $\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; б) $\operatorname{tg}\left(\arccos\frac{\sqrt{3}}{2}\right) = \operatorname{tg}\frac{\pi}{6} = \frac{\sqrt{3}}{3}$;

в) $\operatorname{ctg}(\arccos 0) = \operatorname{ctg}\frac{\pi}{2} = 0$; г) $\sin\left(\arccos\frac{\sqrt{2}}{2}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

293. а) $\cos t = \frac{1}{2}$, $t = \pm\frac{\pi}{3} + 2\pi n$; б) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi n$;

в) $\cos t = 1$, $t = 2\pi n$; г) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm\frac{\pi}{6} + 2\pi n$.

294. а) $\cos t = -1$, $t = \pi + 2\pi n$; б) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm\frac{5\pi}{6} + 2\pi n$;

в) $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$; г) $\cos t = -\frac{\sqrt{2}}{2}$, $t = \pm\frac{3\pi}{4} + 2\pi n$.

295. а) $\cos t = \frac{1}{3}$, $t = \pm\arccos\frac{1}{3} + 2\pi n$; б) $\cos t = -1{,}1$, решений нет;

в) $\cos t = -\frac{3}{7}$, $t = \pm\arccos\left(-\frac{3}{7}\right) + 2\pi n$; г) $\cos t = 2{,}04$, решений нет.

296. а) $\cos t\left(2\arccos\frac{1}{2}-3\arccos 0-\arccos\left(-\frac{1}{2}\right)\right)=\cos\left(\frac{2\pi}{3}-\frac{3\pi}{2}-\frac{2\pi}{3}\right)=\cos\frac{3\pi}{2}=0$;

б) $\frac{1}{3}\left(\arccos\frac{1}{3}+\arccos\left(-\frac{1}{3}\right)\right)=\frac{1}{3}\pi=\frac{\pi}{3}$.

297. а) $x\in[-1;1]$; **б)** $|x|\le\frac{1}{2}$; **в)** $x\in[0;2]$; **г)** $x\in[1;2]$.

298. а) $\sqrt{5}>1$, нет; **б)** $s\frac{\sqrt{2}}{3}<1$, да; **в)** $\frac{\pi}{5}<1$, да; **г)** $-\sqrt{3}<-1$, нет.

299. $\operatorname{tg}(\arccos 0{,}1+\arccos(-0{,}1)+x)=\operatorname{tg}x$, $\operatorname{tg}(\pi+x)=\operatorname{tg}x$.

300. а) $\frac{8\cos t-3}{3\cos t+2}=1$, $\frac{8\cos t-3-3\cos t-2}{3\cos t+2}=0$, $\begin{cases}5\cos t-5=0\\ \cos t\ne-\frac{2}{3}\end{cases}$, $\cos t=1$, $t=2\pi n$;

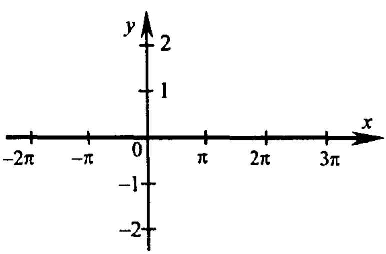
б) $\frac{3\cos t+1}{2}+\frac{5\cos t-1}{3}=1\frac{3}{4}$, $9\cos t+3+10\cos t-2=\frac{7}{4}\cdot 6=\frac{21}{2}$,

$19\cos t=\frac{19}{2}$, $\cos t=\frac{1}{2}$, $t=\pm\frac{\pi}{3}+2\pi n$.

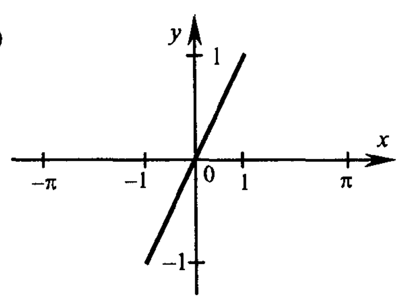
301. $6\cos^2 t+5\cos t+1=0$, $\cos t=\frac{-5+1}{12}=-\frac{1}{3}$, $t=\pm\arccos\left(-\frac{1}{3}\right)+2\pi n$,

$\cos t=\frac{-5-1}{12}=-\frac{1}{2}$, $t=\pm\frac{2\pi}{3}+2\pi n$.

302. а)



б)



303. а) $\cos t>\frac{1}{2}$, $t\in\left(-\frac{\pi}{3}+2\pi k;\frac{\pi}{3}+2\pi k\right)$;

б) $\cos t\le-\frac{\sqrt{2}}{2}$, $t\in\left[\frac{3\pi}{4}+2\pi k;\frac{5\pi}{4}+2\pi k\right]$;

в) $\cos t\ge-\frac{\sqrt{2}}{2}$, $t\in\left[-\frac{5\pi}{4}+2\pi k;-\frac{3\pi}{4}+2\pi k\right]$;

г) $\cos t<\frac{1}{2}$, $t\in\left(\frac{\pi}{3}+2\pi k;\frac{5\pi}{3}+2\pi k\right)$.

304. а) $\cos t < \frac{2}{3}$, $t \in \left(\arccos\frac{2}{3} + 2\pi k; 2\pi - \arccos\frac{2}{3} + 2\pi k\right)$;

б) $\cos t > -\frac{1}{7}$, $t \in \left(-\arccos\left(-\frac{1}{7}\right) + 2\pi k; \arccos\left(-\frac{1}{7}\right) + 2\pi k\right)$;

в) $\cos t > \frac{2}{3}$, $t \in \left(-\arccos\frac{2}{3} + 2\pi k; \arccos\frac{2}{3} + 2\pi k\right)$;

г) $\cos t < -\frac{1}{7}$, $t \in \left(\arccos\left(-\frac{1}{7}\right) + 2\pi k; 2\pi - \arccos\left(-\frac{1}{7}\right) + 2\pi k\right)$.

305. а) $3\cos^2 t - 4\cos t \geq 4$, $3\cos^2 t - 4\cos t - 4 = 0$.

Найдем корни квадратного уравнения:

$\cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{4 \pm 8}{6}$, $\cos t = -\frac{2}{3}$, $\cos t = 2$ не подходит,

$\cos t \leq -\frac{2}{3}$, $t \in \left(\arccos\left(-\frac{2}{3}\right) + 2\pi k; 2\pi - \arccos\left(-\frac{2}{3}\right) + 2\pi k\right)$;

б) $6\cos^2 t + 1 > 5\cos t$.

Найдем корни квадратного уравнения:

$6\cos^2 t - 5\cos t + 1 = 0$, $\cos t = \frac{5+1}{12} = \frac{1}{2}$, $\cos t = \frac{1}{3}$,

$t \in \left(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k\right) \cup \left(\arccos\frac{1}{3} + 2\pi k; 2\pi - \arccos\frac{1}{3} + 2\pi k\right)$;

в) $3\cos^2 t - 4\cos t < 4$, $3\cos^2 t - 4\cos t - 4 < 0$.

Найдем корни квадратного уравнения:

$3\cos^2 t - 4\cos t - 4 = 0$, $\cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{2 \pm 4}{3}$,

$\cos t = 2$ – не подходит, $\cos t = -\frac{2}{3} \rightarrow \cos t > -\frac{2}{3}$,

$t \in \left(-\arccos\left(-\frac{2}{3}\right) + 2\pi k; -\left(-\frac{2}{3}\right) + 2\pi k\right)$;

г) $6\cos^2 t + 1 \leq 5\cos t$, $6\cos^2 t - 5\cos t + 1 \leq 0$.

Найдем корни квадратного уравнения:

$6\cos^2 t - 5\cos t + 1 = 0$, $\cos t = \frac{5 \pm \sqrt{25 - 4 \cdot 6 \cdot 1}}{12} = \frac{5 \pm 1}{12}$, $\cos t = \frac{1}{2}$, $\cos t = \frac{1}{3}$,

$t \in \left(-\arccos\frac{1}{3} + 2\pi k; -\frac{\pi}{3} + 2\pi k\right) \cup \left(\frac{\pi}{3} + 2\pi k; \arccos\frac{1}{3} + 2\pi k\right)$.

306. а) $4\cos^2 t < 1$, $\cos^2 t < \frac{1}{4}$, $\cos t \in \left(-\frac{1}{2}; \frac{1}{2}\right)$, $t \in \left(\frac{\pi}{3} + \pi k; \frac{2\pi}{3} + \pi k\right)$;

б) $3\cos^2 t < \cos t$, $\cos t(3\cos t - 1) < 0$, $\cos t \in \left(0; \frac{1}{3}\right)$,

$$t\in\left(-\frac{\pi}{2}+2\pi n;-\arccos\frac{1}{3}+2\pi n\right)\cup\left(\arccos\frac{1}{3}+2\pi n;\frac{\pi}{2}+2\pi n\right).$$

307. а) $\sin\left(\arccos\frac{3}{5}\right)=\sqrt{1-\cos^2\left(\arccos\frac{3}{5}\right)}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$;

б) $\sin\left(\arccos\left(-\frac{4}{5}\right)\right)=\sqrt{1-\cos^2\left(\arccos\left(-\frac{4}{5}\right)\right)}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$.

308. а) $\operatorname{tg}\left(\arccos\left(-\frac{15}{3}\right)\right)=\dfrac{\sqrt{1-\cos^2\left(\arccos\left(-\frac{5}{13}\right)\right)}}{\cos\left(\arccos\left(-\frac{5}{13}\right)\right)}=\dfrac{\sqrt{1-\frac{25}{169}}}{\frac{-5}{13}}=-\frac{12}{5}$;

б) $\operatorname{ctg}\left(\arccos\frac{4}{5}\right)=\dfrac{\cos\left(\arccos\frac{4}{5}\right)}{\sqrt{1-\cos^2\left(\arccos\frac{4}{5}\right)}}=\frac{4\cdot 5}{5\cdot 3}=\frac{4}{3}$.

§18. Арксинус и решение уравнения sin t = a

309. а) $\arcsin\frac{\sqrt{3}}{2}=\frac{\pi}{3}$; **б)** $\arcsin 1=\frac{\pi}{2}$; **в)** $\arcsin\frac{\sqrt{2}}{2}=\frac{\pi}{4}$; **г)** $\arcsin 0=0$.

310. а) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$; **б)** $\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$;

в) $\arcsin(-1)=-\frac{\pi}{2}$; **г)** $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$.

311. а) $\arcsin 0+\arccos 0=\frac{\pi}{2}$; **б)** $\arcsin\frac{\sqrt{3}}{2}+\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{2}$;

в) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)+\arccos\frac{1}{2}=\frac{\pi}{12}$; **г)** $\arcsin(-1)+\arccos\frac{\sqrt{3}}{2}=-\frac{\pi}{3}$.

312. а) $\arccos\left(-\frac{1}{2}\right)+\arcsin\left(-\frac{1}{2}\right)=\frac{\pi}{2}$; **б)** $\arccos\left(-\frac{\sqrt{2}}{2}\right)-\arcsin(-1)=\frac{5\pi}{4}$;

в) $\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arcsin\left(-\frac{\sqrt{3}}{2}\right)=\frac{\pi}{2}$; **г)** $\arccos\frac{\sqrt{2}}{2}-\arcsin\left(-\frac{\sqrt{3}}{2}\right)=\frac{7\pi}{12}$.

313. а) $\sin t=\frac{\sqrt{3}}{2},\ t=(-1)^k\frac{\pi}{3}+\pi k$; **б)** $\sin t=\frac{\sqrt{2}}{2},\ t=(-1)^k\frac{\pi}{4}+\pi k$;

в) $\sin t=1,\ t=\frac{\pi}{2}+2\pi n$; **г)** $\sin t=\frac{1}{2},\ t=(-1)^k\frac{\pi}{6}+\pi k$.

314. **а)** $\sin t=-1,\ t=-\frac{\pi}{2}+2\pi n$; **б)** $\sin t=-\frac{\sqrt{2}}{2},\ t=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

в) $\sin t=-\frac{1}{2},\ t=(-1)^{k+1}\frac{\pi}{6}+\pi k$; **г)** $\sin t=-\frac{\sqrt{3}}{2},\ t=(-1)^{k+1}\frac{\pi}{3}+\pi k$.

315. **а)** $\sin t=\frac{1}{4},\ t=(-1)^k\arcsin\frac{1}{4}+\pi k$; **б)** $\sin t=1{,}02$, решения нет;

в) $\sin t=-\frac{1}{7},\ t=(-1)^k\arcsin\left(-\frac{1}{7}\right)+\pi k$; **г)** $\sin\frac{\pi}{3}$, решения нет.

316. **а)** $\sin(\arccos x+\arccos(-x))=0,\ \sin\pi=0$;

б) $\cos(\arcsin x+\arcsin(-x))=1,\ \cos 0=1$.

317. **а)** $\sin\left(2\arcsin\frac{1}{2}-3\arccos\left(-\frac{1}{2}\right)\right)=\sin\left(\frac{\pi}{3}-2\pi\right)=\frac{\sqrt{3}}{2}$;

б) $\cos\left(\frac{1}{2}\arcsin 1+\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)=\cos\left(\frac{\pi}{4}-\frac{\pi}{4}\right)=1$.

318. **а)** $\operatorname{tg}\left(\arcsin\frac{\sqrt{3}}{2}+2\arccos\frac{\sqrt{2}}{2}\right)=\operatorname{tg}\left(\frac{\pi}{3}+\frac{\pi}{2}\right)=-\frac{\sqrt{3}}{3}$;

б) $\operatorname{ctg}\left(3\arccos(-1)-\arcsin\left(-\frac{1}{2}\right)\right)=\operatorname{ctg}\left(3\pi+\frac{\pi}{6}\right)=\sqrt{3}$.

319. **а)** $\arcsin x,\ x\in[-1;1]$; **б)** $\arcsin(5-2x),\ x\in[2;3]$;

в) $\arcsin\frac{x}{2},\ x\in[-2;2]$; **г)** $\arcsin\left(x^2-3\right),\ x\in\left[-2;-\sqrt{2}\right]\cup\left[\sqrt{2};2\right]$.

320. **а)** $-\frac{2}{3}>-1$, да; **б)** $1{,}5>1$, нет; **в)** $3-\sqrt{20}<-1$, нет; **г)** $4-\sqrt{20}<1$, да.

321. **а)** $(2\cos x+1)\left(2\sin x-\sqrt{3}\right)=0$,

$$\left[\begin{aligned}&\cos x=-\frac{1}{2}\\&\sin x=\frac{\sqrt{3}}{2}\end{aligned}\right.,\quad x=\pm\frac{2\pi}{3}+2\pi k,\quad x=(-1)^k\frac{\pi}{3}+2\pi k;$$

б) $2\cos x-3\sin x\cos x=0,\ \cos x(2-3\sin x)=0$,

$$\left[\begin{aligned}&\cos x=0\\&\sin x=\frac{2}{3}\end{aligned}\right.,\quad x=\frac{\pi}{2}+\pi n,\quad x=(-1)^k\arcsin\frac{2}{3}+\pi k;$$

в) $4\sin^2 x-3\sin x=0,\ \sin x(4\sin x-3)=0,\ \sin x=0,\ \sin x=\frac{3}{4}$,

$x=\pi n,\ x=(-1)^k\arcsin\frac{3}{4}+\pi k$;

г) $2\sin^2 x-1=0,\ \sin x=\pm\frac{\sqrt{2}}{2},\ x=\frac{\pi}{4}+\frac{\pi n}{2}$.

322. **а)** $6\sin^2 x+\sin x-2=0,\ \sin x=\frac{-1+7}{12}=\frac{1}{2},\ (-1)^n\arcsin\frac{\pi}{6}+\pi n$,

$\sin x=-\frac{2}{3},\ (-1)^{k+1}\arcsin\frac{2}{3}+\pi k$;

б) $3\cos^2 x=7(\sin x+1),\ 3-3\sin^2 x=7\sin x+7,\ 3\sin^2 x+7\sin x+4=0$,

$\sin x=\frac{-7\pm\sqrt{49-4\cdot3\cdot4}}{6}=\frac{-7\pm1}{6}$,

$\sin x=\frac{-8}{6}$ – не подходит, $\sin x=-1,\ x=-\frac{\pi}{2}+2\pi n$.

323. **а)** $\sin t>\frac{\sqrt{3}}{2},\ t\in\left(\frac{\pi}{3}+2\pi k;\frac{2\pi}{3}+2\pi k\right)$; **б)** $\sin t>-\frac{1}{2},\ t\in\left(-\frac{\pi}{6}+2\pi k;\frac{7\pi}{6}+2\pi k\right)$;

в) $\sin t<\frac{\sqrt{3}}{2},\ t\in\left(-\frac{4\pi}{3}+2\pi k;\frac{\pi}{3}+2\pi k\right)$; **г)** $\sin t\le-\frac{1}{2},\ t\in\left(\frac{7\pi}{6}+2\pi k;\frac{11\pi}{6}+2\pi k\right)$.

324. **а)** $\sin t<\frac{1}{3},\ t\in\left(\pi-\arcsin\frac{1}{3}+2\pi k;\arcsin\frac{1}{3}+2\pi k\right)$;

б) $\sin t\ge-\frac{3}{5},\ t\in\left(-\arcsin\frac{3}{5}+2\pi k;\pi+\arcsin\frac{3}{5}+2\pi k\right)$;

в) $\sin t\ge\frac{1}{3},\ t\in\left(\arcsin\frac{1}{3}+2\pi k;\pi-\arcsin\frac{1}{3}+2\pi k\right)$;

г) $\sin t<-\frac{3}{5},\ t\in\left(\pi+\arcsin\frac{3}{5}+2\pi k;2\pi-\arcsin\frac{3}{5}+2\pi k\right)$.

325. **а)** $5\sin^2 t>11\sin t+12,\ 5\sin^2 t-11\sin t-12=0$,

$\sin t=\frac{11+19}{10}$, не подходит, $\sin t=-\frac{8}{10}$,

$t\in\left(\pi+\arcsin\frac{4}{5}+2\pi k;2\pi-\arcsin\frac{4}{5}+2\pi k\right)$;

б) $5\sin^2 t\le11t+12,\ 5\sin^2 t-11t-12=0$,

$\sin t=-\frac{4}{5},\ t\in\left(-\arcsin\frac{4}{5}+2\pi k;\pi+\arcsin\frac{4}{5}+2\pi k\right)$.

326. **а)** $6\cos^2 t+\sin t>4,\ 6-6\sin^2 t+\sin t-4>0,\ 6\sin^2 t-\sin t-2<0$,

$\sin t=\frac{1+7}{12}=\frac{3}{4},\ \sin t=-\frac{1}{2}$,

$t \in \left(-\frac{\pi}{6}+2\pi k; \arcsin\frac{2}{3}+2\pi k\right) \cup \left(\pi-\arcsin\frac{2}{3}+2\pi k; \frac{7\pi}{6}+2\pi k\right)$;

б) $6\cos^2 t+\sin t \le 4$, $6\sin^2 t-\sin t-2=0$, $\sin t=\frac{3}{4}$, $\sin t=-\frac{1}{2}$,

$t \in \left[\arcsin\frac{2}{3}+2\pi k; \pi-\arcsin\frac{2}{3}+2\pi k\right]$, $t \in \left[\frac{7\pi}{6}+2\pi k; \frac{11\pi}{6}+2\pi k\right]$.

327. **а)** $\cos\left(\arcsin\left(-\frac{5}{13}\right)\right)=\sqrt{1-\sin^2\left(\arcsin\left(-\frac{5}{13}\right)\right)}=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$;

б) $\operatorname{tg}\left(\arcsin\frac{3}{5}\right)=\frac{3}{5}\cdot\frac{5}{4}=\frac{3}{4}$; **в)** $\cos\left(\arcsin\frac{8}{17}\right)=\sqrt{1-\sin^2\left(\arcsin\left(-\frac{8}{17}\right)\right)}=\frac{15}{17}$

г) $\operatorname{ctg}\left(\arcsin\left(-\frac{4}{5}\right)\right)=\dfrac{\sqrt{1-\sin^2\left(\arcsin\left(-\frac{4}{5}\right)\right)}}{\sin\left(\arcsin\left(-\frac{4}{5}\right)\right)}=-\frac{3}{5}\cdot\frac{5}{4}=-\frac{3}{4}$.

§19. Арктангенс и решение уравнения tg x = a. Арккотангенс и решение уравнения ctg x = a

328. **а)** $\operatorname{arctg}\frac{\sqrt{3}}{3}=\frac{\pi}{6}$; **б)** $\operatorname{arctg}1=\frac{\pi}{4}$; **в)** $\operatorname{arctg}\sqrt{3}=\frac{\pi}{3}$; **г)** $\operatorname{arctg}0=0$.

329. **а)** $\operatorname{arctg}(-1)=-\frac{\pi}{4}$; **б)** $\operatorname{arctg}(-\sqrt{3})=-\frac{\pi}{3}$; **в)** $\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{6}$; **г)** $\operatorname{arctg}\left(-\frac{1}{3}\right)=-\frac{\pi}{6}$.

330. **а)** $\operatorname{arctg}1-\operatorname{arctg}\sqrt{3}=-\frac{\pi}{12}$; **б)** $\operatorname{arctg}(-\sqrt{3})+\operatorname{arctg}0=-\frac{\pi}{3}$;

в) $\operatorname{arctg}1-\operatorname{arctg}(-1)=\frac{\pi}{2}$; **г)** $\operatorname{arctg}\frac{\sqrt{3}}{3}+\operatorname{arctg}\sqrt{3}=\frac{\pi}{2}$.

331. **а)** $\operatorname{arcctg}\frac{\sqrt{3}}{3}=\frac{\pi}{3}$; **б)** $\operatorname{arcctg}1=\frac{\pi}{4}$; **в)** $\operatorname{arcctg}\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{3}$; **г)** $\operatorname{arcctg}0=\frac{\pi}{2}$.

332. **а)** $\operatorname{arcctg}(-1)-\operatorname{arctg}(-1)=\frac{\pi}{2}$; **б)** $\arcsin\left(\frac{\sqrt{2}}{2}\right)+\operatorname{arctg}(-\sqrt{3})=\frac{7\pi}{12}$;

в) $\operatorname{arcctg}\left(-\frac{\sqrt{3}}{3}\right)-\operatorname{arctg}\frac{\sqrt{3}}{3}=\frac{\pi}{2}$; **г)** $\arccos\left(-\frac{1}{2}\right)-\operatorname{arcctg}(-\sqrt{3})=-\frac{\pi}{6}$.

333. **а)** $\operatorname{tg}x=1$, $x=\frac{\pi}{4}+\pi n$; **б)** $\operatorname{tg}x=-\frac{\sqrt{3}}{3}$, $x=-\frac{\pi}{6}+\pi n$;

в) $\operatorname{tg}x=-1$, $x=-\frac{\pi}{4}+\pi n$; **г)** $\operatorname{tg}x=\frac{\sqrt{3}}{3}$, $x=\frac{\pi}{6}+\pi n$.

334. **а)** $\operatorname{tg} x = 0,\ x = \pi n$; **б)** $\operatorname{tg} x = -2,\ x = -\operatorname{arctg} 2 + \pi n$;

в) $\operatorname{tg} x = -3,\ x = -\operatorname{arctg} 3 + \pi n$; **г)** $\operatorname{tg} x = \frac{1}{2},\ x = \operatorname{arctg}\frac{1}{2} + \pi n$.

335. **а)** $\operatorname{ctg} x = 1,\ x = \frac{\pi}{4} + \pi n$; **б)** $\operatorname{ctg} x = \sqrt{3},\ x = \frac{\pi}{6} + \pi n$;

в) $\operatorname{ctg} x = 0,\ x = \frac{\pi}{2} + \pi n$; **г)** $\operatorname{ctg} x = \frac{\sqrt{3}}{2},\ x = \frac{\pi}{3} + \pi n$.

336. **а)** $\operatorname{ctg} x = -\sqrt{3},\ x = -\frac{\pi}{6} + \pi n$; **б)** $\operatorname{ctg} x = -1,\ x = -\frac{\pi}{4} + \pi n$;

б) $\operatorname{ctg} x = -\frac{\sqrt{3}}{3},\ x = -\frac{\pi}{3} + \pi n$; **г)** $\operatorname{ctg} x = -5,\ x = -\operatorname{arctg} 5 + \pi n$.

337. **а)** $2\arcsin\left(-\frac{\sqrt{3}}{2}\right) + \operatorname{arctg}(-1) + \arccos\frac{\sqrt{2}}{2} = -\frac{2\pi}{3} - \frac{\pi}{4} + \frac{\pi}{4} = -\frac{2\pi}{3}$;

б) $3\arcsin\frac{1}{2} + 4\arccos\left(-\frac{\sqrt{2}}{2}\right) - \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = \frac{\pi}{2} + 3\pi + \frac{\pi}{3} = \frac{11\pi}{3}$;

в) $\operatorname{arctg}\left(-\sqrt{3}\right) + \arccos\left(-\frac{\sqrt{3}}{2}\right) + \arcsin 1 = -\frac{\pi}{3} + \frac{5\pi}{6} + \frac{\pi}{2} = \pi$;

г) $\arcsin(-1) - \frac{3}{2}\arccos\frac{1}{2} + 3\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{2} + \frac{\pi}{2} + \pi = \pi$.

338. **а)** $\sin\left(\operatorname{arctg}\left(-\sqrt{3}\right)\right) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$; **б)** $\operatorname{tg}\left(\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)\right) = \operatorname{tg}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$;

в) $\cos(\operatorname{arctg} 0) = \cos 0 = 1$; **г)** $\operatorname{ctg}(\operatorname{arctg}(-1)) = \operatorname{ctg}\left(-\frac{\pi}{4}\right) = -1$.

339. **а)** $\operatorname{tg}(\operatorname{arcctg} 1) = \operatorname{tg}\frac{\pi}{4} = 1$; **б)** $\sin\left(\operatorname{arcctg}\sqrt{3}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$;

в) $\cos(\operatorname{arcctg}(-1)) = \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$; **г)** $\operatorname{ctg}\left(2\operatorname{arcctg}\left(-\frac{\sqrt{3}}{3}\right)\right) = \operatorname{ctg}\left(\frac{2\pi}{3}\right) = \sqrt{3}$.

340. **а)** $\operatorname{tg}^2 x - 3 = 0,\ \operatorname{tg} x = \pm\sqrt{3},\ x = \pm\frac{\pi}{3} + \pi n$;

б) $2\operatorname{tg}^2 x + 3\operatorname{tg} x = 0,\ \operatorname{tg} x = 0,\ \operatorname{tg} x = -\frac{3}{2},\ x = \pi n,\ x = -\operatorname{arcctg}\frac{3}{2} + \pi n$;

в) $4\operatorname{tg}^2 x - 9 = 0,\ \operatorname{tg} x = \pm\frac{3}{2},\ \operatorname{arcctg}\frac{3}{2} + \pi n$;

г) $3\operatorname{tg}^2 x - 2\operatorname{tg} x = 0,\ \operatorname{tg} x = 0,\ \operatorname{tg} = \frac{2}{3},\ x = \pi n,\ x = \operatorname{arcctg}\frac{2}{3} + \pi n$.

341. а) $\operatorname{tg}^2 x - 6\operatorname{tg} x + 5 = 0$, $\operatorname{tg} x = 5$, $\operatorname{tg} = 1$, $x = \operatorname{arctg} 5 + \pi n$, $x = \dfrac{\pi}{4} + \pi n$;

б) $\operatorname{tg}^2 x - 2\operatorname{tg} x - 3 = 0$, $\operatorname{tg} x = 3$, $\operatorname{tg} = -1$, $x = \operatorname{arctg} 3 + \pi n$, $x = -\dfrac{\pi}{4} + \pi n$.

342. а) $\operatorname{tg}(\pi + x) = \sqrt{3}$, $\operatorname{tg} x = \sqrt{3}$, $x = \dfrac{\pi}{3} + \pi n$;

б) $2\operatorname{ctg}(2\pi + x) - \operatorname{tg}\left(\dfrac{\pi}{2} + x\right) = \sqrt{3}$, $2\operatorname{ctg} x + \operatorname{ctg} x = \sqrt{3}$, $\operatorname{ctg} x = \dfrac{\sqrt{3}}{3}$, $x = \dfrac{\pi}{3} + \pi n$;

в) $-\sqrt{3}\operatorname{tg}(\pi - x) = 1$, $\operatorname{tg} x = \dfrac{\sqrt{3}}{3}$, $x = \dfrac{\pi}{6} + \pi n$;

г) $\operatorname{ctg}(2\pi - x) + \operatorname{ctg}(\pi - x) = 2$, $\operatorname{ctg} x = -1$, $x = \dfrac{\pi}{4} + \pi n$.

343. а) $\operatorname{tg} x < \sqrt{3}$, $x \in \left(-\dfrac{\pi}{2} + \pi k; \dfrac{\pi}{3} + \pi k\right)$; **б)** $\operatorname{ctg} x > 0$, $x \in \left(\pi k; \dfrac{\pi}{2} + \pi k\right)$;

в) $\operatorname{tg} x < 0$, $x \in \left(-\dfrac{\pi}{2} + \pi k; \pi k\right)$; **г)** $\operatorname{ctg} x > -1$, $x \in \left(\pi k; \dfrac{3\pi}{4} + \pi k\right)$.

344. а) $\operatorname{tg} x < 3$, $x \in \left(-\dfrac{\pi}{2} + \pi k; \operatorname{arctg} 3 + \pi k\right)$;

б) $3\operatorname{ctg} x - 1 > 0$, $\operatorname{ctg} x > \dfrac{1}{3}$, $x \in \left(\pi k; \operatorname{arctg}\dfrac{1}{3} + \pi k\right)$;

в) $\operatorname{ctg} x \le 2$, $x \in (\operatorname{arctg} 2 + \pi k; \pi + \pi k)$;

г) $2\operatorname{tg} x + 1 \ge 0$, $\operatorname{tg} x \ge \dfrac{1}{2}$, $x \in \left(-\operatorname{arctg}\dfrac{1}{2} + \pi k; \dfrac{\pi}{2} + \pi k\right)$.

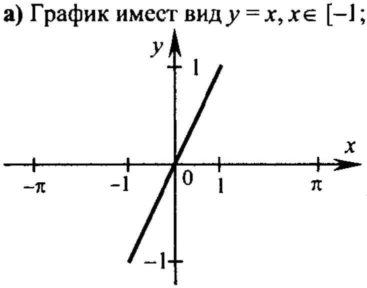
345. а) $\operatorname{tg}^2 x > 9$, $\left[\begin{array}{l}\operatorname{tg} x < -3\\ \operatorname{tg} x > 3\end{array}\right.$, $x \in \left(-\dfrac{\pi}{2} + \pi k; \operatorname{arctg} 3 + \pi k\right) \cup \left(\operatorname{arctg} 3 + \pi k; \dfrac{\pi}{2} + \pi k\right)$;

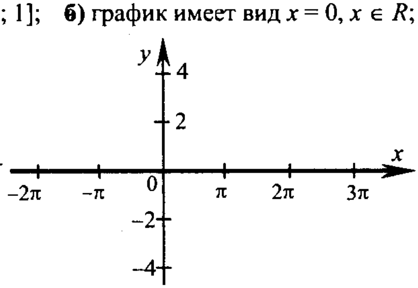
б) $\operatorname{tg}^2 x > \operatorname{tg} x$, $\operatorname{tg} x(\operatorname{tg} x - 1) > 0$, $\left[\begin{array}{l}\operatorname{tg} x < 0\\ \operatorname{tg} x > 1\end{array}\right.$, $x \in \left(-\dfrac{\pi}{2} + \pi k; \pi k\right) \cup \left(\dfrac{\pi}{4} + \pi k; \dfrac{\pi}{2} + \pi k\right)$

в) $\operatorname{tg}^2 x < 9$, $\operatorname{tg} x \in (-3;3)$, $x \in (-\operatorname{arctg} 3 + \pi k; \operatorname{arctg} 3 + \pi k)$;

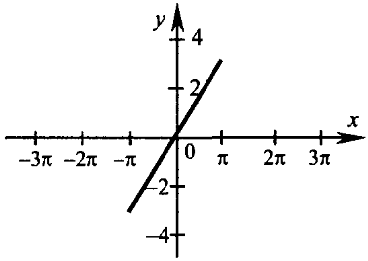
г) $\operatorname{tg}^2 x < 2$, $\operatorname{tg} x(\operatorname{tg} x - 2) < 0$, $\operatorname{tg} x \in (0;2)$, $x \in (\pi k; \operatorname{arctg} 2 + \pi k)$.

346. а) График имеет вид $y = x, x \in [-1; 1]$; **б)** график имеет вид $x = 0$, $x \in R$;

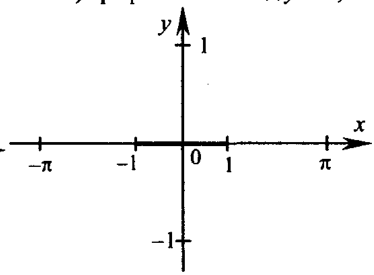




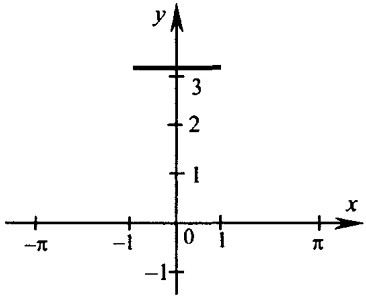
в) график имеет вид $y = x, x \in R$;



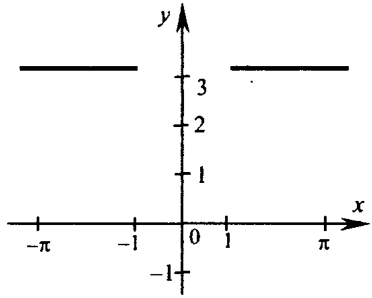
г) график имеет вид $y = 0,\ x \in [-1;\ 1]$.



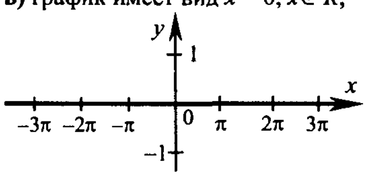
347. График имеет вид $y = \pi, x \in [-1;\ 1]$;



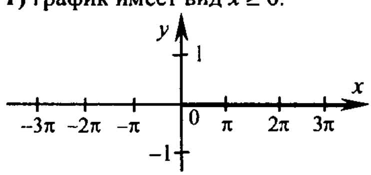
б) график имеет вид $y = \pi,\ x \notin [-1;\ 1]$;



в) график имеет вид $x = 0, x \in R$;



г) график имеет вид $x \geq 0$.



348. а) $\sin\left(\operatorname{arctg}\frac{3}{4}\right)$, $\operatorname{arctg}\frac{3}{4} = x$, $x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$, $\operatorname{tg} x = \frac{3}{4}$,

$\sin x = \frac{3}{4}\sqrt{1-\sin^2 x}$, $16\sin^2 x = 9 - 9\sin^2 x$, $\sin^2 x = \frac{9}{25}$,

$\sin x = \pm\frac{3}{5} \Rightarrow \sin\left(\operatorname{arctg}\frac{3}{4}\right) = \frac{3}{5}$;

б) $\cos\left(\operatorname{arcctg}\frac{12}{5}\right)$, $\operatorname{arcctg}\frac{12}{5} = x$, $x \in (0; \pi)$, $\operatorname{ctg} x = \frac{12}{5}$,

$\frac{12}{5} = \frac{\cos x}{\sqrt{1-\cos^2 x}}$, $144 - 144\cos^2 x = 25\cos^2 x$, $\cos^2 x = \frac{144}{169}$,

$\cos x = \pm\frac{12}{13} \Rightarrow \left(\operatorname{arcctg}\frac{12}{5}\right) = \frac{12}{13}$;

в) $\sin\left(\operatorname{arcctg}\left(-\frac{4}{3}\right)\right)$, $\operatorname{arcctg}\left(-\frac{4}{3}\right) = x$, $x \in (0; \pi)$, $-\frac{4}{3} = \operatorname{ctg} x$, $-\frac{4}{3} = \frac{\cos x}{\sin x}$,

$16\sin^2 x = 9 - 9\sin^2 x$, $\sin^2 x = \frac{9}{25}$, $\sin x = \pm\frac{3}{5} \Rightarrow \sin\left(\operatorname{arctg}\left(-\frac{4}{3}\right)\right) = \frac{3}{5}$;

г) $\cos\left(\operatorname{arctg}\left(-\frac{5}{12}\right)\right)$, $\operatorname{arctg}\left(-\frac{5}{12}\right)=x$, $x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$, $-\frac{5}{12}=\operatorname{tg}x$,

$25\cos^2 x=\frac{144}{160}$, $\cos x=\pm\frac{12}{13}\Rightarrow\cos\left(\operatorname{arctg}\left(-\frac{5}{12}\right)\right)=\frac{12}{13}$.

§20. Тригонометрические уравнения

349. а) $2\cos x+\sqrt{3}=0$, $\cos x=-\frac{\sqrt{3}}{2}$, $x=\pm\frac{5\pi}{6}+2\pi n$;

б) $2\sin x-1=0$, $\sin x=\frac{1}{2}$, $x=(-1)^n\frac{\pi}{6}+\pi k$;

в) $2\cos x-1=0$, $\cos x=\frac{1}{2}$, $x=\pm\frac{\pi}{3}+2\pi n$;

г) $2\sin x+\sqrt{2}=0$, $\sin x=-\frac{\sqrt{2}}{2}$, $x=(-1)^{k+1}\frac{\pi}{6}+\pi k$.

350. а) $\operatorname{tg}x+\sqrt{3}=0$, $\operatorname{tg}x=-\sqrt{3}$, $x=-\frac{\pi}{3}+\pi n$;

б) $\sqrt{3}\operatorname{tg}x-1=0$, $\operatorname{tg}x=\frac{\sqrt{2}}{2}$, $x=\frac{\pi}{6}+\pi n$;

в) $\operatorname{ctg}x+1=0$, $\operatorname{ctg}x=-1$, $x=-\frac{\pi}{4}+\pi n$;

г) $\sqrt{3}\operatorname{ctg}x-1=0$, $\operatorname{tg}x=\frac{\sqrt{3}}{3}$, $x=\frac{\pi}{3}+\pi n$.

351. а) $\sin 2x=\frac{\sqrt{2}}{2}$, $2x=(-1)^k\frac{\pi}{4}+\pi k$, $x=(-1)^k\frac{\pi}{8}+\frac{\pi k}{2}$;

б) $\cos\frac{x}{3}=-\frac{1}{2}$, $\frac{x}{3}=\pm\frac{2\pi}{3}+2\pi n$, $x=\pm2\pi+6\pi n$;

в) $\sin\frac{x}{4}=\frac{1}{2}$, $\frac{x}{4}=(-1)^k\frac{\pi}{6}+2\pi k$, $x=(-1)^k\frac{2\pi}{3}+4\pi k$;

г) $\cos 4x=0$, $4x=\frac{\pi}{2}+\pi n$, $x=\frac{\pi}{8}+\frac{\pi n}{4}$.

352. а) $\sin\left(-\frac{x}{3}\right)=\frac{\sqrt{2}}{2}$, $\sin\frac{x}{3}=-\frac{\sqrt{2}}{2}$, $\frac{x}{3}=(-1)^{k+1}\frac{\pi}{4}+\pi k$, $x=(-1)^{k+1}\frac{3\pi}{4}+3\pi k$;

б) $\cos(-2x)=-\frac{\sqrt{3}}{2}$, $2x=\pm\frac{5\pi}{6}+2\pi n$, $x=\pm\frac{5\pi}{12}+\pi n$;

в) $\operatorname{tg}(-4x)=\frac{\sqrt{3}}{3}$, $\operatorname{tg}4x=-\frac{\sqrt{3}}{3}$, $4x=-\frac{\pi}{6}+\pi n$, $x=-\frac{\pi}{24}+\frac{\pi n}{4}$;

г) $\operatorname{ctg}\left(-\frac{x}{2}\right)=1$, $\operatorname{ctg}\frac{x}{2}=-1$, $\frac{x}{2}=-\frac{\pi}{4}+\pi n$, $x=-\frac{\pi}{2}+2\pi n$.

353. а) $2\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)=\sqrt{3}$, $\frac{x}{2}-\frac{\pi}{6}=\pm\frac{\pi}{6}+2\pi n$, $x=\pm\frac{\pi}{3}+\frac{\pi}{3}+4\pi n$;

б) $\sqrt{3}\,\mathrm{tg}\left(\frac{x}{3}+\frac{\pi}{6}\right)=3$, $\mathrm{tg}\left(\frac{x}{3}+\frac{\pi}{6}\right)=\sqrt{3}$, $\frac{x}{3}+\frac{\pi}{6}=\frac{\pi}{3}+\pi n$, $x=\frac{\pi}{2}+3\pi n$;

в) $2\sin\left(3x-\frac{\pi}{4}\right)=-\sqrt{2}$, $3x-\frac{\pi}{4}=(-1)^{k+1}\frac{\pi}{4}+\pi k$, $x=(-1)^{k+1}\frac{\pi}{12}+\frac{\pi}{12}+\frac{\pi k}{3}$;

г) $\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)+1=0$, $\frac{x}{2}-\frac{\pi}{6}=-\frac{\pi}{2}+2\pi n$, $x=-\frac{2\pi}{3}+4\pi n$.

354. а) $\cos\left(\frac{\pi}{6}-2x\right)=-1$, $2x-\frac{\pi}{6}=\pi+2\pi n$, $x=\frac{\pi}{12}+\pi n$;

б) $\mathrm{tg}\left(\frac{\pi}{4}-\frac{x}{2}\right)=-1$, $\frac{x}{2}-\frac{\pi}{4}=\frac{\pi}{4}+\pi n$, $x=\pi+2\pi n$;

в) $2\sin\left(\frac{\pi}{3}-\frac{x}{4}\right)=\sqrt{3}$, $\frac{x}{4}-\frac{\pi}{3}=(-1)^{k+1}\frac{\pi}{3}+\pi n$, $x=(-1)^{k+1}\frac{4\pi}{3}+\frac{4\pi}{3}+4\pi k$;

г) $2\cos\left(\frac{\pi}{4}-3x\right)=\sqrt{2}$, $3x-\frac{\pi}{4}=\pm\frac{\pi}{4}+2\pi n$, $x=\pm\frac{\pi}{12}+\frac{\pi}{12}+\frac{2\pi n}{3}$.

355. а) $3\sin^2 x-5\sin x-2=0$, $\sin x=\frac{5+7}{6}$ – не подходит,

$\sin=-\frac{1}{3}$, $x=(-1)^{k+1}\arcsin\frac{1}{3}+\pi n$;

б) $3\sin^2 2x+10\sin 2x+3=0$, $\sin 2x=\frac{-10\pm\sqrt{100-4\cdot3\cdot3}}{6}=\frac{-5\pm4}{3}$,

$\sin 2x=\frac{-5-4}{3}$ – не подходит, $\sin 2x=-\frac{1}{3}$, $x=(-1)^{k+1}\frac{\arcsin\frac{1}{3}}{2}+\frac{\pi k}{2}$;

г) $2\sin^2\frac{x}{2}-3\sin\frac{x}{2}+1=0$, $\sin\frac{x}{2}=\frac{-3\pm\sqrt{9-4\cdot2\cdot1}}{4}=\frac{3\pm1}{4}$,

$\sin\frac{x}{2}=\frac{3+1}{4}=1$, $\frac{x}{2}=\frac{\pi}{2}+2\pi n$, $x=\pi+4\pi n$, $\sin\frac{x}{2}=\frac{1}{2}$,

$\frac{x}{2}=(-1)^k\frac{\pi}{6}+\pi k$, , $x=(-1)^k\frac{\pi}{3}+2\pi k$.

356. а) $6\cos^2 x+\cos x-1=0$, $\cos x=\frac{-1\pm\sqrt{1+4\cdot6\cdot1}}{12}=\frac{-1\pm5}{12}$,

$\cos x=-\frac{1}{2}$, $x=\pm\frac{2\pi}{3}+2\pi n$, $\cos x=\frac{1}{3}$, $x=\pm\arccos\frac{1}{3}+2\pi n$;

б) $2\cos^2 3x-5\cos 3x-3=0$, $\cos 3x=\frac{5+7}{4}$ – не подходит,

$\cos 3x=-\frac{1}{2}$, $3x=\pm\frac{2\pi}{3}+2\pi n$, $x=\pm\frac{2\pi}{9}+\frac{2\pi n}{3}$;

в) $2\cos^2 x - \cos x - 3 = 0$, $\cos x = \dfrac{1+5}{4}$ – не подходит,

$\cos x = -1$, $x = \pi + 2\pi n$;

г) $2\cos^2 \dfrac{x}{3} + 3\cos\dfrac{x}{3} - 2 = 0$, $\cos\dfrac{x}{3} = \dfrac{-3-5}{4}$ – не подходит,

$\cos\dfrac{x}{3} = \dfrac{1}{2}$, $\dfrac{x}{3} = \pm\dfrac{\pi}{3} + 2\pi n$, $x = \pm\pi + 6\pi n$.

357. а) $2\sin^2 x + 3\cos x = 0$, $2 - 2\cos^2 x - 3\cos x - 2 = 0$,

$\cos x = \dfrac{3+5}{4}$ – не подходит, $\cos x = -\dfrac{1}{2}$, $x = \pm\dfrac{2\pi}{3} + 2\pi n$;

б) $8\sin^2 2x + \cos 2x + 1 = 0$, $8 - 8\cos^2 x + \cos 2x + 1 = 0$, $8\cos^2 x - \cos 2x - 9 = 0$,

$\cos 2x = \dfrac{1+17}{16}$ – не подходит, $\cos 2x = -1$, $2x = \pi + 2\pi n$, $x = \dfrac{\pi}{2} + \pi n$;

в) $5\cos^2 x + 6\sin x - 6 = 0$, $5 - 5\sin^2 x + 6\sin x - 6 = 0$, $5\sin^2 x - 6\sin x + 1 = 0$,

$\sin x = \dfrac{6 \pm \sqrt{36 - 4\cdot 5\cdot 1}}{10} = \dfrac{3\pm 2}{5}$, $\sin x = 1$, $x = \dfrac{\pi}{2} + 2\pi n$,

$\sin x = \dfrac{1}{5}$, $x = (-1)^n \arcsin\dfrac{1}{5} + \pi n$;

г) $4\sin 3x + \cos^2 3x = 4$, $\sin^2 3x - 4\sin 3x + 3 = 0$, $\sin 3x = 3$ – не подходит,

$\sin 3x = 1$, $3x = \dfrac{\pi}{2} + 2\pi n$, $x = \dfrac{\pi}{6} + \dfrac{2\pi n}{3}$.

358. а) $3\,\mathrm{tg}^2 x + 2\,\mathrm{tg}\, x - 1 = 0$, $\mathrm{tg}\, x = \dfrac{-1+2}{3} = \dfrac{1}{3}$, $x = \mathrm{arctg}\dfrac{1}{3} + \pi n$, $\mathrm{tg}\, x = -1$, $x = -\dfrac{\pi}{4} + \pi n$;

б) $\mathrm{ctg}^2 2x - 6\,\mathrm{ctg}\, 2x + 5 = 0$, $\mathrm{ctg}\, 2x = 5$, $2x = \mathrm{arcctg}\, 5 + \pi n$,

$x = \dfrac{\mathrm{arctg}\, 5}{2} + \dfrac{\pi n}{2}$, $\mathrm{ctg}\, 2x = 1$, $2x = \dfrac{\pi}{4} + \pi n$, $x = \dfrac{\pi}{8} + \dfrac{\pi n}{2}$;

в) $2\,\mathrm{tg}^2 x + 3\,\mathrm{tg}\, x - 2 = 0$, $\mathrm{tg}\, x = \dfrac{-3+5}{4} = \dfrac{1}{2}$, $x = -\mathrm{arctg}\dfrac{1}{2} + \pi n$,

$\mathrm{tg} = -2$, $x = -\mathrm{arctg}\, 2 + \pi n$;

г) $7\,\mathrm{ctg}^2 \dfrac{x}{2} + 2\,\mathrm{ctg}\dfrac{x}{2} = 5$, $7\,\mathrm{ctg}^2 \dfrac{x}{2} + 2\,\mathrm{ctg}\dfrac{x}{2} - 5 = 0$,

$\mathrm{ctg}\dfrac{x}{2} = \dfrac{-1-6}{7} = -1$, $\dfrac{x}{2} = -\dfrac{\pi}{4} + \pi n$, $x = -\dfrac{\pi}{2} + 2\pi n$,

$\mathrm{ctg}\dfrac{x}{2} = \dfrac{-1+6}{7} = \dfrac{5}{7}$, $\dfrac{x}{2} = \mathrm{arcctg}\dfrac{5}{7} + \pi n$, $x = 2\,\mathrm{arcctg}\dfrac{5}{7} + 2\pi n$.

359. а) $\left(\sin x - \dfrac{1}{2}\right)(\sin x + 1) = 0$, $\sin x = \dfrac{1}{2}$, $x = (-1)^k \dfrac{\pi}{6} + \pi k$, $\sin x = -1$, $x = -\dfrac{\pi}{2} + 2\pi n$;

б) $\left(\cos x + \dfrac{1}{2}\right)(\cos x - 1) = 0$, $\cos x = -\dfrac{1}{2}$, $x = \pm\dfrac{2\pi}{3} + 2\pi n$, $\cos x = 1$, $x = 2\pi n$;

в) $\left(\cos x-\frac{\sqrt{2}}{2}\right)\left(\sin x+\frac{\sqrt{2}}{2}\right)=0,\ \cos x=\frac{\sqrt{2}}{2},\ x=\pm\frac{\pi}{4}+2\pi n,$

$\sin x=-\frac{\sqrt{2}}{2},\ x=(-1)^{n+1}\frac{\pi}{4}+\pi n;$

г) $(1+\cos x)\left(\sqrt{2}\sin x-1\right)=0,\ \cos x=-1,\ x=\pi+2\pi n,$

$\sin x=\frac{\sqrt{2}}{2},\ x=(-1)^n\frac{\pi}{4}+\pi n.$

360. а) $\sin x+\sqrt{3}\cos x=0,\ \operatorname{tg} x=-\sqrt{3},\ \cos x\neq 0,\ x=-\frac{\pi}{3}+\pi n;$

б) $\sin x+\cos x=0,\ \operatorname{tg} x=-1,\ \cos x\neq 0,\ x=-\frac{\pi}{4}+\pi n;$

в) $\sin x-3\cos x=0,\ \operatorname{tg} x=3,\ \cos x\neq 0,\ x=\operatorname{arctg}3+\pi n;$

г) $\sqrt{3}\sin x+\cos x=0,\ \operatorname{tg} x=-\frac{\sqrt{3}}{3},\ \cos x\neq 0,\ x=-\frac{\pi}{6}+\pi n.$

361. а) $\sin^2 x+\sin x\cos x=0,\ \sin x(\sin x+\cos x)=0,\ \sin x=0,\ x=\pi n,$

$\sin x+\cos x=0,\ x=-\frac{\pi}{4}+\pi n;$

б) $\sqrt{3}\sin x\cos x+\cos^2 x=0,\ \cos x\left(\sqrt{3}\sin x+\cos x\right)=0,\ \cos x=0,$

$x=\frac{\pi}{2}+\pi n,\ \sqrt{3}\sin x+\cos x=0,\ x=-\frac{\pi}{6}+\pi n;$

в) $\sin^2 x=3\sin x\cos x,\ \sin x(\sin x-3\cos x)=0,\ \sin x=0,\ x=\pi n,$

$\sin x-3\cos x=0,\ x=\operatorname{arctg}3+\pi n;$

г) $\sqrt{3}\cos^2 x=\sin x\cos x,\ \cos x\left(\sqrt{3}\cos x-\sin x\right)=0,\ \cos x=0,$

$x=\frac{\pi}{2}+\pi n,\ \sqrt{3}\cos x-\sin x=0,\ \operatorname{tg} x=\sqrt{3},\ x=\frac{\pi}{3}+\pi n.$

362. а) $\sin^2 x+\sin x\cos x-3\cos^2 x=0,\ \operatorname{tg}^2 x+2\operatorname{tg} x-3=0,\ \operatorname{tg} x=-3,$

$x=-\operatorname{arctg}3+\pi n,\ \operatorname{tg} x=1,\ x=\frac{\pi}{4}+\pi n;$

б) $\sin^2 x-4\sin x\cos x+3\cos^2 x=0,\ \operatorname{tg}^2 x-4\operatorname{tg} x+3=0,\ \operatorname{tg} x=3,$

$x=\operatorname{arctg}3+\pi n,\ \operatorname{tg} x=1,\ x=\frac{\pi}{4}+\pi n;$

в) $\sin^2 x+\sin x\cos x-2\cos^2 x=0,\ \operatorname{tg}^2 x+\operatorname{tg} x-2=0,\ \operatorname{tg} x=\frac{-1\pm 3}{2},$

$\operatorname{tg} x=1,\ x=\frac{\pi}{4}+\pi n,\ \operatorname{tg} x=-2,\ x=\operatorname{arctg}2+\pi n;$

г) $3\sin^2 x+\sin x\cos x-2\cos^2 x=0,\ 3\operatorname{tg}^2 x+\operatorname{tg} x-2=0,\ \cos x\neq 0,$

$\operatorname{tg} x=\frac{-1\pm\sqrt{1+4\cdot 3\cdot 2}}{6}=\frac{-1\pm 5}{6},\ \operatorname{tg} x=-1,\ x=-\frac{\pi}{4}+\pi n,\ \operatorname{tg} x=\frac{2}{3},\ x=\operatorname{arctg}\frac{2}{3}+\pi n.$

363. а) $\sin^2\frac{3x}{4}-\frac{\sqrt{2}}{2}=\sin x-\cos^2\frac{3x}{4}+1$, $1-\frac{\sqrt{2}}{2}-1=\sin x$,

$\sin x=-\frac{\sqrt{2}}{2}$, $x=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

б) $\cos^2 2x-1-\cos x=\frac{\sqrt{3}}{2}-\sin^2 2x$, $\cos x=-\frac{\sqrt{3}}{2}$, $x=\pm\frac{5\pi}{6}+2\pi n$.

364. а) $\sin x=\frac{1}{2}$, $x\in[0;\pi]$; $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$;

б) $\cos x=-\frac{1}{2}$, $x\in[-2\pi;3\pi]$; $x=-\frac{4\pi}{3},-\frac{2\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{8\pi}{3}$.

365. а) $\sin 3x=\frac{\sqrt{2}}{2}$, $x\in[0;2\pi]$, $3x=(-1)^k\frac{\pi}{4}+\pi k$,

$3x=\frac{\pi}{4},\frac{3\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4},\frac{17\pi}{4},\frac{19\pi}{4}$; $x=\frac{\pi}{12},\frac{\pi}{4},\frac{3\pi}{4},\frac{11\pi}{12},\frac{17\pi}{12},\frac{19\pi}{12}$;

б) $\cos 3x=\frac{\sqrt{3}}{2}$, $x\in[-\pi;\pi]$, $3x=\pm\frac{\pi}{6}+2\pi n$,

$3x=-\frac{13\pi}{6},-\frac{11\pi}{6},-\frac{\pi}{6},\frac{11\pi}{6},\frac{13\pi}{6}$; $x=-\frac{13\pi}{18},-\frac{11\pi}{18},-\frac{\pi}{18},\frac{11\pi}{18},\frac{13\pi}{18}$;

в) $\operatorname{tg}\frac{x}{2}=\frac{\sqrt{3}}{3}$, $x\in[-3\pi;3\pi]$, $\frac{x}{2}=\frac{\pi}{6}+\pi n$, $x=\frac{\pi}{3}+2\pi n$, $x=-\frac{5\pi}{3},\frac{\pi}{3},\frac{7\pi}{3}$;

г) $\operatorname{ctg}4x=-1$, $x\in[0;\pi]$, $4x=-\frac{\pi}{4}+\pi n$,

$4x=\frac{3\pi}{4},\frac{7\pi}{4},\frac{11\pi}{4},\frac{15\pi}{4}$; $x=\frac{3\pi}{16},\frac{7\pi}{16},\frac{11\pi}{16},\frac{15\pi}{16}$.

366. а) $\sin 3x=-\frac{1}{2}$, $x\in[-4;4]$, $3x=(-1)^{k+1}\frac{\pi}{6}+\pi k$,

$x=(-1)^{k+1}\frac{\pi}{18}+\frac{\pi k}{3}$, $x=-\frac{\pi}{6},-\frac{5\pi}{6},\frac{7\pi}{6}$;

б) $\cos x=1$, $x\in[-6;16]$, $x=2\pi n$, $x=0;2\pi;4\pi$.

367. а) $\sin x=\frac{x}{2}$, $x\in[-12;18]$, $\frac{x}{2}=\pi n$, $x=-2\pi;0;2\pi;4\pi$;

б) $\cos x=-\frac{\sqrt{2}}{2}$, $x\in[1;7]$, $3x=\pm\frac{3\pi}{4}+2\pi n$, $x=\pm\frac{\pi}{4}+\frac{2\pi n}{3}$, $x=\frac{11\pi}{12};\frac{19\pi}{12};\frac{5\pi}{12};\frac{13\pi}{12};\frac{7\pi}{4}$

368. $\sin\left(2x-\frac{\pi}{4}\right)=-1$, $2x-\frac{\pi}{4}=-\frac{\pi}{2}+2\pi n$, $x=-\frac{\pi}{8}+\pi n$,

а) $x=\frac{7\pi}{8}$; б) $-\frac{\pi}{8};\frac{7\pi}{8}$; в) $-\frac{\pi}{8}$; г) $-\frac{\pi}{8}$.

369. $\cos\left(\frac{\pi}{3}-2x\right)=\frac{1}{2},\ 2x-\frac{\pi}{3}=\pm\frac{\pi}{3}+2\pi n,\ x=\frac{\pi}{3}+\pi n,\ x=\pi n$,

а) $\frac{\pi}{3}$; **б)** $0;\frac{\pi}{3};\pi;\frac{4\pi}{3}$; **в)** $-\frac{2\pi}{3}$; **г)** $-\frac{2\pi}{3};0;\frac{\pi}{3}$.

370. а) $\sqrt{16-x^2}\sin x=0,\ |x|\le 4,\ x=4,\ x=-4,\ \sin x=0,\ x=\pi n,\ n=0,\pm1$.

Ответ: $x=\pm4; x=\pi n, n=0,\pm1...$

б) $\sqrt{7x-x^2}(2\cos x-1)=0,\ 0\le x\le 7,\ 7x-x^2=0,\ x=0,\ x=7$,

$2\cos x-1=0,\ \cos x=\frac{1}{2},\ x=\frac{\pi}{3},\ x=\frac{5\pi}{3}$.

Ответ: $x=0;\frac{\pi}{3};\frac{5\pi}{3};7$.

371. а) $\left(\sqrt{2}\cos x-1\right)\sqrt{4x^2-7x+3}=0,\ 4x^2-7x+3\ge 0$,

$x\ge\frac{7+1}{8}=1,\ x\le\frac{3}{4},\ \sqrt{2}\cos x-1=0,\ x=\pm\frac{\pi}{4}+2\pi n$.

Ответ: $x=1, x=\frac{3}{4}, x=-\frac{\pi}{4}, x=\pm\frac{\pi}{4}+2\pi n; n=\pm1;\pm2;\pm3...$

б) $\left(2\sin x-\sqrt{3}\right)\sqrt{3x^2-7x+4}=0,\ 3x^2-7x+4\ge 0;\ x\le 1;\ x\ge\frac{4}{3}$,

$2\sin x-\sqrt{3}=0,\ \sin x=\frac{\sqrt{3}}{2},\ x=\frac{2\pi}{3}+2\pi k;\ k=\pm1;\pm2...$

$3x^2-7x+4=0,\ x=1;\ x=\frac{4}{3}$.

Ответ: $1;\frac{4}{3};\frac{2\pi}{3}+2\pi k;\frac{\pi}{3}+2\pi k;\ k=\pm1;\pm2...$

372. а) $\operatorname{tg}x-2\operatorname{ctg}x+1=0,\ \operatorname{tg}^2x-\operatorname{tg}x-2=0$,

$\operatorname{tg}x=-2,\ x=-\operatorname{arctg}2+\pi n,\ \operatorname{tg}x=1,\ x=\frac{\pi}{4}+\pi n$;

б) $\frac{\operatorname{tg}x+5}{2}=\frac{1}{\cos^2x},\ 2\operatorname{tg}^2x-\operatorname{tg}x-3=0$,

$\operatorname{tg}x=\frac{1+5}{4}=\frac{3}{2},\ x=\operatorname{arctg}\frac{3}{2}+\pi k,\ \operatorname{tg}x=-1,\ x=-\frac{\pi}{4}+\pi k$;

в) $2\operatorname{ctg}x-3\operatorname{tg}x+5=0,\ 2\operatorname{ctg}^2x+5\operatorname{ctg}x-3=0,\ \operatorname{ctg}x=\frac{-5+7}{4}=\frac{1}{2}$,

$x=\operatorname{arcctg}\frac{1}{2}+\pi n,\ \operatorname{ctg}x=-3,\ x=-\operatorname{arcctg}3+\pi n$;

г) $\frac{7-\operatorname{ctg}x}{4}=\frac{1}{\sin^2x},\ 7-\operatorname{ctg}x=4\operatorname{ctg}^2x+4,\ 4\operatorname{ctg}^2x+\operatorname{ctg}x-3=0$,

$\operatorname{ctg}x=\frac{-1+7}{8}=\frac{3}{4},\ x=\operatorname{arctg}\frac{3}{4}+\pi n,\ \operatorname{ctg}x=-1,\ x=-\frac{\pi}{4}+\pi n$.

373. а) $2\cos^2\frac{x}{2}+\sqrt{3}\cos\frac{x}{2}=0,\ \cos\frac{x}{2}\left(2\cos\frac{x}{2}+\sqrt{3}\right)=0$,

$\cos\frac{x}{2}=0,\ \frac{x}{2}=\frac{\pi}{2}+\pi k,\ x=\pi+2\pi k$,

$\cos\frac{\sqrt{3}}{2},\ \frac{x}{2}=\pm\frac{5\pi}{6}+2\pi n,\ x=\pm\frac{5\pi}{3}+4\pi n$;

б) $4\cos^2\left(x-\frac{\pi}{6}\right)-3=0,\ \cos\left(x-\frac{\pi}{6}\right)=\pm\frac{\sqrt{3}}{2}$,

$x-\frac{\pi}{6}=\pm\frac{\pi}{6}+2\pi n,\ x=\frac{\pi}{3}+2\pi n,\ x=2\pi n$,

$x-\frac{\pi}{6}=\pm\frac{5\pi}{6}+2\pi n,\ x=\pi+2\pi n,\ x=-\frac{2\pi}{3}+2\pi n,\ x=\frac{\pi}{3}+\pi n,\ x=\pi n$;

в) $\sqrt{3}\,\mathrm{tg}^2 3x-3\,\mathrm{tg}\,3x=0,\ \mathrm{tg}\,3x\left(\sqrt{3}\,\mathrm{tg}\,3x-3\right)=0,\ \mathrm{tg}\,3x=0$,

$3x=\pi n,\ x=\frac{\pi n}{3},\ \mathrm{tg}\,3x=\sqrt{3},\ 3x=\frac{\pi}{3}+\pi n,\ x=\frac{\pi}{9}+\frac{\pi n}{3}$;

г) $4\sin^2\left(2x+\frac{\pi}{3}\right)-1=0,\ \sin\left(2x+\frac{\pi}{3}\right)=\pm\frac{1}{2}$,

$\left(2x+\frac{\pi}{3}\right)=(-1)^n\frac{\pi}{6}+\pi n,\ x=(-1)^n\frac{\pi}{12}-\frac{\pi}{6}+\frac{\pi n}{2}$,

$2x+\frac{\pi}{3}=(-1)^{n+1}\frac{\pi}{6}+\pi n,\ x=(-1)^n\frac{\pi}{12}-\frac{\pi}{6}+\frac{\pi n}{2}$.

374. а) $\sin^2 x-\frac{12-\sqrt{2}}{2}\cdot\sin x-3\sqrt{2}=0,\ \sin x=6$, решений нет,

$\sin x=-\frac{\sqrt{2}}{2},\ x=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

б) $\cos^2 x-\frac{8-\sqrt{3}}{2}\cos x-2\sqrt{3}=0,\ \cos x=4$, не подходит,

$\cos x=-\frac{\sqrt{2}}{2},\ x=\pm\frac{5\pi}{6}+2\pi n$.

375. а) $\sin 2x=\cos 2x,\ \mathrm{tg}\,2x=1,\ \cos 2x\neq 0,\ 2x=\frac{\pi}{4}+\pi n,\ x=\frac{\pi}{8}+\frac{\pi n}{2}$;

б) $\sqrt{3}\sin 3x=\cos 3x,\ \mathrm{ctg}\,3x=\sqrt{3},\ \sin 3x\neq 0,\ 3x=\frac{\pi}{6}+\pi n,\ x=\frac{\pi}{18}+\frac{\pi n}{3}$;

в) $\sin\frac{x}{2}=\sqrt{3}\cos\frac{x}{2},\ \mathrm{tg}\frac{x}{2}=\sqrt{3},\ \cos\frac{x}{2}\neq 0,\ \frac{x}{2}=\frac{\pi}{3}+\pi n,\ x=\frac{2\pi}{3}+2\pi n$;

г) $\sqrt{3}\sin 17x=\sqrt{6}\cos 17x,\ \mathrm{tg}\,17x=\sqrt{3},\ \cos 17x\neq 0,\ 17x=\frac{\pi}{3}+\pi n,\ x=\frac{\pi}{51}+\frac{\pi n}{17}$.

376. а) $2\sin^2 2x-5\sin 2x\cos 2x+\cos^2 2x=0$, $2\,\mathrm{tg}^2 2x-5\,\mathrm{tg}\,2x+1=0,\ \cos 2x\neq 0$,

$\operatorname{tg}2x=\frac{5-\sqrt{17}}{4}$, $2x=\operatorname{arctg}\frac{5-\sqrt{17}}{4}+\pi n$, $x=\frac{1}{2}\operatorname{arctg}\frac{5-\sqrt{17}}{4}+\frac{\pi n}{2}$;

б) $3\sin^2 3x+10\sin 3x\cos 3x+3\cos^2 3x=0$, $2\sin 3x\cos 3x=-\frac{3}{5}$,

$3\operatorname{tg}^2 3x+10\operatorname{tg}3x+3=0$, $\operatorname{tg}3x=\frac{-10\pm\sqrt{100-4\cdot 3\cdot 3}}{6}=\frac{-10\pm 8}{6}$, $\operatorname{tg}3x=-3$,

$x=\frac{1}{3}\operatorname{arctg}(-3)+\frac{\pi n}{3}$, $\operatorname{tg}3x=-\frac{1}{3}$, $x=\frac{1}{3}\operatorname{arctg}\left(-\frac{1}{3}\right)+\frac{\pi n}{3}$, $6x=(-1)^{k+1}\arcsin\frac{3}{5}+\frac{\pi k}{6}$.

377. а) $\sin^2\frac{x}{2}=3\cos^2\frac{x}{2}$, $\cos^2\frac{x}{2}=\frac{1}{4}$, $\cos\frac{x}{2}=\pm\frac{1}{2}$, $\frac{x}{2}=\pm\frac{2\pi}{3}+2\pi n$, $x=\pm\frac{4\pi}{3}+4\pi n$;

б) $\sin^2 4x=\cos^2 4x$, $\operatorname{tg}^2 4x=1$, $\cos 4x\neq 0$, $\operatorname{tg}4x=\pm 1$,

$4x=\frac{\pi}{4}+\pi n$, $x=\frac{\pi}{16}+\frac{\pi n}{4}$, $4x=-\frac{\pi}{4}+\pi n$, $x=-\frac{\pi}{16}+\frac{\pi n}{4}$.

378. а) $5\sin^2 x-14\sin x\cos x-3\cos^2 x=2$,

$3\sin^2 x-14\sin x\cos x-5\cos^2 x=0$, $3\operatorname{tg}^2 x-14\operatorname{tg}x-5=0$, $\cos\neq 0$,

$\operatorname{tg}^2 x=\frac{7+8}{3}=5$, $x=\operatorname{arctg}5+\pi k$, $\operatorname{tg}x=-\frac{1}{3}$, $x=-\operatorname{arctg}\frac{1}{3}+\pi k$;

б) $3\sin^2 x-\sin x\cos x=2$, $\sin^2 x-\sin x\cos x-2\cos^2 x=0$, $\operatorname{tg}^2 x-\operatorname{tg}x-2=0$,

$\cos x\neq 0$, $\operatorname{tg}x=2$, $x=\operatorname{arctg}2+\pi n$, $\operatorname{tg}x=-1$, $x=-\frac{\pi}{4}+\pi n$;

в) $2\cos^2 x-\sin x\cos x+5\sin^2 x=3$, $2\sin^2 x-\sin x\cos x-\cos^2 x=0$,

$2\operatorname{tg}^2 x-\operatorname{tg}x-1=0$, $\cos x\neq 0$, $\operatorname{tg}x=1$, $x=\frac{\pi}{4}+\pi k$, $\operatorname{tg}x=-\frac{1}{2}$, $x=-\operatorname{arctg}\frac{1}{2}+\pi k$;

г) $4\sin^2 x-2\sin x\cos x=3$, $\sin^2 x-2\sin x\cos x-3\cos^2 x=0$,

$\operatorname{tg}^2 x-2\operatorname{tg}x-3=0$, $\cos x\neq 0$, $\operatorname{tg}x=-1$, $x=-\frac{\pi}{4}+\pi k$, $\operatorname{tg}x=3$, $x=\operatorname{arctg}3+\pi k$.

379. а) $\sqrt{3}\sin x\cos x+\cos^2 x=0$, $\cos x\left(\sqrt{3}\sin x+\cos x\right)=0$, $\cos x=0$,

$x=\frac{\pi}{2}+\pi n$, $\sqrt{3}\sin x+\cos x=0$, $\sqrt{3}\operatorname{tg}x=-1$, $\operatorname{tg}x=\frac{\sqrt{3}}{3}$, $x=-\frac{\pi}{6}+\pi n$;

б) $2\sin^2 x-3\sin x\cos x+4\cos^2 x=4$, $3\sin x\cos x=2-2\cos^2 x$,

$3\sin x\cos x=2\sin^2 x$, $\sin x(3\cos x-2\sin x)=0$, $\sin x=0$, $x=\pi n$,

$3\cos x-2\sin x=0$, $\operatorname{tg}x=\frac{3}{2}$, $\cos x\neq 0$, $x=\operatorname{arctg}\frac{3}{2}+\pi n$.

380. а) $3\sin^2 2x-2=\sin 2x\cos 2x$, $\sin^2 2x-\sin 2x\cos 2x-2\cos^2 2x=0$,

$\operatorname{tg}^2 2x-\operatorname{tg}2x-2=0$, $\cos^2 2x\neq 0$, $\operatorname{tg}2x=2$, $2x=\operatorname{arctg}2+\pi n$,

$x=\frac{1}{2}\operatorname{arctg}2+\frac{\pi n}{2}$, $\operatorname{tg}2x=-1$, $2x=-\frac{\pi}{4}+\pi n$, $x=-\frac{\pi}{8}+\frac{\pi n}{2}$;

б) $2\sin^2 4x-4-3\sin 4x\cos 4x-4\cos^2 4x=0,\ 2+2\cos^2 4x-4=3\sin 4x\cos 4x$,

$2\sin^2 4x+3\sin 4x\cos 4x=0,\ \sin 4x(2\sin 4x+3\cos 4x)=0,\ \sin 4x=0$,

$4x=\pi n,\ x=\dfrac{\pi n}{4},\ 2\sin 4x+3\cos 4x=0,\ 2\operatorname{tg}4x=-3,\ \cos 4x\neq 0$,

$x=-\dfrac{1}{4}\operatorname{arctg}\dfrac{3}{2}+\dfrac{\pi n}{4}$.

381. а) $\sin^2\dfrac{x}{2}-3=2\sin\dfrac{x}{2}\cos\dfrac{x}{2},\ 2\sin^2\dfrac{x}{2}+2\sin\dfrac{x}{2}\cos\dfrac{x}{2}+3\cos^2\dfrac{x}{2}=0$,

$2\operatorname{tg}^2\dfrac{x}{2}+2\operatorname{tg}\dfrac{x}{2}+3=0,\ \cos\dfrac{x}{2}\neq 0$, т.е. решений нет;

б) $3\sin^2\dfrac{x}{3}+4\cos^2\dfrac{x}{3}=3+\sqrt{3}\sin\dfrac{x}{3}\cos\dfrac{x}{3},\ \cos^2\dfrac{x}{3}-3\sqrt{3}\sin\dfrac{x}{3}\cos\dfrac{x}{3}=0$,

$\cos\dfrac{x}{3}\left(\cos\dfrac{x}{3}-\sqrt{3}\sin\dfrac{x}{3}\right)=0,\ \cos\dfrac{x}{3}=0,\ \dfrac{x}{3}=\dfrac{\pi}{2}+\pi n,\ x=\dfrac{3\pi}{2}+3\pi n$,

$\cos\dfrac{x}{3}-\sqrt{3}\sin\dfrac{x}{3}=0,\ \operatorname{ctg}\dfrac{x}{3}=\sqrt{3},\ \sin\dfrac{x}{3}\neq 0,\ \dfrac{x}{3}=\dfrac{\pi}{6}+\pi n,\ x=\dfrac{\pi}{2}+3\pi n$.

382. а) $\sin\left(\dfrac{\pi}{2}+2x\right)+\cos\left(\dfrac{\pi}{2}-2x\right)=0,\ \cos 2x+\sin 2x=0$,

$\operatorname{tg}2x=-1,\ \cos 2x\neq 0,\ 2x=-\dfrac{\pi}{4}+\pi n,\ x=-\dfrac{\pi}{8}+\dfrac{\pi n}{2}$;

б) $2\sin(\pi-3x)+\cos(2\pi-3x)=0,\ 2\sin 3x+\cos 3x=0$,

$\operatorname{tg}3x=-\dfrac{1}{2},\ \cos 3x\neq 0,\ 3x=-\operatorname{arctg}\dfrac{1}{2}+\pi n,\ x=-\dfrac{1}{3}\operatorname{arctg}\dfrac{1}{2}+\pi n$.

383. а) $\cos\left(\dfrac{\pi}{2}-\dfrac{x}{2}\right)-3\cos\left(\pi-\dfrac{x}{2}\right)=0,\ \sin\dfrac{x}{2}+3\cos\dfrac{x}{2}=0$,

$\operatorname{tg}\dfrac{x}{2}=-3,\ \cos\dfrac{x}{2}\neq 0,\ \dfrac{x}{2}=-\operatorname{arctg}3x+\pi n,\ x=-2\operatorname{arctg}3+2\pi n$;

б) $\sqrt{3}\sin\left(\pi-\dfrac{x}{3}\right)+\sin\left(\dfrac{\pi}{2}-\dfrac{x}{3}\right)=0,\ \sqrt{3}\sin\dfrac{x}{3}+3\cos\dfrac{x}{3}=0$,

$\operatorname{tg}\dfrac{x}{3}=-\sqrt{3},\ \cos\dfrac{x}{3}\neq 0,\ x=-\pi+3\pi n$.

384. а) $|\sin x|=|\cos x|,\ \sin x=\pm\cos x,\ \operatorname{tg}x=\pm 1,\ \cos x\neq 0,\ x=\pm\dfrac{\pi}{4}+\pi n$;

б) $|\sin 2x|=\left|\sqrt{3}\cos 2x\right|,\ \sin 2x=\pm\sqrt{3}\cos 2x,\ \operatorname{tg}2x=\pm\sqrt{3},\ \cos 2x\neq 0$,

$2x=\pm\dfrac{\pi}{3}+\pi n,\ x=\pm\dfrac{\pi}{6}+\dfrac{\pi n}{2}$.

385. а) $\sin\left(2x-\dfrac{\pi}{6}\right)+\cos\left(\dfrac{13\pi}{6}-2x\right)=0,\ \sin\left(2x-\dfrac{\pi}{6}\right)+\cos\left(2x-\dfrac{\pi}{6}\right)=0$,

$\operatorname{tg}\left(2x-\frac{\pi}{6}\right)=-1,\ \cos\left(2x-\frac{\pi}{6}\right)\neq 0,\ 2x-\frac{\pi}{6}=-\frac{\pi}{4}+\pi n,\ x=-\frac{\pi}{24}+\frac{\pi n}{2};$

б) $\sin\left(\frac{x}{2}+\frac{\pi}{3}\right)=\sqrt{3}\cos\left(\frac{47\pi}{3}-\frac{x}{2}\right),\ \sin\left(\frac{x}{2}+\frac{\pi}{3}\right)=\sqrt{3}\cos\left(\frac{x}{2}+\frac{\pi}{3}\right),$

$\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{3}\right)=\sqrt{3},\ \cos\left(\frac{x}{2}+\frac{\pi}{3}\right)\neq 0,\ \frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{3}+\pi n,\ x=2\pi n.$

386. а) $\sin^2 x-5\cos x=\sin x\cos x-5\sin x,\ \sin x(\sin x+5)-\cos x(\sin x+5)=0,$

$(\sin x+5)(\sin x-\cos x)=0,\ \sin x-\cos x=0,\ x=\frac{\pi}{4}+\pi n;$

б) $\cos^2 x-7\sin x+\sin x\cos x=7\cos x,\ \cos x(\cos x-7)+\sin x(\cos x-7)=0,$

$(\cos x-7)(\cos x+\sin x)=0,\ \cos x+\sin x=0,\ x=-\frac{\pi}{4}+\pi n.$

387. а) $\sin^2 x+\cos\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)-2\cos^2 x,\ \operatorname{tg}^2 x+\operatorname{tg} x-2=0,\ \cos x\neq 0,$

$\operatorname{tg} x=-2,\ x=-\operatorname{arctg} 2+\pi n,\ \operatorname{tg} x=1,\ x=\frac{\pi}{4}+\pi n;$

б) $\sin^2 3x+3\cos^2 3x-4\sin\left(\frac{\pi}{2}+3x\right)\cos\left(\frac{\pi}{2}+3x\right)=0,$

$\sin^2 3x+3\cos^2 3x+4\sin 3x\cos 3x=0,\ \operatorname{tg}^2 3x+4\operatorname{tg} 3x+3=0,\ \cos 3x\neq 0,$

$\operatorname{tg} 3x=-1,\ x=-\frac{\pi}{12}+\frac{\pi k}{3},\ \operatorname{tg} 3x=-3,\ x=-\frac{1}{3}\operatorname{arctg} 3+\frac{\pi k}{3};$

в) $\sin^2 x+2\sin(\pi-x)\cos x-3\cos^2(2\pi-x)=0,\ \operatorname{tg}^2 x+2\operatorname{tg} x-3=0,$

$\cos x\neq 0,\ \operatorname{tg} x=-3,\ x=-\operatorname{arctg} 3+\pi n,\ \operatorname{tg} x=1,\ x=\frac{\pi}{4}+\pi n;$

г) $\sin^2(2\pi-3x)+5\sin(\pi-3x)\cos 3x+4\sin^2\left(\frac{3\pi}{2}-3x\right)=0,$

$\operatorname{tg}^2 3x+5\operatorname{tg} 3x+4=0,\ \operatorname{tg} 3x=-4,\ 3x=-\operatorname{arctg} 4+\pi n,\ x=-\frac{1}{3}\operatorname{arctg} 4+\frac{\pi n}{3},$

$\operatorname{tg} 3x=-1,\ 3x=-\frac{\pi}{4}+\pi n,\ x=-\frac{\pi}{12}+\frac{\pi n}{3}.$

388. а) $3\sin^2\frac{x}{2}+\sin\frac{x}{2}\sin\left(\frac{\pi}{2}-\frac{x}{2}\right)=2,\ \sin^2\frac{x}{2}+\sin\frac{x}{2}\cos\frac{x}{2}-2\cos^2 x=0,$

$\operatorname{tg}^2\frac{x}{2}+\operatorname{tg}\frac{x}{2}-2=0,\ \cos\frac{x}{2}\neq 0,\ \operatorname{tg}\frac{x}{2}=-2,\ \frac{x}{2}=-\operatorname{arctg} 2+\pi n,\ x=-2\operatorname{arctg} 2+2\pi n,$

$\operatorname{tg}\frac{x}{2}=1;\ \frac{x}{2}=\frac{\pi}{4}+\pi n,\ x=\frac{\pi}{2}+2\pi n;$

б) $2\cos^2\frac{x}{2}-3\sin\left(\pi-\frac{x}{2}\right)\cos\left(2\pi-\frac{x}{2}\right)+7\sin^2\frac{x}{2}=3,$

$4\sin^2\frac{x}{2}-3\sin\frac{x}{2}\cos\frac{x}{2}-\cos^2\frac{x}{2}=0,\ 4\,\mathrm{tg}^2\frac{x}{2}-3\,\mathrm{tg}\frac{x}{2}-1=0$,

$\cos\frac{x}{2}\neq 0,\ \mathrm{tg}\frac{x}{2}=\frac{3+5}{8}=1,\ \frac{x}{2}=\frac{\pi}{4}+\pi n,\ x=\frac{\pi}{2}+2\pi n,\ \mathrm{tg}\frac{x}{2}=-\frac{1}{4}$,

$\frac{x}{2}=-\mathrm{arctg}\frac{1}{4}+\pi n,\ x=-2\,\mathrm{arctg}\frac{1}{4}+2\pi n$;

в) $4\cos^2\left(\frac{\pi}{2}+x\right)+\sqrt{3}\sin\left(\frac{3\pi}{2}-x\right)\sin(\pi+x)+3\cos^2(\pi+x)=3$,

$\sin^2 x+\sqrt{3}\sin x\cos x=0,\ \sin x\left(\sin x+\sqrt{3}\cos x\right)=0,\ \sin x=0,\ x=\pi n$,

$\sin x+\sqrt{3}\cos x=0,\ \mathrm{tg}\,x=-\sqrt{3},\ \cos x\neq 0,\ x=-\frac{\pi}{3}+\pi n$;

г) $3\sin^2\left(x-\frac{3\pi}{2}\right)-2\cos\left(\frac{3\pi}{2}+x\right)\cos(\pi+x)+2\sin^2(x-\pi)=2$,

$\cos^2 x+2\sin x\cos x=0,\ \cos x(\cos x+2\sin x)=0,\ \cos x=0$,

$x=\frac{\pi}{2}+\pi n,\ \cos x+2\sin x=0,\ \mathrm{tg}\,x=-\frac{1}{2},\ \cos x\neq 0,\ x=-\mathrm{arctg}\frac{1}{2}+\pi n$.

389. а) $2\sin^2(\pi+x)-5\cos\left(\frac{\pi}{2}+x\right)+2=0,\ 2\sin^2 x+5\sin x+2=0$,

$\sin x=\frac{-5-3}{4}$ – не подходит, $\sin x=-\frac{1}{2},\ x=(-1)^{k+1}\frac{\pi}{6}+\pi k$;

б) $2\cos^2 x+5\cos\left(\frac{\pi}{2}-x\right)-4=0,\ 2\sin^2 x-5\sin x+2=0$,

$\sin x=\frac{5+3}{4}$ – не подходит, $\sin x=\frac{1}{2},\ x=(-1)^k\frac{\pi}{6}+\pi k$;

в) $2\cos^2 x+\sin\left(\frac{\pi}{2}-x\right)-1=0,\ 2\cos^2 x+\cos x-1=0$,

$\cos x=\frac{-1-3}{4}=-1,\ x=\pi+2\pi k,\ \cos x=\frac{1}{2},\ x=\pm\frac{\pi}{3}+2\pi n$;

г) $5-5\sin(3(\pi-x))=\cos^2(\pi-3x),\ 5-5\sin(3\pi-3x)=\cos^2 3x$,

$\sin^2 3x-5\sin 3x+4=0,\ \sin 3x=4$ – не подходит, $\sin 3x=1$,

$3x=\frac{\pi}{2}+2\pi n,\ x=\frac{\pi}{6}+\frac{2\pi n}{3}$.

390. а) $2\,\mathrm{tg}^2 2x+3\,\mathrm{tg}(\pi+2x)=0,\ 2\,\mathrm{tg}^2 2x+3\,\mathrm{tg}\,2x=0$,

$\mathrm{tg}\,2x(2\,\mathrm{tg}\,2x+3)=0,\ \mathrm{tg}\,2x=0,\ 2x=\pi n,\ x=\frac{\pi n}{2},\ \mathrm{tg}\,2x=-\frac{3}{2}$,

$2x=-\mathrm{arctg}\frac{3}{2}+\pi n,\ x=-\frac{1}{2}\mathrm{arctg}\frac{3}{2}+\frac{\pi n}{2}$;

б) $\mathrm{tg}^2 3x-6\,\mathrm{ctg}\left(\frac{\pi}{2}-3x\right)=0,\ \mathrm{tg}^2 3x-6\,\mathrm{tg}\,3x=0,\ \mathrm{tg}\,3x(\mathrm{tg}\,3x-6)=0$,

$\operatorname{tg} 3x = 0,\ x = \frac{\pi n}{3},\ \operatorname{tg} 3x = 6,\ x = \frac{1}{3}\operatorname{arctg} 6 + \frac{\pi n}{3}.$

391. а) $3\operatorname{tg}^2 \frac{x}{2} - 2\operatorname{ctg}\left(\frac{3\pi}{2} + \frac{x}{2}\right) - 1 = 0,\ 3\operatorname{tg}^2 \frac{x}{2} + 2\operatorname{tg}\frac{x}{2} - 1 = 0,$

$\operatorname{tg}\frac{x}{2} = \frac{-1-2}{3} = -1,\ \frac{x}{2} = -\frac{\pi}{4} + \pi n,\ x = -\frac{\pi}{2} + 2\pi n,\ \operatorname{tg}\frac{x}{2} = \frac{1}{3},\ x = 2\operatorname{arctg}\frac{1}{3} + 2\pi n;$

б) $3\operatorname{tg}^2 4x - 2\operatorname{ctg}\left(\frac{\pi}{2} - 4x\right) = 1,\ 3\operatorname{tg}^2 4x - 2\operatorname{tg} 4x - 1 = 0,$

$\operatorname{tg} 4x = 1,\ x = \frac{\pi}{16} + \frac{\pi n}{4},\ \operatorname{tg} 4x = -\frac{1}{3},\ x = -\frac{1}{4}\operatorname{arctg}\frac{1}{3} + \frac{\pi n}{4}.$

392. а) $\operatorname{tg}(\pi + x) + 2\operatorname{tg}\left(\frac{\pi}{2} + x\right) + 1 = 0,\ \operatorname{tg} x - 2\operatorname{ctg} x + 1 = 0,$

$\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0,\ \operatorname{tg} x = -2,\ x = -\operatorname{arctg} 2 + \pi n,\ \operatorname{tg} x = 1,\ x = \frac{\pi}{4} + \pi n;$

б) $2\operatorname{ctg} x - 3\operatorname{ctg}\left(\frac{\pi}{2} - x\right) + 5 = 0,\ 2\operatorname{ctg} x - 3\operatorname{tg} x + 5 = 0,\ 3\operatorname{tg}^2 x - 5\operatorname{tg} x - 2 = 0,$

$\operatorname{tg} x = \frac{5 \pm \sqrt{25 + 4\cdot 3\cdot 2}}{6} = \frac{5 \pm 7}{6},\ \operatorname{tg} x = 2,\ x = \operatorname{arctg} 2 + \pi n,\ \operatorname{tg} x = -\frac{1}{3},$

$x = -\operatorname{arctg}\frac{1}{3} + \pi n.$

393. а) $\sin^2 x + \cos^2 2x + \cos^2\left(\frac{3\pi}{2} + 2x\right) + 2\cos x \operatorname{tg} x = 1,$

$\sin^2 x + \cos^2 2x + \sin^2 2x + 2\sin x - 1 = 0,\ \sin^2 x + 2\sin x = 0,\ \sin x(\sin x + 2) = 0,$
$\sin x = 0,\ x = \pi n;$

б) $2\cos^2 x - \sin\left(x - \frac{\pi}{2}\right) + \operatorname{tg} x \operatorname{tg}\left(x + \frac{\pi}{2}\right) = 0,\ 2\cos^2 x + \cos x - 1 = 0,$

$\cos x = \frac{-1-3}{4} = -1,\ x = \pi + \pi n,\ \cos x = \frac{1}{2},\ x = \pm\frac{\pi}{3} + 2\pi n.$

394. а) $\sin 2x < \frac{1}{2},\ 2x \in \left(-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n\right),\ x \in \left(-\frac{7\pi}{12} + \pi n; \frac{\pi}{12} + \pi n\right);$

б) $3\cos 4x < 1,\ \cos 4x < \frac{1}{3},\ 4x \in \left(\arccos\frac{1}{3} + 2\pi n; 2\pi - \arccos\frac{1}{3} + 2\pi n\right),$

$x \in \left(\frac{1}{4}\arccos\frac{1}{3} + \frac{\pi n}{2}; \frac{\pi}{2} - \frac{1}{4}\arccos\frac{1}{3} + \frac{\pi n}{2}\right);$

в) $\cos 3x > \frac{\sqrt{3}}{2},\ 3x \in \left(-\frac{\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n\right),\ x \in \left(-\frac{\pi}{18} + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{2\pi n}{3}\right);$

г) $7\sin\frac{x}{2} > -1,\ \sin\frac{x}{2} > -\frac{1}{7},\ \frac{x}{2} \in \left(-\arcsin\frac{1}{7} + 2\pi n; \arcsin\frac{1}{7} + \pi + 2\pi n\right)$,

$x \in \left(-2\arcsin\frac{1}{7} + 4\pi n; 2\arcsin\frac{1}{7} + 2\pi + 4\pi n\right)$.

395. а) $\sin\left(2x - \frac{\pi}{3}\right) > \frac{1}{3},\ 2x - \frac{\pi}{3} \in \left(\arcsin\frac{1}{3} + 2\pi n; \pi - \arcsin\frac{1}{3} + 2\pi n\right)$,

$x \in \left(\frac{1}{2}\arcsin\frac{1}{3} + \frac{\pi}{6} + \pi n; \frac{2\pi}{3} - \frac{1}{2}\arcsin\frac{1}{3} + \pi n\right)$;

б) $\cos\left(\frac{\pi}{4} - x\right) < \frac{\sqrt{2}}{2},\ x - \frac{\pi}{4} \in \left(\frac{\pi}{4} + 2\pi n; \frac{7\pi}{4} + 2\pi n\right),\ x \in \left(\frac{\pi}{2} + 2\pi n; 2\pi + 2\pi n\right)$;

в) $\cos\left(3x - \frac{\pi}{6}\right) > -\frac{1}{4},\ 3x - \frac{\pi}{6} \in \left(-\arccos\left(-\frac{1}{4}\right) + 2\pi n; \arccos\left(-\frac{1}{4}\right) + 2\pi n\right)$,

$x \in \left(\frac{\pi}{18} - \frac{1}{3}\arccos\left(-\frac{1}{4}\right) + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{1}{3}\arccos\left(-\frac{1}{4}\right) + \frac{2\pi n}{3}\right)$;

г) $\sin\left(\frac{3\pi}{4} - x\right) < \frac{\sqrt{3}}{2},\ \sin\left(x - \frac{3\pi}{4}\right) > -\frac{\sqrt{3}}{2}$,

$x - \frac{3\pi}{4} \in \left(-\frac{\pi}{3} + 2\pi n; \frac{4\pi}{3} + 2\pi n\right),\ x \in \left(-\frac{\pi}{3} + \frac{3\pi}{4} + 2\pi n; \frac{4\pi}{3} + \frac{3\pi}{4} + 2\pi n\right)$.

396. а) $\sin^2 x - 6\sin x\cos x + 5\cos^2 x > 0,\ \operatorname{tg}^2 x - 6\operatorname{tg} x + 5 > 0,\ \cos x \neq 0$,

$\operatorname{tg} x < 1,\ \operatorname{tg} x > 5,\ x \in \left(-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n\right),\ \left(\operatorname{arctg} 5 + \pi n; \frac{\pi}{2} + \pi n\right)$;

б) $\sin^2 x - 6\sin x\cos x + 5\cos^2 x < 0,\ \operatorname{tg}^2 x - 6\operatorname{tg} x + 5 < 0,\ \operatorname{tg} x \in (1;5)$,

$x \in \left(\frac{\pi}{4} + \pi n; \operatorname{arctg} 5 + \pi n\right)$.

397. а) $y = \sin x + \sqrt{-\cos^2 x},\ \cos^2 x \geq 0,\ x = \frac{\pi}{2} + \pi n$,

Область значений функции: $\{-1;\ 1\}$.

б) $y = \cos x + \sqrt{-\sin^2 x},\ \sin^2 x \geq 0,\ \sin x = 0,\ x = \pi n$,

Область значений функции: $\{-1;\ 1\}$.

398. а) $y = \cos 3x + \sqrt{\cos^2 3x - 1} = \cos 3x + \sqrt{-\sin^2 x},\ \sin^2 x \geq 0,\ \sin - 3x = 0,\ x = \frac{\pi n}{3}$,

Область значений функции: $\{-1;\ 1\}$.

б) $y = \sin 2x + \sqrt{\sin^2 4x - 1} = \sin 2x + \sqrt{-\cos^2 4x},\ \cos^2 4x \geq 0,\ \cos 4x = 0$,

$2x = \frac{\pi}{4} + \frac{\pi n}{2},\ x = \frac{\pi}{8} + \frac{\pi n}{4}$. Область определения функции: $\left\{\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right\}$.

Глава 3. Преобразование тригонометрических выражений

§21. Синус и косинус суммы аргументов

399. а) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$;

б) $\cos 105^\circ = \cos(60^\circ + 45^\circ) = \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$.

400. а) $\sin(\alpha + \beta) - \sin\alpha\cos\beta = \sin\alpha\cos\beta + \sin\beta\cos\alpha - \sin\alpha\cos\beta = \sin\beta\cos\alpha$;

б) $\sin\left(\frac{\pi}{3} + \alpha\right) - \frac{1}{2}\sin\alpha = \sin\frac{\pi}{3}\cos\alpha + \cos\frac{\pi}{3}\sin\alpha - \frac{1}{2}\sin\alpha = \frac{\sqrt{3}}{2}\cos\alpha$;

в) $\sin\alpha\sin\beta + \cos(\alpha + \beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos\alpha\cos\beta$;

г) $\cos\left(\alpha + \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha$.

401. а) $\sin(\alpha + \beta) + \sin(-\alpha)\cos(-\beta) = \sin\alpha\cos\beta$,
$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin\beta\cos\alpha$ тождество неверно;

б) $\cos(\alpha + \beta) + \sin(-\alpha)\sin(-\beta) = \cos\alpha\cos\beta$,
$\cos(\alpha + \beta) + \sin(-\alpha)\sin(-\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta + \sin\alpha\sin\beta = \cos\alpha\cos\beta$.

402. а) $\sin 74^\circ \cos 16^\circ + \cos 74^\circ \sin 16^\circ = \sin(74^\circ + 16^\circ) = \sin 90^\circ = 1$;

б) $\cos 23^\circ \cos 22^\circ - \sin 23^\circ \sin 22^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$;

в) $\sin 89^\circ \cos 1^\circ + \cos 89^\circ \sin 1^\circ = \sin(89^\circ + 1^\circ) = \sin 90^\circ = 1$;

г) $\cos 178^\circ \cos 2^\circ - \sin 178^\circ \sin 2^\circ = \cos(178^\circ + 2^\circ) = -1$.

403. а) $\sin\frac{\pi}{5}\cos\frac{\pi}{20} + \cos\frac{\pi}{5}\sin\frac{\pi}{20} = \sin\left(\frac{\pi}{5} + \frac{\pi}{20}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$;

б) $\cos\frac{2\pi}{7}\cos\frac{5\pi}{7} - \sin\frac{2\pi}{7}\sin\frac{5\pi}{7} = \cos\left(\frac{2\pi}{7} + \frac{5\pi}{7}\right) = \cos\pi = -1$;

в) $\sin\frac{\pi}{12}\cos\frac{11\pi}{12} + \cos\frac{\pi}{12}\sin\frac{11\pi}{12} = \sin\left(\frac{\pi}{12} + \frac{11\pi}{12}\right) = \sin\pi = 0$;

г) $\cos\frac{2\pi}{15}\cos\frac{\pi}{5} - \sin\frac{2\pi}{15}\sin\frac{\pi}{5} = \cos\left(\frac{2\pi}{15} + \frac{\pi}{5}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$.

404. а) $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \sin\left(\frac{\pi}{3} - x\right)$, $\sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x = \sin\left(\frac{\pi}{3} + x\right)$;

б) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos\left(x + \frac{\pi}{3}\right)$.

405. а) $\sin 5x \cos 3x + \cos 5x \sin 3x = \sin 8x$, $\sin(5x + 3x) = \sin 8x$;

б) $\cos 5x \cos 3x - \sin 5x \sin 3x = \cos 8x$, $\cos(5x + 3x) = \cos 8x$;

в) $\sin 7x\cos 4x + \cos 7x\sin 4x = \sin 11x$, $\sin(7x + 4x) = \sin 11x$;

г) $\cos 2x\cos 12x - \sin 2x\sin 12x = \cos 14x$.

406. а) $\sin 2x\cos x + \cos 2x\sin x = 1$, $\sin 3x = 1$, $3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$;

б) $\cos 3x\cos 5x = \sin 3x\sin 5x$, $\cos 3x\cos 5x - \sin 3x\sin 5x = 0$,

$\cos 8x = 0$, $8x = \frac{\pi}{2} + \pi n$, $x = \frac{\pi}{16} + \frac{\pi n}{8}$.

407. а) $\sin 6x\cos x + \cos 6x\sin x = \frac{1}{2}$, $\sin 7x = \frac{1}{2}$,

$7x = (-1)^k \frac{\pi}{6} + \pi k$, $x = (-1)^k \frac{\pi}{42} + \frac{\pi k}{7}$;

б) $\cos 5x\cos 7x - \sin 5x\sin 7x = -\frac{\sqrt{3}}{2}$, $\cos 12x = -\frac{\sqrt{3}}{2}$,

$12x = \pm\frac{5\pi}{6} + 2\pi n$, $x = \pm\frac{5\pi}{72} + \frac{\pi n}{6}$.

408. $\sin t = \frac{3}{5}$, $0 < t < \frac{\pi}{2}$, $\cos t = \frac{4}{5}$.

а) $\sin\left(\frac{\pi}{3} + t\right) = \sin\frac{\pi}{3}\cos t + \sin t\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\frac{4}{5} + \frac{1}{2}\frac{3}{5} = \frac{2\sqrt{3}}{5} + \frac{3}{10} = \frac{4\sqrt{3}+3}{10}$;

б) $\cos\left(\frac{\pi}{2} + t\right) = -\sin t = -\frac{3}{5}$; **в)** $\sin\left(\frac{\pi}{2} + t\right) = \cos t = \frac{4}{5}$;

г) $\cos\left(\frac{\pi}{3} + t\right) = \cos\frac{\pi}{3}\cos t - \sin\frac{\pi}{3}\sin t = \frac{1}{2}\frac{4}{5} - \frac{\sqrt{3}}{2}\frac{3}{5} = \frac{4 - 3\sqrt{3}}{10}$.

409. $\cos t = -\frac{5}{13}$, $\frac{\pi}{2} < t < \pi$, $\sin t = \frac{12}{13}$.

а) $\sin\left(t + \frac{\pi}{6}\right) = \sin t\frac{\sqrt{3}}{2} + \cos t\frac{1}{2} = \frac{\sqrt{3}}{2}\frac{12}{13} - \frac{5}{13}\frac{1}{2} = \frac{12\sqrt{3} - 5}{26}$;

б) $\cos\left(t + \frac{3\pi}{2}\right) = \sin t = \frac{12}{13}$;

в) $\cos\left(t + \frac{\pi}{6}\right) = \cos t\cos\frac{\pi}{6} - \sin t\sin\frac{\pi}{6} = -\frac{5}{13}\frac{\sqrt{3}}{2} - \frac{12}{13}\frac{1}{2} = \frac{-5\sqrt{3} - 12}{26}$;

г) $\sin\left(t + \frac{3\pi}{2}\right) = -\cos t = \frac{5}{13}$.

410. $\cos\alpha = \frac{15}{17}$, $\cos\beta = \frac{4}{5}$, $0 < \alpha < \frac{\pi}{2}$,

$0 < \beta < \frac{\pi}{2}$, $\sin\alpha = \frac{8}{17}$, $\sin\beta = \frac{3}{5}$.

а) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\frac{8}{17}\frac{4}{5}+\frac{3}{5}\frac{15}{17}=\frac{77}{85}$;

б) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{5}{17}\frac{4}{5}-\frac{8}{17}\frac{3}{5}=\frac{60-24}{85}=\frac{36}{85}$.

411. $\sin\alpha=\frac{4}{5}$, $\cos\beta=-\frac{15}{17}$, $\frac{\pi}{2}<\alpha<\pi$, $\frac{\pi}{2}<\beta<\pi$, $\cos\alpha=-\frac{3}{5}$, $\sin\beta=\frac{8}{17}$.

а) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\frac{4}{5}\left(-\frac{15}{17}\right)+\frac{8}{17}\left(-\frac{3}{5}\right)=-\frac{84}{85}$;

б) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{13}{85}$.

412. $\sin\alpha=\frac{9}{41}$, $\sin\beta=-\frac{40}{41}$, $0<\alpha<\frac{\pi}{2}$, $\frac{3\pi}{2}<\beta<2\pi$, $\cos\alpha=\frac{40}{41}$, $\cos\beta=\frac{9}{41}$.

а) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\left(\frac{9}{41}\right)^2-\left(\frac{40}{41}\right)^2=\frac{9-40}{41}\cdot\frac{9+40}{41}=\frac{1519}{1681}$;

б) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{40}{41}\frac{9}{41}+\frac{9}{41}\frac{40}{41}=\frac{720}{1681}$.

413. а) $\sin 75°\cos 75°=\frac{1}{4}$,

$\sin 75°\cos 75°=\frac{1}{2}(\sin 75°\cos 75°+\sin 75°\cos 75°)=\frac{1}{2}\sin 150°=\frac{1}{2}\sin 30°=\frac{1}{4}$;

б) $\cos^2 75°-\sin^2 75°-\sin 75°\sin 75°=\cos(75°+75°)=\cos 150°=-\cos 30°=-\frac{\sqrt{3}}{2}$;

в) $\sin 105°\cos 105°=-\frac{1}{4}$, $\sin 105°\cos 105°=\frac{1}{2}(\sin 105°\cos 105°+$

$+\sin 105°\cos 105°)=\sin 210°=-\frac{1}{2}\sin 30°=-\frac{1}{4}$;

г) $\cos^2 75°+\sin^2 75°=1$.

414. а) $\sin 2x=2\sin x\cos x$, $\sin 2x=\sin(x+x)=\sin x\cos x+\sin x\cos x=2\sin x\cos x$;

б) $\cos 2x=\cos^2 x-\sin^2 x$, $\cos 2x=\cos(x+x)=\cos x\cos x-\sin x\sin x=\cos^2 x-\sin^2 x$.

415. а) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$,

$\sin(\alpha-\beta)=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$;

б) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$,

$\cos(\alpha+(-\beta))=\cos\alpha\cos(-\beta)-\sin\alpha\sin(-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

416. а) $\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x=1$, $\sin\frac{\pi}{4}\sin x+\cos\frac{\pi}{4}\cos x=1$,

$\cos\left(x-\frac{\pi}{4}\right)=1$, $x-\frac{\pi}{4}=2\pi n$, $x=\frac{\pi}{4}+2\pi n$;

б) $\sin x+\cos x=1,\ \sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}$,

$x+\dfrac{\pi}{4}=(-1)^k\dfrac{\pi}{4}+\pi n,\ x=(-1)^k\dfrac{\pi}{4}-\dfrac{\pi}{4}+\pi n$;

в) $\dfrac{\sqrt{3}}{2}\cos x-\dfrac{1}{2}\sin x=1,\ \cos\dfrac{\pi}{6}\cos x-\sin\dfrac{\pi}{6}\sin x=1$,

$x+\dfrac{\pi}{6}=2\pi n,\ x=-\dfrac{\pi}{6}+2\pi n$;

г) $\sqrt{3}\cos x-\sin x=1,\ \dfrac{\sqrt{3}}{2}\cos x-\dfrac{1}{2}\sin x=\dfrac{1}{2}$,

$x+\dfrac{\pi}{6}=\pm\dfrac{\pi}{3}+2\pi n,\ x=\pm\dfrac{\pi}{3}-\dfrac{\pi}{6}+2\pi n$.

417. а) $\sin x\cos 3x+\cos x\sin 3x>\dfrac{1}{2},\ \sin 4x>\dfrac{1}{2}$,

$4x\in\left(\dfrac{\pi}{6}+2\pi n;\dfrac{5\pi}{6}+2\pi n\right),\ x\in\left(\dfrac{\pi}{24}+\dfrac{\pi n}{2};\dfrac{5\pi}{24}+\dfrac{\pi n}{2}\right)$;

б) $\cos 2x\cos 5x-\sin 2x\sin 5x<-\dfrac{1}{3},\ \cos 7x<-\dfrac{1}{3}$,

$7x\in\left(\pi-\arccos\dfrac{1}{3}+2\pi n;\pi+\arccos\dfrac{1}{3}+2\pi n\right)$,

$x\in\left(\dfrac{\pi}{7}-\dfrac{1}{7}\arccos\dfrac{1}{3}+\dfrac{2\pi n}{7};\dfrac{\pi}{7}+\dfrac{1}{7}\arccos\dfrac{1}{3}+\dfrac{2\pi n}{7}\right)$;

в) $\sin x\cos\dfrac{x}{2}+\cos x\sin\dfrac{x}{2}\le-\dfrac{2}{7},\ \sin\dfrac{3x}{2}\le-\dfrac{2}{7}$,

$\dfrac{3x}{2}\in\left[-\pi+\arcsin\dfrac{2}{7}+2\pi n;-\arcsin\dfrac{2}{7}+2\pi n\right]$,

$x\in\left[-\dfrac{2\pi}{3}+\dfrac{2}{3}\arcsin\dfrac{2}{7}+\dfrac{4\pi n}{3};-\dfrac{2}{3}\arcsin\dfrac{2}{7}+\dfrac{4\pi n}{3}\right]$;

г) $\cos\dfrac{x}{2}\cos\dfrac{x}{4}-\sin\dfrac{x}{2}\sin\dfrac{x}{4}>\dfrac{\sqrt{2}}{2},\ \cos\dfrac{3x}{4}>\dfrac{\sqrt{2}}{2}$,

$\dfrac{3x}{4}\in\left(-\dfrac{\pi}{4}+2\pi n;\dfrac{\pi}{4}+2\pi n\right),\ x\in\left(-\dfrac{\pi}{3}+\dfrac{8\pi n}{3};\dfrac{\pi}{3}+\dfrac{8\pi n}{3}\right)$.

§22. Синус и косинус разности аргументов

418. а) $\sin(60^\circ-\beta)=\sin 60^\circ\cos\beta-\cos 60^\circ\sin\beta=\dfrac{\sqrt{3}}{2}\cos\beta-\dfrac{1}{2}\sin\beta$;

б) $\cos(\beta-30^\circ)=\cos\beta\cos 30^\circ+\sin\beta\sin 30^\circ=\dfrac{\sqrt{3}}{2}\cos\beta+\dfrac{1}{2}\sin\beta$;

в) $\sin(\alpha-30^\circ)=\sin\alpha\cos30^\circ-\sin30^\circ\cos\alpha=-\frac{\sqrt{3}}{2}\sin\alpha+\frac{1}{2}\cos\alpha$;

г) $\cos(60^\circ-\alpha)=\cos60^\circ\cos\alpha+\sin60^\circ\sin\alpha=\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha$.

419. а) $\sin\left(\frac{5\pi}{6}-\alpha\right)-\frac{1}{2}\cos\alpha=\sin\frac{5\pi}{6}\cos\alpha-\sin\alpha\cos\frac{5\pi}{6}-\frac{1}{2}\cos\alpha=$

$=\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha-\frac{1}{2}\cos\alpha=\frac{\sqrt{3}}{2}\sin\alpha$;

б) $\sqrt{3}\cos\alpha-2\cos\left(\alpha-\frac{\pi}{6}\right)=\sqrt{3}\cos\alpha-2\cos\alpha\cos\frac{\pi}{6}-2\sin\alpha\sin\frac{\pi}{6}=-\sin\alpha$;

в) $\frac{\sqrt{3}}{2}\sin\alpha+\cos\left(\alpha-\frac{5\pi}{3}\right)=\frac{\sqrt{3}}{2}\sin\alpha+\cos\alpha\cos\frac{5\pi}{3}+\sin\alpha\sin\frac{5\pi}{3}=\frac{1}{2}\cos\alpha$;

г) $\sqrt{2}\sin\left(\alpha-\frac{\pi}{4}\right)-\sin\alpha=\sqrt{2}\sin\alpha\cos\frac{\pi}{4}-\sqrt{2}\cos\alpha\sin\frac{\pi}{4}-\sin\alpha=-\cos\alpha$.

420. а) $\cos(\alpha-\beta)-\cos\alpha\cos\beta=\cos\alpha\cos\beta+\sin\alpha\sin\beta-\cos\alpha\cos\beta=\sin\alpha\sin\beta$;
б) $\sin(\alpha+\beta)+\sin(\alpha-\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha+\sin\alpha\cos\beta-\sin\beta\cos\alpha=$
$=2\sin\alpha\cos\beta$;
в) $\sin\alpha\cos\beta-\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\alpha\cos\beta+\sin\beta\cos\alpha=\sin\beta\cos\alpha$;
г) $\cos(\alpha-\beta)-\cos(\alpha+\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta-\cos\alpha\cos\beta+\sin\alpha\sin\beta=$
$=2\sin\alpha\sin\beta$.

421. а) $\cos36^\circ\cos24^\circ-\sin36^\circ\sin24^\circ=\cos(36^\circ+24^\circ)=\cos60^\circ=\frac{1}{2}$;

б) $\cos36^\circ\cos24^\circ-\sin36^\circ\sin24^\circ=\cos(36^\circ+24^\circ)=\cos60^\circ=\frac{1}{2}$;

в) $\sin63^\circ\cos27^\circ+\cos63^\circ\sin27^\circ=\sin(63^\circ+27^\circ)=\sin90^\circ=1$;

г) $\sin51^\circ\cos21^\circ-\cos51^\circ\sin21^\circ=\sin(51^\circ-21^\circ)=\frac{1}{2}$.

422. а) $\cos\frac{5\pi}{8}\cos\frac{3\pi}{8}+\sin\frac{5\pi}{8}\sin\frac{3\pi}{8}=\cos\left(\frac{5\pi}{8}-\frac{3\pi}{8}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

б) $\sin\frac{2\pi}{15}\cos\frac{\pi}{5}+\cos\frac{2\pi}{15}\sin\frac{\pi}{5}=\sin\left(\frac{2\pi}{15}+\frac{\pi}{5}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$;

в) $\cos\frac{\pi}{12}\cos\frac{\pi}{4}-\sin\frac{\pi}{12}\sin\frac{\pi}{4}=\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$;

г) $\sin\frac{\pi}{12}\cos\frac{\pi}{4}-\cos\frac{\pi}{12}\sin\frac{\pi}{4}=\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$.

423. а) $\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=\sin\left(\frac{\pi}{3}-x\right)$, $\sin\frac{\pi}{3}\cos x-\cos x\frac{\pi}{3}\sin x=\sin\left(\frac{\pi}{3}-x\right)$;

б) $\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=\cos\left(\frac{\pi}{3}-x\right)$, $\cos\frac{\pi}{3}\cos x+\sin\frac{\pi}{3}\sin x=\cos\left(\frac{\pi}{3}-x\right)$.

424. а) $\cos(\alpha-\beta)+\sin(-\alpha)\sin\beta=\cos\alpha\cos\beta$,

$\cos\alpha\cos\beta+\sin\alpha\sin\beta-\sin\alpha\sin\beta=\cos\alpha\cos\beta$;

б) $\sin(30°-\alpha)-\cos(60°-\alpha)=-\sqrt{3}\sin\alpha$,

$\sin 30°\cos\alpha-\sin\alpha\cos 30°-\cos 60°\cos\alpha-\sin 60°\sin\alpha=$

$=\frac{1}{2}\cos\alpha-\frac{\sqrt{3}}{2}\sin\alpha-\frac{1}{2}\cos\alpha-\frac{\sqrt{3}}{2}\sin\alpha=-\sqrt{3}\sin\alpha$;

в) $\sin(\alpha-\beta)-\cos\alpha\sin(-\beta)=\sin\alpha\cos\beta$,

$\sin\alpha\cos\beta-\sin\beta\cos\alpha+\sin\beta\cos\alpha=\sin\alpha\cos\beta$;

г) $\sin(30°-\alpha)+\sin(30°+\alpha)=\cos\alpha$,

$\frac{1}{2}\cos\alpha-\frac{\sqrt{3}}{2}\sin\alpha+\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha=\cos\alpha$.

425. а) $\sin(\alpha-\beta)-\sin(\alpha+\beta)=-2\cos\alpha\sin\beta$,

$\sin\alpha\cos\beta-\sin\beta\cos\alpha-\sin\alpha\cos\beta-\sin\beta\cos\alpha=-2\cos\alpha\sin\beta$;

б) $\cos(\alpha-\beta)+\cos(\alpha+\beta)=2\cos\alpha\cos\beta$,

$\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\alpha\cos\beta-\sin\alpha\sin\beta=2\cos\alpha\cos\beta$.

426. а) $\cos 6x\cos 5x+\sin 6x\sin 5x=-1$, $\cos(6x-5x)=-1$, $\cos x=-1$, $x=\pi+2\pi n$;

б) $\sin 3x\cos 5x-\sin 5x\cos 3x=\frac{1}{2}$, $\sin(3x-5x)=\frac{1}{2}$, $\sin 2x=-\frac{1}{2}$,

$2x=(-1)^{k+1}\frac{\pi}{6}+\pi k$, $x=(-1)^{k+1}\frac{\pi}{12}+\frac{\pi k}{2}$.

427. а) $\sin 15°=\sin(45°-30°)=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\frac{1}{2}=\frac{\sqrt{6}-\sqrt{2}}{2}$;

б) $\cos 15°=\cos(45°-30°)=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{2}$;

в) $\sin 15°\cos 15°=\frac{1}{2}(\sin(15°+15°))=\frac{1}{2}\sin 30°=\frac{1}{4}$;

г) $\cos^2 15°-\sin^2 15°=\cos(15°+15°)=\frac{\sqrt{3}}{2}$.

428. а) Опечатка в условии:

$\sin 77°\cos 17°-\sin 13°\cos 73°=\sin 77°\cos 17°-\cos 77°\sin 17°=$

$=\sin(77°-17°)=\sin 60°=\frac{\sqrt{3}}{2}$;

б) $\cos 125°\cos 5°+\sin 55°\cos 85°=-\cos 55°\cos 5°+\sin 55°\cos 5°=-\cos 60°=-\frac{1}{2}$.

429. а) $\sin\left(\frac{\pi}{6}+t\right)\cos\left(\frac{\pi}{3}-t\right)+\sin\left(\frac{2\pi}{3}+t\right)\sin\left(\frac{\pi}{3}-t\right)=$

$\sin\left(\frac{\pi}{6}+t\right)\cos\left(\frac{\pi}{3}-t\right)+\sin\left(\frac{\pi}{3}-t\right)\sin\left(\frac{\pi}{3}-t\right)=\cos^2 t\left(\frac{\pi}{3}-t\right)+\sin^2\left(\frac{\pi}{3}-t\right)=1$;

б) $\cos\left(\frac{\pi}{4}+t\right)\cos\left(\frac{\pi}{12}-t\right)-\cos\left(\frac{\pi}{4}-t\right)\cos\left(\frac{5\pi}{12}+t\right)=$

$=\cos\left(\frac{\pi}{12}-t\right)\sin\left(\frac{\pi}{4}-t\right)-\sin\left(\frac{\pi}{12}-t\right)\cos\left(\frac{\pi}{4}-t\right)=\sin\left(\frac{\pi}{4}-t-\frac{\pi}{12}+t\right)=\sin\frac{\pi}{6}=\frac{1}{2}$.

430. а) $\frac{\cos 105^\circ\cos 55^\circ+\sin 105^\circ\cos 85^\circ}{\sin 95^\circ\cos 5^\circ-\cos 95^\circ\sin 185^\circ}=\frac{\sin 15^\circ\cos 5^\circ+\cos 15^\circ\sin 15^\circ}{\cos^2 5^\circ-\sin^2 5^\circ}=$

$=\frac{-\sin 10^\circ}{\cos 10^\circ}=-\operatorname{tg}10^\circ$;

б) $\frac{\sin 25^\circ\cos 5^\circ-\cos 25^\circ\cos 85^\circ}{\cos 375^\circ\cos 5^\circ-\sin 15^\circ\sin 365^\circ}=\frac{\sin(25^\circ-5^\circ)}{\cos 15^\circ\cos 5^\circ-\sin 15^\circ\sin 5^\circ}=\frac{\sin 20^\circ}{\cos 20^\circ}=\operatorname{tg}20^\circ$.

431. а) $\frac{\sin(\alpha+\beta)-\cos\alpha\sin\beta}{\sin(\alpha-\beta)+\cos\alpha\sin\beta}=\frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta}=1$;

б) $\frac{\sin(\alpha-\beta)+2\cos\alpha\sin\beta}{2\cos\alpha\cos\beta-\cos(\alpha-\beta)}=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\operatorname{tg}(\alpha+\beta)$;

в) $\frac{\cos(\alpha+\beta)+\sin\alpha\sin\beta}{\cos(\alpha-\beta)-\sin\alpha\sin\beta}=\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}=1$;

г) $\frac{\cos(\alpha-\beta)+2\sin\alpha\sin\beta}{2\sin\alpha\cos\beta-\sin(\alpha-\beta)}=\operatorname{ctg}(\alpha+\beta)$.

432. $\sin t=\frac{5}{13}$, $\frac{\pi}{2}<t<\pi$, $\cos t=-\frac{12}{13}$,

а) $\sin\left(\frac{\pi}{3}-t\right)=\sin\frac{\pi}{3}\cos t-\cos\frac{\pi}{3}\sin t=-\frac{\sqrt{3}}{2}\cdot\frac{12}{13}-\frac{1}{2}\cdot\frac{5}{13}=\frac{-12\sqrt{3}-5}{26}$;

б) $\cos\left(t-\frac{\pi}{2}\right)=\sin t=\frac{5}{13}$; в) $\sin\left(\frac{\pi}{2}-t\right)=\sin\frac{\pi}{2}\cos t-\cos\frac{\pi}{2}\sin t=\cos t=-\frac{12}{13}$;

г) $\cos\left(\frac{\pi}{3}-t\right)=\cos\frac{\pi}{3}\cos t+\sin\frac{\pi}{3}\sin t=-\frac{1}{2}\cdot\frac{12}{13}+\frac{\sqrt{3}}{2}\cdot\frac{5}{13}=\frac{5\sqrt{3}-12}{26}$.

433. $\cos t=\frac{3}{5}$, $\frac{3\pi}{2}<t<2\pi$

а) $\sin\left(t-\frac{\pi}{5}\right)=\sin t\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos t=-\frac{4}{5}\frac{\sqrt{3}}{2}-\frac{1}{2}\frac{3}{5}=-\frac{4\sqrt{3}+3}{10}$;

б) $\sin\left(t-\frac{3\pi}{2}\right)=\sin t\cos\frac{3\pi}{2}-\sin\frac{3\pi}{2}\cos t=\cos t=\frac{3}{5}$;

в) $\cos\left(t-\frac{3\pi}{2}\right)=\cos t\cos\frac{3\pi}{2}+\sin t\sin\frac{3\pi}{2}=-\sin t=\frac{4}{5}$;

г) $\cos\left(t-\frac{\pi}{5}\right)=\cos t\cos\frac{\pi}{6}+\sin t\sin\frac{\pi}{6}=\cos t\frac{\sqrt{3}}{2}+\sin t\frac{1}{2}=\frac{3}{5}\frac{\sqrt{3}}{2}-\frac{4}{5}\frac{1}{2}=\frac{3\sqrt{3}-4}{10}$.

434. $\sin\alpha = \frac{4}{5},\ \cos\beta = -\frac{15}{17},\ \frac{\pi}{2} < \alpha < \pi,\ \frac{\pi}{2} < \beta < \pi$

а) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha = -\frac{4}{5}\cdot\frac{15}{17} + \frac{8}{17}\cdot\frac{3}{5} = \frac{-60+24}{85} = -\frac{36}{85}$;

б) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\beta\sin\alpha = \frac{3}{5}\cdot\frac{15}{17} + \frac{4}{5}\cdot\frac{8}{17} = \frac{77}{85}$.

435. $\sin\beta = -\frac{12}{13},\ \cos\alpha = -\frac{4}{5},\ \pi < \beta < \frac{3\pi}{2},\ \cos\beta = -\frac{5}{13},\ \sin\alpha = \frac{3}{5},\ \frac{\pi}{2} < \alpha < \pi$

а) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha = -\frac{3}{5}\cdot\frac{5}{13} - \frac{12}{13}\cdot\frac{4}{5} = \frac{-15-48}{65} = -\frac{63}{65}$;

б) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$.

436. а) $\dfrac{\sqrt{2}\cos\alpha - 2\cos\left(\frac{\pi}{4}-\alpha\right)}{2\sin\left(\frac{\pi}{6}+\alpha\right) - \sqrt{3}\sin\alpha} = -\sqrt{2}\,\mathrm{tg}\,\alpha$,

$$\frac{\sqrt{2}\cos\alpha - \sqrt{2}\cos\alpha - \sqrt{2}\sin\alpha}{\cos\alpha + \sqrt{3}\sin\alpha - \sqrt{3}\sin\alpha} = \frac{-\sqrt{2}\sin\alpha}{\cos\alpha} = -\sqrt{2}\,\mathrm{tg}\,\alpha;$$

б) $\dfrac{\cos\alpha - 2\cos\left(\frac{\pi}{3}+\alpha\right)}{2\sin\left(-\frac{\pi}{6}+\alpha\right) - \sqrt{3}\sin\alpha} = -\sqrt{3}\,\mathrm{tg}\,\alpha$,

$$\frac{\cos\alpha - \cos\alpha + \sqrt{3}\sin\alpha}{2\sin\alpha\cos\frac{\pi}{6} - 2\sin\frac{\pi}{6}\cos\alpha - \sqrt{3}\sin\alpha} = \frac{\sqrt{3}\sin\alpha}{-\cos\alpha} = -\sqrt{3}\,\mathrm{tg}\,\alpha.$$

437. а) $\sqrt{2}\cos\left(\frac{\pi}{4}-x\right) - \cos x = \frac{1}{2},\ \cos x + \sin x - \cos x = \frac{1}{2}$,

$\sin x = \frac{1}{2},\ x = (-1)^k\frac{\pi}{6} + \pi k$;

б) $\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right) + \sin\frac{x}{2} = \frac{\sqrt{3}}{2},\ \cos\frac{x}{2} - \sin\frac{x}{2} + \sin\frac{x}{2} = \frac{\sqrt{3}}{2}$,

$\cos\frac{x}{2} = \frac{\sqrt{3}}{2},\ \frac{x}{2} = \pm\frac{\pi}{6} + 2\pi n,\ x = \pm\frac{\pi}{3} + 4\pi n$.

438. а) $\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = 1,\ \sin\left(x-\frac{\pi}{4}\right) = 1,\ x - \frac{\pi}{4} = \frac{\pi}{2} + 2\pi n,\ x = \frac{3\pi}{4} + 2\pi n$;

б) $\sin x - \cos x = 1,\ \sin\left(x-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$,

$x-\frac{\pi}{4}=(-1)^n\frac{\pi}{4}+2\pi n,\ \ x=(-1)^n\frac{\pi}{4}+\frac{\pi}{4}+2\pi n$;

в) $\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1,\ \cos\left(x-\frac{\pi}{6}\right)=1,\ \ x-\frac{\pi}{6}=2\pi n,\ \ x=\frac{\pi}{6}+2\pi n$;

г) $\sqrt{3}\cos x+\sin x=1,\ \cos\left(x-\frac{\pi}{6}\right)=\frac{1}{2},\ \ x-\frac{\pi}{6}=\pm\frac{\pi}{3}+2\pi n,\ \ x=\pm\frac{\pi}{3}+\frac{\pi}{6}+2\pi n$.

439. а) $\sin 5x\cos 3x-\cos 5x in 3x>\frac{1}{2},\ \sin 2x>\frac{1}{2}$,

$2x\in\left(\frac{\pi}{6}+2\pi n;\frac{5\pi}{6}+2\pi n\right),\ \ x\in\left(\frac{\pi}{12}+\pi n;\frac{5\pi}{12}+\pi n\right)$;

б) $\cos x\cos\frac{x}{2}+\sin x\sin\frac{x}{2}<-\frac{2}{7},\ \cos\frac{x}{2}<-\frac{2}{7}$,

$\frac{x}{2}\in\left(\pi-\arccos\frac{2}{7}+2\pi n;\pi-\arccos\frac{2}{7}+2\pi n\right)$,

$x\in\left(2\pi-2\arccos\frac{2}{7}+4\pi n;2\pi-2\arccos\frac{2}{7}+4\pi n\right)$;

в) $\sin\frac{x}{4}\cos\frac{x}{2}-\cos\frac{x}{4}\sin\frac{x}{2}<\frac{1}{3},\ \sin\left(\frac{x}{4}-\frac{x}{2}\right)<\frac{1}{3},\ \sin\frac{x}{4}>-\frac{1}{3}$,

$\frac{x}{4}\in\left(-\arcsin\frac{1}{3}+2\pi n;\pi+\arcsin\frac{1}{3}+2\pi n\right)$,

$x\in\left(-4\arcsin\frac{1}{3}+8\pi n;4\pi+4\arcsin\frac{1}{3}+8\pi n\right)$;

г) $\sin 2x\sin 5x+\cos 2x\cos 5x>-\frac{\sqrt{3}}{2},\ \cos 3x>-\frac{\sqrt{3}}{2}$,

$3x\in\left(-\frac{5\pi}{6}+2\pi n;\frac{5\pi}{6}+2\pi n\right),\ \ x\in\left(-\frac{5\pi}{18}+\frac{2\pi n}{3};\frac{5\pi}{18}+\frac{2\pi n}{3}\right)$.

§23. Тангенс суммы и разности аргументов

440. а) $\operatorname{tg}15°=\operatorname{tg}(45°-30°)=\frac{\operatorname{tg}45°-\operatorname{tg}30°}{1+\operatorname{tg}45°\operatorname{tg}30°}=\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$;

б) $\operatorname{tg}75°=\operatorname{tg}(45°+30°)=\frac{\operatorname{tg}45°+\operatorname{tg}30°}{1-\operatorname{tg}45°\operatorname{tg}30°}=\frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}}$;

в) $\operatorname{tg}105°=\operatorname{tg}(-60°+45°)=\frac{\operatorname{tg}60°+\operatorname{tg}45°}{1-\operatorname{tg}60°\operatorname{tg}45°}=\frac{\sqrt{3}+1}{1-\sqrt{3}}$;

г) $\operatorname{tg}165^\circ = \operatorname{tg}-15^\circ = -\dfrac{1-\dfrac{\sqrt{3}}{3}}{1+\dfrac{\sqrt{3}}{3}}$.

441. а) $\dfrac{\operatorname{tg}25^\circ + \operatorname{tg}20^\circ}{1-\operatorname{tg}25^\circ\operatorname{tg}20^\circ} = \operatorname{tg}45^\circ = -1$; б) $\dfrac{1-\operatorname{tg}70^\circ\operatorname{tg}65^\circ}{\operatorname{tg}70^\circ + \operatorname{tg}65^\circ} = \operatorname{ctg}135^\circ = -1$;

в) $\dfrac{\operatorname{tg}9^\circ + \operatorname{tg}51^\circ}{1-\operatorname{tg}9^\circ\operatorname{tg}51^\circ} = \operatorname{tg}60^\circ = \sqrt{3}$; г) $\dfrac{1+\operatorname{tg}54^\circ\operatorname{tg}9^\circ}{\operatorname{tg}54^\circ - \operatorname{tg}9^\circ} = \operatorname{ctg}45^\circ = 1$.

442. а) $\operatorname{tg}\left(\dfrac{\pi}{4}-\alpha\right)$, $\operatorname{tg}\alpha = \dfrac{2}{3}$, $\operatorname{tg}\left(\dfrac{\pi}{4}-\alpha\right) = \dfrac{1-\operatorname{tg}\alpha}{1+\operatorname{tg}\alpha} = \dfrac{1}{3}\cdot\dfrac{3}{5} = \dfrac{1}{5}$;

б) $\operatorname{tg}\alpha = \dfrac{4}{5}$, $\operatorname{tg}\left(\alpha+\dfrac{\pi}{4}\right) = \dfrac{\operatorname{tg}\alpha + \operatorname{tg}\dfrac{\pi}{3}}{1-\operatorname{tg}\alpha\operatorname{tg}\dfrac{\pi}{3}} = \dfrac{\dfrac{4}{5}+\dfrac{\sqrt{3}}{3}}{1-\dfrac{4}{5}\cdot\dfrac{\sqrt{3}}{3}} = \dfrac{12+5\sqrt{3}}{15-4\sqrt{3}}$;

в) $\operatorname{ctg}\alpha = \dfrac{4}{3}$, $\operatorname{tg}\left(\dfrac{\pi}{2}+\alpha\right) = -\operatorname{ctg}\alpha = -\dfrac{4}{3}$;

г) $\operatorname{ctg}\alpha = \dfrac{8}{5}$, $\operatorname{tg}\left(\alpha-\dfrac{\pi}{6}\right) = \dfrac{\operatorname{tg}\alpha - \operatorname{tg}\dfrac{\pi}{3}}{1+\operatorname{tg}\alpha\operatorname{tg}\dfrac{\pi}{3}} = \dfrac{\dfrac{8}{5}-\dfrac{\sqrt{3}}{3}}{1+\dfrac{8}{5}\cdot\dfrac{\sqrt{3}}{3}} = \dfrac{24-5\sqrt{3}}{15+8\sqrt{3}}$.

443. $\operatorname{tg}\alpha = \dfrac{1}{2}$, $\operatorname{tg}\beta = \dfrac{1}{3}$;

а) $\operatorname{tg}(\alpha+\beta) = \dfrac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta} = \dfrac{\dfrac{1}{2}+\dfrac{1}{3}}{1-\dfrac{1}{6}} = \dfrac{5}{6}\cdot\dfrac{6}{5} = 1$;

б) $\operatorname{tg}(\alpha-\beta) = \dfrac{\dfrac{1}{8}-\dfrac{1}{3}}{1+\dfrac{1}{6}} = \dfrac{1}{6}\cdot\dfrac{6}{7} = \dfrac{1}{7}$.

444. $\operatorname{tg}\alpha = \dfrac{2}{5}$, $\operatorname{tg}\left(\dfrac{\pi}{2}+\beta\right) = -3$, $-\operatorname{ctg}\beta = -3$, $\operatorname{tg}\beta = \dfrac{1}{3}$;

а) $\operatorname{tg}(\alpha+\beta) = \dfrac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta} = \dfrac{\dfrac{2}{3}+\dfrac{1}{3}}{1-\dfrac{2}{15}} = \dfrac{11}{15}\cdot\dfrac{15}{13} = \dfrac{11}{13}$;

б) $\operatorname{tg}(\alpha-\beta) = \dfrac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta} = \dfrac{\dfrac{2}{5}-\dfrac{1}{3}}{1+\dfrac{2}{15}} = \dfrac{1}{15}\cdot\dfrac{15}{17} = \dfrac{1}{17}$.

445. а) $\frac{\mathrm{tg}\,2{,}22+\mathrm{tg}\,0{,}92}{1-\mathrm{tg}\,2{,}22\cdot\mathrm{tg}\,0{,}92}=\mathrm{tg}(2{,}22+0{,}92)=\mathrm{tg}\,3{,}14$;

б) $\frac{\mathrm{tg}\,1{,}47-\mathrm{tg}\,0{,}69}{1+\mathrm{tg}\,1{,}47\cdot\mathrm{tg}\,0{,}69}=\mathrm{tg}(1{,}47-0{,}69)=\mathrm{tg}\,0{,}78$.

446. а) $\frac{\mathrm{tg}\,x+\mathrm{tg}\,3x}{1-\mathrm{tg}\,x\,\mathrm{tg}\,3x}=1,\ \mathrm{tg}\,4x=1,\ 4x=\frac{\pi}{4}+\pi n,\ x=\frac{\pi}{16}+\frac{\pi n}{4}$;

б) $\frac{\mathrm{tg}\,5x-\mathrm{tg}\,3x}{1+\mathrm{tg}\,5x\,\mathrm{tg}\,3x}=\sqrt{3},\ 2x=\frac{\pi}{3}+\pi n,\ x=\frac{\pi}{6}+\frac{\pi n}{2}$.

447. а) $\mathrm{tg}\left(\alpha-\frac{\pi}{4}\right)=3,\ \frac{\mathrm{tg}\,\alpha-1}{1+\mathrm{tg}\,\alpha}=3,\ \mathrm{tg}\,\alpha-1=3+3\,\mathrm{tg}\,\alpha,\ 2\,\mathrm{tg}\,\alpha=-4,\ \mathrm{tg}\,\alpha=-2$;

б) $\mathrm{tg}\left(\alpha+\frac{\pi}{4}\right)=\frac{1}{5},\ \frac{\mathrm{tg}\,\alpha+1}{1-\mathrm{tg}\,\alpha}=\frac{1}{5},\ 5\,\mathrm{tg}\,\alpha+5=1-\mathrm{tg}\,\alpha,\ \mathrm{tg}\,\alpha=-\frac{2}{3},\ \mathrm{ctg}\,\alpha=-\frac{3}{2}$.

448. а) $\mathrm{tg}\,\alpha=3,\ \mathrm{tg}(\alpha+\beta)=1,\ \frac{\mathrm{tg}\,\alpha+\mathrm{tg}\,\beta}{1-\mathrm{tg}\,\alpha\,\mathrm{tg}\,\beta}=1,\ 3+\mathrm{tg}\,\beta=1-3\,\mathrm{tg}\,\beta,\ \mathrm{tg}\,\beta=-\frac{1}{2}$;

б) $\mathrm{tg}\alpha=\frac{1}{4},\mathrm{tg}(\alpha-\beta)=2,\frac{\mathrm{tg}\alpha-\mathrm{tg}\beta}{1+\mathrm{tg}\alpha\,\mathrm{tg}\beta}=2,\ \frac{1}{4}-\mathrm{tg}\beta=2+\frac{1}{2}\mathrm{tg}\beta,\ \frac{3}{2}\mathrm{tg}\beta=-\frac{7}{4},\ \mathrm{tg}\beta=-\frac{7}{6}$.

449. $\sin\alpha=-\frac{12}{13},\qquad \pi<\alpha<\frac{3\pi}{2},\qquad \cos\alpha=-\frac{5}{13},\qquad \mathrm{tg}\,\alpha=\frac{12}{5}$;

а) $\mathrm{tg}\left(\alpha+\frac{\pi}{4}\right)=\frac{\mathrm{tg}\,\alpha+1}{1-\mathrm{tg}\,\alpha}=\frac{\frac{12}{5}+1}{1-\frac{12}{5}}=-\frac{17}{5}\cdot\frac{5}{7}=-\frac{17}{7}$,

б) $\mathrm{tg}\left(\alpha-\frac{\pi}{4}\right)=\frac{\mathrm{tg}\,\alpha-1}{1+\mathrm{tg}\,\alpha}=\frac{\frac{12}{5}-1}{1+\frac{12}{5}}=\frac{17}{5}\cdot\frac{5}{7}=\frac{17}{7}$.

450. $\cos\alpha=\frac{3}{5},\qquad 0<\alpha<\frac{\pi}{2},\qquad \sin\alpha=\frac{4}{5},\qquad \mathrm{tg}\,\alpha=\frac{4}{3}$;

а) $\mathrm{tg}\left(\alpha+\frac{\pi}{3}\right)=\frac{\mathrm{tg}\,\alpha+\sqrt{3}}{1-\sqrt{3}\,\mathrm{tg}\,\alpha}=\frac{\frac{4}{3}+\sqrt{3}}{1-\frac{4}{\sqrt{3}}}=\frac{4+3\sqrt{3}}{3}$.

$\frac{\sqrt{3}}{\sqrt{3}-4}=\frac{4\sqrt{3}+9}{3\sqrt{3}-12}=-\frac{48+25\sqrt{3}}{39}$;

б) $\mathrm{tg}\left(\alpha-\frac{\pi}{3}\right)=\frac{\mathrm{tg}\,\alpha-\sqrt{3}}{1+\sqrt{3}\,\mathrm{tg}\,\alpha}=\frac{\frac{4}{3}-\sqrt{3}}{3}\cdot\frac{3}{3+4\sqrt{3}}=\frac{4-3\sqrt{3}}{3+4\sqrt{3}}$.

451. а) $\dfrac{\operatorname{tg}\left(\dfrac{\pi}{8}+\alpha\right)+\operatorname{tg}\left(\dfrac{\pi}{8}-\alpha\right)}{1-\operatorname{tg}\left(\dfrac{\pi}{8}+\alpha\right)\operatorname{tg}\left(\dfrac{\pi}{8}-\alpha\right)}=\operatorname{tg}\left(\dfrac{\pi}{8}+\alpha+\dfrac{\pi}{8}-\alpha\right)=\operatorname{tg}\dfrac{\pi}{4}=1$;

б) $\dfrac{\operatorname{tg}(45^\circ+\alpha)-\operatorname{tg}\alpha}{1+\operatorname{tg}(45^\circ+\alpha)\operatorname{tg}\alpha}=\operatorname{tg}(45^\circ+\alpha-\alpha)=\operatorname{tg}45^\circ=1$.

452. а) $\dfrac{1-\operatorname{tg}\alpha}{1+\operatorname{tg}\alpha}=\operatorname{tg}(45^\circ-\alpha)$, $\operatorname{tg}(45^\circ-\alpha)=\dfrac{1-\operatorname{tg}\alpha}{1+\operatorname{tg}\alpha}$;

б) $\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{tg}(\alpha+\beta)}+\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{\operatorname{tg}(\alpha-\beta)}=2$,

$\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{tg}(\alpha+\beta)}+\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{\operatorname{tg}(\alpha-\beta)}=1-\operatorname{tg}\alpha\operatorname{tg}\beta+1+\operatorname{tg}\alpha\operatorname{tg}\beta=2$.

453. а) $\operatorname{tg}\left(\dfrac{3\pi}{4}-x\right)+\operatorname{tg}x=\operatorname{tg}\left(\dfrac{3\pi}{4}-x\right)\operatorname{tg}x-1$,

$\dfrac{\operatorname{tg}x+1}{\operatorname{tg}x-1}+\operatorname{tg}x=\dfrac{\operatorname{tg}x+1}{\operatorname{tg}x-1}\operatorname{tg}x-1$, $\dfrac{\operatorname{tg}^2x+1}{\operatorname{tg}x-1}=\dfrac{\operatorname{tg}^2x+\operatorname{tg}x-\operatorname{tg}x+1}{\operatorname{tg}x-1}$;

б) $\operatorname{tg}\left(\alpha+\dfrac{\pi}{4}\right)-\operatorname{tg}\alpha=1+\operatorname{tg}\left(\dfrac{\pi}{4}+\alpha\right)\operatorname{tg}\alpha$,

$\dfrac{\operatorname{tg}\alpha+1}{1-\operatorname{tg}\alpha}-\operatorname{tg}\alpha=\dfrac{-\operatorname{tg}\alpha+\operatorname{tg}^2\alpha+\operatorname{tg}\alpha+1}{1-\operatorname{tg}\alpha}=\dfrac{1-\operatorname{tg}\alpha+\operatorname{tg}^2\alpha+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha}$.

$\dfrac{\operatorname{tg}^2\alpha+1}{1-\operatorname{tg}\alpha}=\dfrac{\operatorname{tg}^2\alpha+1}{1-\operatorname{tg}\alpha}$.

454. а) $\operatorname{tg}(\alpha+\beta)-(\operatorname{tg}\alpha+\operatorname{tg}\beta)=\operatorname{tg}(\alpha+\beta)\operatorname{tg}\alpha\operatorname{tg}\beta$,

$\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta+\operatorname{tg}^2\alpha\operatorname{tg}\beta-\operatorname{tg}\beta+\operatorname{tg}\alpha\operatorname{tg}^2\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}=\dfrac{\operatorname{tg}^2\alpha\operatorname{tg}\beta+\operatorname{tg}\alpha\operatorname{tg}^2\beta}{1-\operatorname{tg}\alpha}=\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}(\alpha+\beta)$;

б) $\operatorname{tg}(\alpha-\beta)-(\operatorname{tg}\alpha-\operatorname{tg}\beta)=\operatorname{tg}(\beta+\alpha)-\operatorname{tg}\alpha\operatorname{tg}\beta$,

$\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta-\operatorname{tg}\alpha-\operatorname{tg}^2\alpha\operatorname{tg}\beta+\operatorname{tg}\beta+\operatorname{tg}\alpha\operatorname{tg}^2\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}=\operatorname{tg}(\beta-\alpha)\operatorname{tg}\alpha\operatorname{tg}\beta$.

455. а) $\dfrac{\sqrt{3}-\operatorname{tg}x}{1+\sqrt{3}\operatorname{tg}x}=1$, $\operatorname{tg}\left(\dfrac{\pi}{3}-x\right)=1$, $x-\dfrac{\pi}{3}=-\dfrac{\pi}{4}+\pi n$, $x=\dfrac{\pi}{12}+\pi n$;

б) $\dfrac{\operatorname{tg}\dfrac{\pi}{4}-\operatorname{tg}2x}{\operatorname{tg}\dfrac{\pi}{4}\operatorname{tg}2x+1}=\sqrt{3}$, $\operatorname{tg}\left(2x-\dfrac{\pi}{5}\right)=-\sqrt{3}$, $2x-\dfrac{\pi}{5}=-\dfrac{\pi}{3}+\pi n$, $x=-\dfrac{\pi}{15}+\dfrac{\pi n}{2}$.

456. $\operatorname{tg}2x=\operatorname{tg}(x+x)=\dfrac{2\operatorname{tg}x}{1-\operatorname{tg}^2x}$.

457. $\alpha-\beta=\dfrac{\pi}{4}$; $\alpha=\dfrac{\pi}{4}+\beta$; $\beta=\alpha-\dfrac{\pi}{4}$;

а) $\dfrac{1+\operatorname{tg}\beta}{1-\operatorname{tg}\beta}=\operatorname{tg}\alpha,\ \operatorname{tg}\alpha=\operatorname{tg}\left(\beta+\dfrac{\pi}{4}\right)=\dfrac{\operatorname{tg}\beta+1}{1-\operatorname{tg}\beta}$;

б) $\dfrac{\operatorname{tg}\alpha-1}{\operatorname{tg}\alpha+1}=\operatorname{tg}\beta,\ \operatorname{tg}\beta=\operatorname{tg}\left(\alpha-\dfrac{\pi}{4}\right)=\dfrac{\operatorname{tg}\alpha-1}{1+\operatorname{tg}\alpha}$.

458. $\dfrac{\operatorname{tg}(\alpha-\beta)-\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{tg}(\alpha-\beta)\operatorname{tg}\beta}=\dfrac{\operatorname{tg}(\beta-\alpha)\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{tg}(\alpha-\beta)\operatorname{tg}\beta}=-\operatorname{tg}\alpha$.

459. $\dfrac{\operatorname{tg}\dfrac{\pi}{5}+\operatorname{tg}x}{1-\operatorname{tg}\dfrac{\pi}{5}\operatorname{tg}x}<1$

а) $\operatorname{tg}\left(x+\dfrac{\pi}{5}\right)<1,\ x+\dfrac{\pi}{5}\in\left(-\dfrac{\pi}{2}+\pi n;\dfrac{\pi}{4}+\pi n\right),\ x\in\left(-\dfrac{7\pi}{10}+\pi n;\dfrac{\pi}{20}+\pi n\right)$;

б) $\dfrac{\operatorname{tg}3x-1}{\operatorname{tg}3x+1}>1,\ \operatorname{tg}\left(3x-\dfrac{\pi}{4}\right)>1,\ 3x-\dfrac{\pi}{4}\in\left(\dfrac{\pi}{4}+\pi n;\dfrac{\pi}{2}+\pi n\right)$

$x\in\left(\dfrac{\pi}{6}+\dfrac{\pi n}{3};\dfrac{\pi}{4}+\dfrac{\pi n}{3}\right)$.

560. $y_1=3x+1$; $y_2=6-2x$

$\operatorname{tg}y_1=3$ – тангенс угла наклона 1-ой прямой;

$\operatorname{tg}y_2=-2$ – тангенс угла наклона 2-ой прямой;

$y_1=\operatorname{arctg}3$

$y_2=-\operatorname{arctg}2$

$y_1-y_2=\operatorname{arctg}3+\operatorname{arctg}2$

$\operatorname{tg}(y_1-y_2)=\dfrac{3+2}{1-6}=-1$

$y_1-y_2=\dfrac{3\pi}{4}$.

461. $\operatorname{tg}\angle KBC=\dfrac{1}{2},\ \angle KBC=\operatorname{arctg}\dfrac{1}{2}$ – см. рис. 144

$\angle BKC=90°-\angle KBC$

$\angle KOC=180°-45°-90°+\angle KBC=45°+\angle KBC.$

$\operatorname{tg}\angle KOC=\operatorname{tg}\left(45°+\operatorname{arctg}\dfrac{1}{2}\right)=\dfrac{1+\dfrac{1}{2}}{1-\dfrac{1}{2}}=\dfrac{3}{2}\cdot2=3$

$\operatorname{tg}\angle KOC=\operatorname{arctg}3$.

§24. Формулы двойного аргумента

462. а) $\dfrac{\sin2t}{\cos t}-\sin t=2\sin t-\sin t=\sin t$; **б)** $\dfrac{\sin6t}{\cos^2 3t}=2\operatorname{tg}3t$;

в) $\cos^2t-\cos2t=\cos^2t-\cos^2t+\sin^2t=\sin^2t$;

г) $\dfrac{\cos 2t}{\cos t-\sin t}-\sin t=\dfrac{(\cos t-\sin t)(\cos t+\sin t)}{\cos t-\sin t}-\sin t=\cos t$.

463. а) $\dfrac{\sin 40^\circ}{\sin 20^\circ}=2\cos 20^\circ$; б) $\dfrac{\cos 80^\circ}{\cos 40^\circ+\sin 40^\circ}=\cos 40^\circ-\sin 40^\circ$;

в) $\dfrac{\sin 100^\circ}{2\cos 50^\circ}=\sin 50^\circ$; г) $\dfrac{\cos 36^\circ+\sin^2 18^\circ}{\cos 18^\circ}=\dfrac{\cos^2 18^\circ}{\cos 18^\circ}=\cos 18^\circ$.

464. а) $2\sin 15^\circ\cos 15^\circ=\sin 30^\circ=\dfrac{1}{2}$;

б) $(\cos 75^\circ-\sin 75^\circ)^2=1-2\sin 75^\circ\cos 75^\circ=1-\sin 150^\circ=\dfrac{1}{2}$;

в) $\cos^2 15^\circ-\sin^2 15^\circ=\cos 30^\circ=\dfrac{\sqrt{3}}{2}$;

г) $(\cos 15^\circ+\sin 15^\circ)^2=1+2\sin 15^\circ\cos 15^\circ=\dfrac{3}{2}$.

465. а) $2\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8}=\sin\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$; б) $\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8}+\dfrac{1}{4}=\dfrac{\sqrt{2}}{2}+\dfrac{1}{4}=\dfrac{\sqrt{2}+1}{4}$;

в) $\cos^2\dfrac{\pi}{8}-\sin^2\dfrac{\pi}{8}=\cos\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$; г) $\dfrac{\sqrt{2}}{2}-\left(\cos\dfrac{\pi}{8}+\sin\dfrac{\pi}{8}\right)^2=\dfrac{\sqrt{2}}{2}-1-\sin\dfrac{\pi}{4}=-1$.

466. а) $\dfrac{2\,\mathrm{tg}\,15^\circ}{1-\mathrm{tg}^2 15^\circ}=\mathrm{tg}\,30^\circ=\dfrac{\sqrt{3}}{3}$; б) $\dfrac{\mathrm{tg}\dfrac{\pi}{8}}{1-\mathrm{tg}^2\dfrac{\pi}{8}}=\dfrac{1}{2}\mathrm{tg}\dfrac{\pi}{8}=\dfrac{1}{2}$;

в) $\dfrac{\mathrm{tg}\,75^\circ}{1-\mathrm{tg}^2 75^\circ}=-\mathrm{tg}\dfrac{\pi}{3}=-\sqrt{3}$; г) $\dfrac{2\,\mathrm{tg}\dfrac{\pi}{6}}{\mathrm{tg}^2\dfrac{\pi}{6}-1}=-\mathrm{tg}\,150^\circ=-\dfrac{\sqrt{3}}{3}$.

467. а) $\sin\dfrac{x}{2}\cos\dfrac{x}{2}=\dfrac{1}{2}\sin x,\ \dfrac{1}{2}\sin x=\sin\dfrac{x}{2}\cos\dfrac{x}{2}$;

б) $\cos^2\dfrac{x}{4}-\sin^2\dfrac{x}{4}=\cos\dfrac{x}{2},\ \cos\dfrac{x}{2}=\cos^2\dfrac{x}{4}-\sin^2\dfrac{x}{4}$;

в) $\sin 2x\cos 2x=\dfrac{1}{2}\sin 4x,\ \dfrac{1}{2}\sin 4x=\sin 2x\cos 2x$;

г) $\cos^2\dfrac{x}{2}-\sin^2\dfrac{x}{2}=\cos x,\ \cos x=\cos^2\dfrac{x}{2}-\sin^2\dfrac{x}{2}$.

468. а) $\cos(2\alpha+2\beta)=\cos^2(\alpha+\beta)-\sin^2(\alpha+\beta)$,
$\cos(2\alpha+2\beta)=\cos 2(\alpha+\beta)=\cos^2(\alpha+\beta)-\sin^2(\alpha+\beta)$;
б) $\sin(2\alpha+2\beta)=2\sin(\alpha+\beta)\cos(\alpha+\beta)$,
$\sin(2\alpha+2\beta)=\sin 2(\alpha+\beta)=2\sin(\alpha+\beta)\cos(\alpha+\beta)$.

469. а) $\mathrm{tg}(2\alpha+2\beta)=\dfrac{2\,\mathrm{tg}(\alpha+\beta)}{1-\mathrm{tg}^2(\alpha+\beta)},\ \mathrm{tg}(2\alpha+2\beta)=\mathrm{tg}\,2(\alpha+\beta)=\dfrac{2\,\mathrm{tg}(\alpha+\beta)}{1-\mathrm{tg}^2(\alpha+\beta)}$;

б) $\operatorname{tg}(\alpha+\beta)=\dfrac{2\operatorname{tg}\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)}{1-\operatorname{tg}^2\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)}$, $\operatorname{tg}(\alpha+\beta)=\operatorname{tg}2\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)=\dfrac{2\operatorname{tg}\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)}{1-\operatorname{tg}^2\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)}$.

470. $\sin t=\dfrac{5}{13}$, $\dfrac{\pi}{2}<t<\pi$, $\cos t=-\dfrac{12}{13}$, $\operatorname{tg}t=\dfrac{5}{13}\left(-\dfrac{13}{12}\right)=-\dfrac{5}{12}$;

а) $\sin 2t=2\sin t\cos t=2\cdot\dfrac{5}{13}\cdot\left(-\dfrac{12}{13}\right)=-\dfrac{120}{169}$;

б) $\cos 2t=\cos^2 t-\sin^2 t=\dfrac{144}{169}-\dfrac{25}{169}=-\dfrac{119}{169}$;

в) $\operatorname{tg}2t=\dfrac{2\operatorname{tg}t}{1-\operatorname{tg}^2 t}=\dfrac{2\cdot\left(-\dfrac{5}{12}\right)}{1-\dfrac{25}{144}}=-\dfrac{5}{6}\cdot\dfrac{144}{119}=-\dfrac{120}{119}$;

г) $\operatorname{ctg}2t=\dfrac{1}{\operatorname{tg}2t}=-\dfrac{119}{120}$.

471. $\cos x=\dfrac{4}{5}$, $0<x<\dfrac{\pi}{2}$, $\sin x=\dfrac{3}{5}$, $\operatorname{tg}x=\dfrac{3}{4}$, $\operatorname{ctg}x=\dfrac{4}{3}$;

а) $\sin 2x=2\cdot\dfrac{4}{5}\cdot\dfrac{3}{5}=\dfrac{24}{25}$; **б)** $\cos 2x=\dfrac{16}{25}-\dfrac{9}{25}=\dfrac{7}{25}$;

в) $\operatorname{tg}2x=\dfrac{2\operatorname{tg}x}{1-\operatorname{tg}^2 x}=\dfrac{\dfrac{3}{2}}{1-\dfrac{9}{16}}=\dfrac{3}{2}\cdot\dfrac{16}{7}=\dfrac{24}{7}$; **г)** $\operatorname{ctg}2x=\dfrac{7}{24}$.

472. а) $\dfrac{\sin t}{2\cos^2\dfrac{t}{2}}=\dfrac{2\sin\dfrac{t}{2}\left(\cos\dfrac{t}{2}\right)}{2\cos^2\dfrac{t}{2}}=\operatorname{tg}\dfrac{t}{2}$;

б) $\dfrac{\cos t}{\cos\dfrac{t}{2}+\sin\dfrac{t}{2}}=\dfrac{\cos^2\dfrac{t}{2}-\sin^2\dfrac{t}{2}}{\cos\dfrac{t}{2}+\sin\dfrac{t}{2}}=\cos\dfrac{t}{2}-\sin\dfrac{t}{2}$;

в) $\dfrac{\sin 4t}{\cos 2t}=\dfrac{2\sin 2t\cos 2t}{\cos 2t}=2\sin 2t$;

г) $\dfrac{\cos 2t-\sin 2t}{\cos 4t}=\dfrac{\cos 2t-\sin 2t}{(\sin 2t-\cos 2t)(\cos 2t+\sin 2t)}=\dfrac{1}{\cos 2t+\sin 2t}$.

473. а) $\dfrac{\sin 2t-2\sin t}{\cos t-1}=\dfrac{2\sin t(\cos t-1)}{\cos t-1}=2\sin t$;

б) $\dfrac{\cos 2t-\cos^2 t}{1-\cos^2 t}=\dfrac{\cos^2 t-\sin^2 t-\cos^2 t}{\sin^2 t}=-1$;

в) $\sin 2t \operatorname{ctg} t - 1 = 2\sin t \cos t \dfrac{\cos t}{\sin t} - 1 = 2\cos^2 t - 1 = \cos 2t$;

г) $\left(\operatorname{tg} t + \operatorname{ctg} t\right)\sin 2t = \dfrac{1}{\sin t \cos t} 2\sin t \cos t = 2$.

474. а) $\dfrac{2}{\operatorname{tg} t + \operatorname{ctg} t} = \dfrac{2}{\dfrac{1}{\sin t \cos t}} = \sin 2t$; **б)** $\dfrac{2}{\operatorname{tg} t - \operatorname{ctg} t} = \dfrac{2}{\dfrac{-\cos 2t}{\sin t \cos t}} = \dfrac{\sin 2t}{-\cos 2t} = -\operatorname{tg} 2t$.

475. а) $(1 - \operatorname{tg}^2 t) \cos^2 t = \cos^2 t - \sin^2 t = \cos 2t$;

б) $2\cos^2 \dfrac{\pi + t}{4} - 2\sin^2 \dfrac{\pi + t}{4} = 2\cos\left(\dfrac{\pi}{2} + \dfrac{t}{2}\right) = -2\sin \dfrac{t}{2}$.

476. а) $(\sin t - \cos t)^2 = 1 - \sin 2t$, $\sin^2 t - 2\sin t \cos t + \cos^2 t = 1 - \sin 2t$;
б) $2\cos^2 t = 1 + \cos 2t$, $1 + \cos 2t = \sin^2 t + \cos^2 t - \sin^2 t + \cos^2 t = 2\cos^2 t$;
в) $(\sin t + \cos t)^2 = 1 + \sin 2t$, $\sin^2 t + 2\sin t \cos t + \cos^2 t = 1 + \sin 2t$;
г) $2\sin^2 t = 1 - \cos 2t$, $1 - \cos 2t = \sin^2 t + \cos^2 t - \cos^2 t + \sin^2 t = 2\sin^2 t$.

477. а) $\cos^4 t - \sin^4 t = \cos 2t$,
$\cos 2t = (\cos^2 t - \sin^2 t) \cdot 1 = (\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t) = \cos^4 t - \sin^4 t$;

б) $\cos^4 t + \sin^4 t = 1 - \dfrac{1}{2}\sin^2 2t$,

$\cos^4 t + \sin^4 t + 2\sin^2 t \cos^2 t - 2\sin^2 t \cos^2 t = \left(\sin^2 t + \cos^2 t\right)^2 - \dfrac{1}{2}\sin^2 2t = 1 - \dfrac{1}{2}\sin^2 2t$.

478. а) $\operatorname{ctg} t - \sin 2t = \operatorname{ctg} t \cos 2t$,
$\dfrac{\cos t - 2\sin^2 t \cos t}{\sin t} = \dfrac{\cos t\left(\cos^2 t - \sin^2 t\right)}{\sin t} = \dfrac{\cos t \cos 2t}{\sin t} = \operatorname{ctg} t \cos 2t$;

б) $\sin 2t - \operatorname{tg} t = \cos 2t \operatorname{tg} t$,
$\dfrac{2\sin t \cos^2 t - \sin t}{\cos t} = \dfrac{\left(\cos^2 t - \sin^2 t\right)\sin t}{\cos t} = \dfrac{\cos 2t \sin t}{\cos t} = \cos 2t \operatorname{tg} t$.

479. а) $\sin 2x - 2\cos x = 0$

$2\cos x = \left(\sin x - 1\right) = 0$, $\cos x = 0$, $x = \dfrac{\pi}{2} + \pi n$, $\sin x = 1$, $x = \dfrac{\pi}{2} + 2\pi n$;

б) $2\sin x = \sin 2x$, $2\sin(\cos x - 1) = 0$,
$\sin x = 0$, $\cos x = 1$, $x = \pi n$, $x = 2\pi n$;
в) $\sin 2x - \sin x = 0$, $\sin x(2\cos x - 1) = 0$, $\sin x(2\cos x - 1) = 0$,

$\sin x = 0$, $x = \pi n$, $\cos x = \dfrac{1}{2}$, $x = \pm\dfrac{\pi}{3} + 2\pi n$;

г) $\sin 2x - \cos x = 0$, $\cos x(2\sin x - 1) = 0$, $\cos x = 0$,

$x = \dfrac{\pi}{2} + \pi n$, $\sin x = \dfrac{1}{2}$, $x = \left(-1\right)^k \dfrac{\pi}{6} + \pi k$.

480. а) $\sin x \cos x = 1$,
$\dfrac{1}{2}\sin x \cos x = \dfrac{1}{2}$, $\sin 2x = \dfrac{1}{2}$, $2x = \left(-1\right)^n \dfrac{\pi}{6} + 2\pi n$, $x = \left(-1\right)^n \dfrac{\pi}{12} + \pi n$,
$\sin 2x = 2$, решений нет;

б) $\sin 4x \cos 4x = \frac{1}{2}$,

$\sin 8x = 1,\ 8x = \frac{\pi}{2} + \pi n,\ x = \frac{\pi}{16} + \frac{\pi n}{8}$;

в) $\cos^2 \frac{x}{3} - \sin^2 \frac{x}{3} = \frac{1}{2}$,

$\cos \frac{2x}{3} = \frac{1}{2},\ \frac{2x}{3} = \pm \frac{\pi}{3} + 2\pi n,\ x = \pm \frac{\pi}{2} + 3\pi n$;

г) $\sin^2 x - \cos^2 x = \frac{1}{2}$,

$\cos 2x = -\frac{1}{2},\ 2x = \pm \frac{2\pi}{3} + 2\pi n,\ x = \pm \frac{\pi}{3} + \pi n$.

481. а) $\cos 2x + 3 \sin x = 1$,
$1 - 2 \sin^2 x + 3 \sin x = 1$, $\sin x\,(2\sin x - 3) = 0$, $\sin x = 0$, $x = \pi n$;
б) $\sin^2 x = -\cos 2x$,

$\sin^2 x + 1 - 2\sin^2 x = 0,\ \sin^2 x = 1,\ \sin x = \pm 1,\ x = \frac{\pi}{2} + \pi n$;

в) $\cos 2x = \cos^2 2x$, $2\cos^2 2x - 1 - \cos^2 x = 0$, $\cos^2 2x = 1$, $\cos x = 1$, $x = \pi n$;

г) $2\cos 2x = 2\sin^2 x,\ 1 - 2\sin^2 x = 2\sin^2 x,\ \sin x = \pm \frac{1}{2}\ \ x = (-1)^{k+1} \frac{\pi}{6} + \pi n$;

482. а) $\sin 11°15' \cos 11°15' \cos 22°30' \cos 45° =$

$= \frac{1}{2} \sin 22°30' \cos 22°30' \cos 45° = \frac{1}{8} \sin 90° = \frac{1}{8}$;

б) $\sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} = \frac{1}{8} \sin \frac{\pi}{6} = \frac{1}{16}$.

483. а) $\frac{1 + \cos 40° + \cos 80°}{\sin 80° + \sin 40°} \operatorname{tg} 40° = \frac{2\cos^2 40° + \cos 40°}{\sin 40°(2\cos 40° + 1)} \operatorname{tg} 40° = \operatorname{tg} 40° \operatorname{ctg} 40° = 1$;

б) $\frac{1 - \cos 25° + \cos 50°}{\sin 50° - \sin 25°} - \operatorname{tg} 65° = \frac{2\cos^2 25° - \cos 25°}{2\sin 25° \cos 25° - \sin 25°} - \operatorname{tg} 25° =$

$= \frac{\cos 25°}{\sin 25°} \frac{2\cos 25° - 1}{2\cos 25° - 1} - \operatorname{tg} 65° = \operatorname{ctg} 25° - \operatorname{tg} 25° = \operatorname{tg} 65° - \operatorname{tg} 65° = 0$.

484. $\operatorname{tg} x = \frac{3}{4},\ \pi < x < \frac{3\pi}{2},\ \cos x = -\frac{4}{5},\ \sin x = -\frac{3}{5}$;

а) $\sin 2x = 2 \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{24}{25}$; б) $\cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$;

в) $\operatorname{tg} 2x = \frac{24}{25} \cdot \frac{25}{7} = \frac{24}{7}$; г) $\operatorname{ctg} 2x = \frac{7}{24}$.

485. $\operatorname{ctg} x = -\frac{4}{3},\ \frac{3\pi}{2} < x < 2\pi,\ \operatorname{tg} x = -\frac{3}{45},\ \cos x = \frac{4}{5},\ \sin x = -\frac{3}{5}$;

а) $\sin 2x = -2 \cdot \frac{4}{5} \cdot \frac{3}{5} = -\frac{24}{25}$; б) $\cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$;

в) $\operatorname{tg} 2x = -\frac{24}{25} \cdot \frac{25}{7} = -\frac{24}{7}$; **г)** $\operatorname{ctg} 2x = -\frac{7}{24}$.

486. а) $\frac{\cos 2t}{\sin t \cos t + \sin^2 t} = \operatorname{ctg}(\pi + t) - 1$,

$\frac{\cos 2t}{\sin t(\cos t + \sin t)} = \frac{\cos t - \sin t}{\sin t} = \operatorname{ctg} t - 1 = \operatorname{ctg}(\pi + t) - 1$;

б) $(\operatorname{ctg} t - \operatorname{tg} t)\sin 2t = 2\cos 2t$,

$\frac{\cos t}{\sin t} 2\sin t \cos t - \frac{\sin t}{\cos t} 2\sin t \cos t = 2\cos^2 t - 2\sin^2 t = 2\cos 2t$.

487. а) $\frac{\sin 2t - 2\sin\left(\frac{\pi}{2} - t\right)}{\cos\left(\frac{\pi}{2} - t\right) - 2\sin^2 t} = -2\operatorname{ctg} t$, $\frac{2\cos t(\sin t - 1)}{-\sin t(\sin t - 1)} = -2\operatorname{ctg} t$;

б) $\frac{1 - \cos 2t + \sin 2t}{1 + \cos 2t + \sin 2t} \operatorname{tg}\left(\frac{\pi}{2} - t\right) = 1$,

$\frac{2\sin^2 2t + \sin 2t}{2\cos^2 t + \sin 2t} \operatorname{ctg} t = \frac{2\sin t(\sin t + \cos t)}{2\cos t(\sin t + \cos t)} \operatorname{ctg} t = 1$.

488. а) $\sin 2\alpha = \frac{1}{3}$,

$\sin^4 \alpha + \cos^4 \alpha = 1 - 2\sin^2 \alpha \cos^2 \alpha = 1 - 2 \cdot \frac{1}{36} = \frac{17}{18}$;

б) $\sin^4 \alpha + \cos^4 \alpha = \frac{49}{50}$, $\frac{\pi}{2} < \alpha < \pi$,

$1 - 2\sin^2 \alpha \cos^2 \alpha = \frac{49}{50}$, $\sin^2 \alpha \cos^2 \alpha = \frac{1}{100}$, $\sin \alpha \cos \alpha = -\frac{1}{10}$, $\sin 2\alpha = -\frac{1}{5}$.

489. а) $\sin 3x = 3\sin x - 4\sin^3 x$,
$\sin 3x = \sin(x + 2x) = \sin x \cos 2x + \sin 2x \cos x =$
$= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$.
б) $\cos 3x = 4\cos^3 x - 3\cos x$,
$\cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x = 2\cos^3 x - \cos x - 2\cos x \sin x =$
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x$.

490. а) $\cos x \cos 2x = \frac{\sin 4x}{4\sin x}$; $\frac{\sin 4x}{4\sin x} = \frac{2\sin 2x \cos 2x}{4\sin x} = \cos x \cos 2x$;

б) $\cos x \cos 2x \cos 4x = \frac{\sin 8x}{8\sin x}$;

$\frac{2\sin 4x \cos 4x}{8\sin x} = \frac{8\sin x \cos x \cos 2x \cos 4x}{8\sin x} = \cos x \cos 2x \cos 4x$;

в) $\sin x \cos 2x = \frac{\sin 4x}{4\cos x}$; $\frac{\sin 4x}{4\cos x} = \frac{4\sin x \cos x \cos 2x}{4\cos x} = \sin x \cos 2x$;

г) $\sin x \cos 2x \cos 4x = \frac{\sin 8x}{8\cos x}$;

$$\frac{\sin 8x}{8\cos x}=\frac{8\sin x\cos x\cos 2x\cos 4x}{8\cos x}=\sin x\cos 2x\cos 4x.$$

491. а) $\sin 18°\cos 18°\cos 36°=\frac{1}{4}\sin 72°$, $\frac{1}{2}\sin 36°\cos 36°=\frac{1}{4}\sin 72°$;

б) $\sin 18°\cos 36°=\frac{1}{4}$; т.к. $\sin 18°\cos 18°\cos 36°=\frac{1}{4}\sin 72°$,

$\sin 18°\cos 18°\cos 36°=\frac{1}{4}\cos 18° \Rightarrow \sin 18°\cos 36°=\frac{1}{4}$.

492. а) $\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}=\frac{\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}}=\frac{1}{32}$;

б) $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}=\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}=-\frac{1}{8}$.

493. а) $2-\cos 2x+3\sin x=0$,
$2\sin^2 x-1+2+3\sin x=0$, $2\sin^2 x+3\sin x+1=0$,
$\sin x=\frac{-3-1}{4}=-1$, $x=-\frac{\pi}{2}+2\pi n$, $\sin x=-\frac{1}{2}$, $x=(-1)^{k+1}\frac{\pi}{6}+\pi k$;
б) $\cos 6x-\cos 3x-2=0$, $2\cos^2 3x-\cos 3x-3=0$,
$\cos 3x=\frac{1+5}{4}$ – решений нет;

$\cos 3x=-1$, $x=\frac{\pi}{3}+\frac{2\pi n}{3}$.

494. а) $26\sin x\cos x-\cos 4x+7=0$,
$13\sin 2x-1+2\sin^2 2x+7=0$, $2\sin^2 2x+13\sin 2x+6=0$,
$\sin 2x=\frac{-13-11}{4}$ – решений нет; $\sin 2x=-\frac{1}{2}$,

$2x=(-1)^{k+1}\frac{\pi}{6}+\frac{\pi k}{2}$;

б) $\sin^4 x+\cos^4 x=\sin x\cos x$

$\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x=2\sin^2 x\cos^2 x+\frac{1}{2}\sin 2x$,

$\frac{1}{2}\sin^2 2x+\frac{1}{2}\sin 2x-1=0$,

1) $\sin 2x=1$, $2x=\frac{\pi}{2}+2\pi n$, $x=\frac{\pi}{4}+\pi n$;

2) $\sin 2x=-2$ –решений нет.

495. а) $3\sin 2x+\cos 2x=1$,
$6\sin x\cos x-2\sin^2 x=0$, $2\sin x\,(3\cos x-\sin x)=0$,
$\sin x=3\cos x$, $\operatorname{tg} x=3$, $x=\operatorname{arctg} 3+\pi n$, $\sin x=0$, $x=\pi n$;

б) $\cos 4x + 2\sin 4x = 1$,

$-2\sin^2 2x + 4\sin 2x \cos 2x = 0$, $\sin 2x\,(2\cos 2x - \sin 2x) = 0$,

$$\cos 2x = \frac{1}{2}\sin 2x,\ \ \mathrm{tg}\,2x = 2,\ \ 2x = \mathrm{arctg}\,2 + \pi n,\ \ x = \frac{1}{2}\mathrm{arctg}\,2 + \frac{\pi n}{2},$$

$$\sin 2x = 0,\ \ 2x = \pi n,\ \ x = \frac{\pi n}{2}.$$

496. а) $4\sin x + \sin 2x = 0$, $x \in [0; 2\pi]$,

$2\sin x(2 + \cos x) = 0$, $\sin x = 0$, $\cos x = -2$ –решений нет

$x = 0$, $x = \pi$, $x = 2\pi$;

б) $\cos^2\left(3x + \frac{\pi}{4}\right) - \sin^2\left(3x + \frac{\pi}{4}\right) + \frac{\sqrt{3}}{2} = 0$, $x \in \left[\frac{3\pi}{4}; \pi\right]$,

$$\cos\left(6x + \frac{2\pi}{4}\right) = -\frac{\sqrt{3}}{2},\ \ \sin 6x = \frac{\sqrt{3}}{2},$$

$$6x = (-1)^k \frac{\pi}{3} + \pi k,\ \ x = (-1)^k \frac{\pi}{18} + \frac{\pi k}{6} \Rightarrow x = \frac{7\pi}{18}.$$

497. а) $(\cos x - \sin x)^2 = 1 - 2\sin x = 2x$, $x \in \left[\frac{20\pi}{9}; \frac{28\pi}{9}\right]$,

$$1 - \sin 2x = 11 - 2\sin 2x,\ \ \sin 2x = 2,\ \ x = \frac{\pi n}{2},\ -2\ \text{корня};$$

б) $2\cos^2\left(2x - \frac{\pi}{4}\right) - 2\sin^2\left(2x - \frac{\pi}{4}\right) + 1 = 0$, $x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$,

$$\cos\left(4x - \frac{\pi}{2}\right) = -\frac{1}{2},\ \ \sin 4x = -\frac{1}{2},\ \ 4x = (-1)^{k+1}\frac{\pi}{6} + \pi k,$$

$$x = (-1)^{k+1}\frac{\pi}{24} + \frac{\pi k}{4}\ -3\ \text{корня}.$$

498. а) $\sin x = \dfrac{2\,\mathrm{tg}\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} = 2\,\mathrm{tg}\frac{x}{2}\cos^2\frac{x}{2} = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$;

б) $\cos x = \dfrac{1 - \mathrm{tg}^2\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} = \left(1 - \mathrm{tg}^2\frac{x}{2}\right)\cos^2\frac{x}{2} = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$.

499. а) $\sin x + 7\cos x = 5$,

$$\frac{2\,\mathrm{tg}\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} + \frac{7 - 7\,\mathrm{tg}^2\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} = 2\,\mathrm{tg}\frac{x}{2} + 7 - 7\,\mathrm{tg}^2\frac{x}{2} = 5 + 5\,\mathrm{tg}^2\frac{x}{2},$$

$$12\,\mathrm{tg}^2\frac{x}{2} - 2\,\mathrm{tg}\frac{x}{2} - 2 = 0,\ \ 6\,\mathrm{tg}^2\frac{x}{2} - \mathrm{tg}\frac{x}{2} - 1 = 0,$$

$\operatorname{tg}\frac{x}{2}=\frac{1+5}{12}=\frac{1}{2}$, $x=\operatorname{arctg}\frac{1}{2}+2\pi n$,

$\operatorname{tg}\frac{x}{2}=\frac{1-5}{12}=-\frac{1}{3}$, $x=2\operatorname{arctg}\left(-\frac{1}{3}\right)+2\pi n$,

$x=-2\operatorname{arctg}\frac{1}{3}+2\pi n$;

б) $5\sin x+10\cos x+2=0$, $\dfrac{10\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}+\dfrac{10-10\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}+2=0$,

$10\operatorname{tg}\frac{x}{2}+10-10\operatorname{tg}^2\frac{x}{2}+2+2\operatorname{tg}^2\frac{x}{2}=0$,

$4\operatorname{tg}^2\frac{x}{2}-5\operatorname{tg}^2\frac{x}{2}-6=0$, $\operatorname{tg}\frac{x}{2}=\frac{5+11}{8}=2$, $x=2\operatorname{arctg}2+2\pi n$,

$\operatorname{tg}\frac{x}{2}=-\frac{3}{4}$, $x=-2\operatorname{arctg}\frac{3}{4}+2\pi n$.

500. а) $\cos\dfrac{1}{x^2-\pi}\sin 2x=8\sin x\cos x$,

$\cos\dfrac{1}{x^2-\pi}\sin 2x=4\sin 2x$, $\sin 2x=0$,

$x=\frac{\pi n}{2}$, $n\neq\pm2$, $\cos\dfrac{1}{x^2-\pi}=4$ – решений нет;

б) $16\sin x\cos x+\sin 2x\sin\frac{1}{x}=0$, $8\sin 2x+\sin 2x\sin\frac{1}{x}=0$,

$\sin 2x=0$, $x=\frac{\pi n}{2}$, $x\neq0$, $\sin\frac{1}{x}=-8$ – решений нет.

501. а) $\sin 2x+2\sin x=2-2\cos x$, $\sin x\cos x+\sin x+\cos x=1$,

$(\sin x+\cos x)=t$, $\sin x\cos x=\frac{1}{2}t^2-\frac{1}{2}$,

$t^2-1+2t-2=0$, $t^2+2t-3=0$, $t=-3$ – решений нет,

$t=1$, $\sin x+\cos x=1$, $\sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$,

$x+\frac{\pi}{4}=(-1)^k\frac{\pi}{4}+\pi k$, $x=(-1)^k\frac{\pi}{4}-\frac{\pi}{4}+\pi k$;

б) $4\sin 2x+8(\sin x-\cos x)=7$,

$\sin x-\cos x=t$, $1-\sin 2x=t^2$, $\sin 2x=1-t^2$,

$4-4t^2+8t-7=0$, $t=\frac{4+2}{4}=\frac{3}{2}$,

$\sin x+\cos x=\frac{3}{2}$ – решений нет,

$t=\frac{1}{2}$, $\sin x+\cos x=\frac{1}{2}$, $\sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{4}$,

$x+\frac{\pi}{4}=(-1)^k\arcsin\frac{\sqrt{2}}{4}+\pi k$, $x=(-1)^k\arcsin\frac{\sqrt{2}}{4}+\pi k-\frac{\pi}{4}$.

502. **а)** $\sin 2x\cos 2x<\frac{1}{4}$, $\sin 4x<\frac{1}{2}$,

$4x\in\left(-\frac{7\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n\right)$, $x\in\left(-\frac{7\pi}{24}+\frac{\pi n}{2};\frac{\pi}{24}+\frac{\pi n}{2}\right)$;

б) $\cos^2\frac{x}{4}-\sin^2\frac{x}{4}>\frac{1}{2}$, $\cos\frac{x}{2}>\frac{1}{2}$,

$\frac{x}{2}\in\left(-\frac{\pi}{3}+2\pi n;\frac{\pi}{3}+2\pi n\right)$, $x\in\left(-\frac{2\pi}{3}+4\pi n;\frac{2\pi}{3}+4\pi n\right)$.

503. **а)** $\cos^2 2x-\sin^2 2x\le -1$,

$\cos 4x\le -1$, $4x=\pi+2\pi n$, $x=\frac{\pi}{4}+\frac{\pi n}{2}$;

б) $\sin 5x\cos 5x\ge\frac{1}{2}$, $\sin 10x\ge 1$, $10x=\frac{\pi}{2}+2\pi n$, $x=\frac{\pi}{20}+\frac{\pi n}{5}$;

в) $\sin^2 3x-\cos^2 3x\le -1$, $\cos 6x\ge 1$, $6x=2\pi n$, $x=\frac{\pi n}{3}$;

г) $\sin\frac{2x}{3}\cos\frac{2x}{3}\le -\frac{1}{2}$, $\sin\frac{4x}{3}\le -1$, $\frac{4x}{3}=-\frac{\pi}{2}+2\pi n$, $x=-\frac{3\pi}{8}+\frac{3\pi n}{2}$.

§25. Формулы понижения степени

504. **а)** $\sin^2 2t=\frac{1-\cos 4t}{2}$; $\frac{1-\cos 4t}{2}=\frac{1}{2}\left(1-\cos^2 2t+\sin^2 2t\right)=\frac{1}{2}\left(2\sin^2 2t\right)=\sin^2 2t$

б) $2\sin^2\frac{t}{2}+\cos t=1$, $2(1-\cos t)\frac{1}{2}+\cos t=1$;

в) $2\sin^2 2t=1+\sin\left(\frac{3t}{2}-4t\right)$, $1-\cos 4t=1+\sin\left(\frac{3t}{2}-4t\right)$;

г) $2\cos^2 t-\cos 2t=1$, $2\cos^2 t-\cos 2t=2\cos^2 t-\cos^2 t+\sin^2 t=1$.

505. **а)** $\cos^2 3t=\frac{1+\sin\left(\frac{\pi}{2}-6t\right)}{2}$, $\cos^2 3t=\frac{1+\cos 6t}{2}=\frac{1+\sin\left(\frac{\pi}{3}-6t\right)}{2}$;

б) $\frac{1-\cos t}{1+\cos t}=\operatorname{tg}^2\frac{t}{2}$, $\operatorname{tg}^2\frac{t}{2}=\frac{\sin^2\frac{t}{2}}{\cos^2\frac{t}{2}}=\frac{1-\cos t}{1+\cos t}$;

в) $\cos^2 3t=\frac{1-\cos(6t+(-3\pi))}{2}=\cos^2 3t=\frac{1+\cos 6t}{2}=\frac{1-\cos(6t-3\pi)}{2}$;

г) $\dfrac{1-\cos t}{\sin t}=\operatorname{tg}\dfrac{t}{2}$, $\dfrac{1-\cos t}{2}=\dfrac{2\sin^2\dfrac{t}{2}}{2\sin\dfrac{t}{2}\cos\dfrac{t}{2}}=\operatorname{tg}\dfrac{t}{2}$.

506. **а)** $1+\sin\alpha=2\cos^2\left(45^\circ-\dfrac{1}{2}\right)$; $2\cos^2\left(45^\circ-\dfrac{\alpha}{2}\right)=1+\cos(90^\circ-\alpha)=1+\sin\alpha$;

б) $2\sin^2(45^\circ-\alpha)+\sin 2\alpha=1$; $1-\cos(90^\circ-2\alpha)+\sin 2\alpha=1$, $1-\sin 2\alpha+\sin 2\alpha=1$;

в) $1-\sin\alpha=2\sin^2\left(45^\circ-\dfrac{\alpha}{2}\right)$; $2\sin^2\left(45^\circ-\dfrac{\alpha}{2}\right)=1-\cos(90^\circ-\alpha)=1-\sin\alpha$;

г) $2\cos^2(45^\circ+\alpha)+\sin 2\alpha=1$; $1+\cos(90^\circ+2\alpha)+\sin 2\alpha=1-\sin 2\alpha+\sin 2\alpha=1$.

507. **а)** $\sin 22{,}5^\circ=\sqrt{\dfrac{1-\cos 45^\circ}{2}}=\sqrt{\dfrac{2-\sqrt{2}}{2}}$; **б)** $\cos 22{,}5^\circ=\sqrt{\dfrac{1+\cos 45^\circ}{2}}=\sqrt{\dfrac{2+\sqrt{2}}{2}}$;

в) $\sin\dfrac{3\pi}{8}=\sqrt{\dfrac{1-\cos\dfrac{3\pi}{8}}{2}}=\sqrt{\dfrac{2-\sqrt{2}}{2}}$; **г)** $\cos\dfrac{3\pi}{8}=\sqrt{\dfrac{1+\cos\dfrac{3\pi}{8}}{2}}=\sqrt{\dfrac{2+\sqrt{2}}{2}}$.

507. **а)** $1-\cos x=2\sin\dfrac{x}{2}$, $1-\left(1-2\sin^2\dfrac{x}{2}\right)=2\sin\dfrac{x}{2}$,

$\sin^2\dfrac{x}{2}=\sin\dfrac{x}{2}$, $\left(\sin\dfrac{x}{2}-1\right)\sin\dfrac{x}{2}=0$,

$$\left[\begin{array}{l}\sin\dfrac{x}{2}=0\\ \sin\dfrac{x}{2}=1\end{array}\right.;\quad \left[\begin{array}{l}x=2\pi n\\ x=\pi+4\pi k\end{array}\right.;$$

б) $1+\cos x=2\cos\dfrac{x}{2}$,

$1+\left(2\cos^2\dfrac{x}{2}-1\right)=2\cos\dfrac{x}{2}$, $\cos^2\dfrac{x}{2}=\cos\dfrac{x}{2}$,

$$\left(\cos\dfrac{x}{2}-1\right)\cos\dfrac{x}{2}=0,\quad \left[\begin{array}{l}\cos\dfrac{x}{2}=0\\ \cos\dfrac{x}{2}=1\end{array}\right.;\quad \left[\begin{array}{l}x=\pi+2\pi n\\ x=4\pi k\end{array}\right..$$

508. **а)** $1-\cos x=\sin x\sin\dfrac{x}{2}$, $1-\left(1-2\sin^2\dfrac{x}{2}\right)=\sin x\sin\dfrac{x}{2}$, $2\sin^2\dfrac{x}{2}=2\sin^2\dfrac{x}{2}\cos\dfrac{x}{2}$,

$$\sin^2\dfrac{x}{2}\left(1-\cos\dfrac{x}{2}\right)=0,\quad \left[\begin{array}{l}\sin\dfrac{x}{2}=0\\ \cos\dfrac{x}{2}=1\end{array}\right.;\quad \left[\begin{array}{l}x=2\pi n\\ x=4\pi k\end{array}\right.;\ x=2\pi n;$$

б) $\sin x = \mathrm{tg}^2\frac{x}{2}(1+\cos x)$, $\sin x = \frac{1-\cos x}{1+\cos x}(1+\cos x)$, $\begin{cases}\sin x = 1-\cos x\\ 1+\cos x \ne 0\end{cases}$,

$\begin{cases}\sqrt{2}\sin x\left(x+\frac{\pi}{4}\right)=1\\ x\ne \pi+2\pi n\end{cases}$; $\begin{cases}\left[\begin{array}{l} x+\frac{\pi}{4}=\frac{\pi}{4}+2\pi k\\ x+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi k\end{array}\right.\\ x\ne\pi+2\pi n\end{cases}$; $\begin{cases}\left[\begin{array}{l} x=2\pi k\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.;\\ x\ne\pi+2\pi n\end{cases}$ $\left[\begin{array}{l} x=2\pi k\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.$.

509. а) $\sin^2 2x = 1$; $\left[\begin{array}{l}\sin 2x = 1\\ \sin 2x = -1\end{array}\right.$; $\left[\begin{array}{l}2x=\frac{\pi}{2}+2\pi n\\ 2x=-\frac{\pi}{2}+2\pi k\end{array}\right.$; $2x=\frac{\pi}{2}+\pi l$; $x=\frac{\pi}{4}+\frac{\pi l}{2}$;

б) $\cos^2 4x=\frac{1}{2}$, $2\cos^2 4x-1=0$, $\cos 8x=0$, $8x=\frac{\pi}{2}+\pi n$, $x=\frac{\pi}{16}+\frac{\pi n}{8}$;

в) $\sin^2\frac{x}{2}=\frac{3}{4}$,

$\left[\begin{array}{l}\sin\frac{x}{2}=\frac{\sqrt{3}}{2}\\ \sin\frac{x}{2}=-\frac{\sqrt{3}}{2}\end{array}\right.$; $\left[\begin{array}{l}\frac{x}{2}=(-1)^n\frac{\pi}{3}+\pi n\\ \frac{x}{2}=(-1)^k\left(-\frac{\pi}{3}\right)+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=(-1)^n\frac{2\pi}{3}+2\pi n\\ x=(-1)^k\left(-\frac{2\pi}{3}\right)+2\pi k\end{array}\right.$; $x=\pm\frac{2\pi}{3}+2\pi n$;

г) $\cos^2\frac{x}{4}=\frac{1}{4}$,

$\left[\begin{array}{l}\cos\frac{x}{4}=\frac{1}{2}\\ \cos\frac{x}{4}=-\frac{1}{2}\end{array}\right.$; $\left[\begin{array}{l}\frac{x}{4}=\pm\frac{\pi}{3}+2\pi n\\ \frac{x}{4}=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.$; $\frac{x}{4}=\pm\frac{\pi}{3}+\pi n$, $x=\pm\frac{4\pi}{3}+4\pi n$.

510. $2\cos^2\frac{x}{2}-\cos\frac{\pi}{9}=1$;

$\left(2\cos^2\frac{x}{2}-1\right)=\cos\frac{\pi}{9}$, $\cos x=\cos\frac{\pi}{9}$, $x=\pm\arccos\left(\cos\frac{\pi}{9}\right)+2\pi n$,

Отсюда имеем, что уравнение имеет отрезок $[-2\pi; 2\pi]$ 4 корня:

$\frac{\pi}{9}-2\pi; -\frac{\pi}{9}; \frac{\pi}{9}; 2\pi-\frac{\pi}{9}$.

511. а) Т.к. $0<t<\frac{\pi}{2}$, то $\cos\frac{t}{2}=\sqrt{\frac{1}{2}(1+\cos t)}=\sqrt{\frac{1}{2}\left(1+\frac{3}{4}\right)}=\frac{\sqrt{7}}{2\sqrt{2}}$,

$\sin\frac{t}{2}=\sqrt{\frac{1}{2}(1-\cos t)}=\frac{1}{2\sqrt{2}}$, $\mathrm{tg}\frac{t}{2}=\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}=\frac{1}{\sqrt{7}}$, $\mathrm{ctg}\frac{t}{2}=\frac{1}{\mathrm{tg}\frac{t}{2}}=\sqrt{7}$;

б) Имеем: $\cos^2 t = \dfrac{\mathrm{ctg}^2 t}{1+\mathrm{ctg}^2 t} = \dfrac{9/16}{1+9/16} = \dfrac{9}{25}$,

Т.к. $\pi < t < \dfrac{3\pi}{2}$ (т.е. $\cos t < 0$), получим $\cos t = -\dfrac{3}{5}$,

Поскольку $\dfrac{\pi}{2} < \dfrac{t}{2} < \dfrac{3\pi}{4}$, то $\sin\dfrac{t}{2} > 0$, $\cos\dfrac{t}{2} < 0$, т.е. имеем:

$$\sin\frac{t}{2} = \sqrt{\frac{1}{2}(1-\cos t)} = \sqrt{\frac{1}{2}\left(1+\frac{3}{5}\right)} = \frac{2}{\sqrt{5}},$$

$$\cos\frac{t}{2} = \sqrt{\frac{1}{2}(1+\cos t)} = -\sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)} = -\frac{1}{\sqrt{5}},$$

$$\mathrm{tg}\frac{t}{2} = \frac{\sin\frac{t}{2}}{\cos\frac{t}{2}} = -2,\ \mathrm{ctg}\frac{t}{2} = \frac{1}{\mathrm{tg}\frac{t}{2}} = -\frac{1}{2}.$$

512. а) Рассмотрим два случая:

1. $\dfrac{\pi}{2} < x < \dfrac{3\pi}{4}$. Тогда $\pi < 2x < \dfrac{3\pi}{2}$ и $\cos 2x = -\sqrt{1-\sin^2 2x} = -\dfrac{4}{5}$,

$$\sin x = \sqrt{\frac{1}{2}(1-\cos 2x)} = \sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = \frac{3}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = -\frac{1}{\sqrt{10}},$$

$$\mathrm{tg}\, x = \frac{\sin x}{\cos x} = -3,\ \mathrm{ctg}\, x = \frac{1}{\mathrm{tg}\, x} = -\frac{1}{3};$$

2. $\dfrac{3\pi}{4} < x < \pi$. Тогда $\dfrac{3\pi}{2} < 2x < 2\pi$ и $\cos 2x = \sqrt{1-\sin^2 2x} = \dfrac{4}{5}$,

$$\sin x = \sqrt{\frac{1}{2}(1-\cos 2x)} = \sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = \frac{1}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = -\frac{3}{\sqrt{10}},$$

$$\mathrm{tg}\, x = \frac{\sin x}{\cos x} = -\frac{1}{3},\ \mathrm{ctg}\, x = \frac{1}{\mathrm{tg}\, x} = -3;$$

б) Т.к. $\pi < x < \dfrac{5\pi}{4}$, то $2\pi < 2x < \dfrac{5\pi}{2}$, т.е. $\cos 2x = \dfrac{1}{\sqrt{1+\mathrm{tg}^2 2x}} = \dfrac{1}{\sqrt{1+9/16}} = \dfrac{4}{5}$,

$$\sin x = -\sqrt{\frac{1}{2}(1-\cos 2x)} = -\sqrt{\frac{1}{2}\left(1-\frac{4}{5}\right)} = -\frac{1}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1+\cos 2x)} = -\sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)} = -\frac{3}{\sqrt{10}},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{1}{3}, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = 3.$$

514. а) $\dfrac{\sin 2t}{1+\cos 2t}\cdot\dfrac{\cos t}{1+\cos t} = \operatorname{tg}\dfrac{t}{2}$,

$$\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t} = \frac{2\sin t\cos^2 t}{2\cos^2 t(1+\cos t)} = \frac{\sin t}{1+\cos t} = \operatorname{tg}\frac{t}{2};$$

б) $\dfrac{\sin 2t}{1+\cos 2t}\cdot\dfrac{\cos t}{1+\cos t}\cdot\dfrac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}} = \operatorname{tg}\dfrac{t}{4}$,

$$\frac{\sin 2t}{1+\cos 2t}\cdot\frac{\cos t}{1+\cos t}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}} = \frac{2\sin t\cos^2 t}{2\cos^2 t(1+\cos t)}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}} =$$

$$= \frac{\sin t}{1+\cos t}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}} = \operatorname{tg}\frac{t}{2}\cdot\frac{\cos\frac{t}{2}}{1+\cos\frac{t}{2}} = \frac{\sin\frac{t}{2}}{1+\cos\frac{t}{2}} = \operatorname{tg}\frac{t}{4}.$$

515. а) $\dfrac{1-\cos 2t+\sin 2t}{1+\cos 2t+\sin 2t} = \operatorname{tg} t$,

$$\frac{1-(1-2\sin^2 t)+2\sin t\cos t}{1+2\sin t\cos t+2\cos^2 t-1} = \frac{\sin t(\sin t+\cos t)}{\cos t(\sin t+\cos t)} = \frac{\sin t}{\cos t} = \operatorname{tg} t;$$

б) $\dfrac{1+\cos 2t-\sin 2t}{1+\sin 2t+\cos 2t} = \operatorname{tg}\left(\dfrac{\pi}{4}-t\right)$,

$$\frac{1+\cos 2t-\sin 2t}{1+\sin 2t+\cos 2t} = \frac{1+2\cos^2 t-1-2\sin t\cos t}{1+2\sin t\cos t+2\cos^2 t-1} = \frac{\cos t-\sin t}{\cos t+\sin t} = \frac{\sqrt{2}\sin\left(\frac{\pi}{4}-t\right)}{\sqrt{2}\cos\left(\frac{\pi}{4}-t\right)} =$$

$$= \operatorname{tg}\left(\frac{\pi}{4}-t\right).$$

516. а) $\cos^2 t-\cos^2\left(\dfrac{\pi}{4}-t\right) = \dfrac{1}{\sqrt{2}}\sin\left(\dfrac{\pi}{4}-2t\right)$,

$$\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}-2t\right) = \frac{1}{\sqrt{2}}\left(\sin\frac{\pi}{4}\cos 2t-\sin 2t\cos\frac{\pi}{4}\right) =$$

$$= \frac{1}{2}(\cos 2t-\sin 2t) = \frac{1}{2}\left(\cos 2t-\cos\left(\frac{\pi}{2}-2t\right)\right) =$$

$$=\frac{1}{2}\left(2\cos^2 t-1-2\cos^2\left(\frac{\pi}{4}-t\right)+1\right)=\cos^2 t-\cos^2\left(\frac{\pi}{4}-t\right);$$

б) $\sin^2 t-\sin^2\left(\frac{\pi}{4}-t\right)=\frac{1}{\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)$,

$$\frac{1}{\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\left(\sin 2t\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\cos 2t\right)=$$

$$=\frac{1}{2}(\sin 2t-\cos 2t)=\frac{1}{2}\left(\cos\left(\frac{\pi}{2}-2t\right)-\cos 2t\right)=$$

$$=\frac{1}{2}\left(1-2\sin^2\left(\frac{\pi}{4}-t\right)-1+2\sin^2 t\right)=\sin^2 t-\sin^2\left(\frac{\pi}{4}-t\right).$$

517. а) $f(x)=2\cos 2x+\sin^2 x=2\cos 2x+\frac{1}{2}(1-2\cos 2x)=\frac{3}{2}\cos 2x+\frac{1}{2}$

Поскольку наибольшее значение функции $y=\cos 2x$ равно 1, а наименьшее -1, то наибольшее значение функции $f(x)$ равно 2, а наименьшее -1;

б) $f(x)=2\sin^2 3x-\cos 6x=(1-\cos 6x)-\cos 6x=1-2\cos 6x$

Поскольку наибольшее значение функции $y=\cos 6x$ равно 1, а наименьшее -1, то наибольшее значение функции $f(x)$ равно 3, а наименьшее -1.

518. а) $t\in\left[\frac{\pi}{2};\pi\right]$, т.е. $\sin t\geq 0,\ \cos t\leq 0$,

$$\sqrt{1-\cos 2t}+\sqrt{1+\cos 2t}=\sqrt{2}(|\sin t|+|\cos t|)=\sqrt{2}(\sin t-\cos t)=2\sin\left(t-\frac{\pi}{4}\right);$$

б) $t\in\left[\frac{3\pi}{2};2\pi\right]$, т.е. $\sin t\leq 0,\ \cos t\geq 0$,

$$\sqrt{1-\cos 2t}+\sqrt{1+\cos 2t}=\sqrt{2}(|\sin t|+|\cos t|)=\sqrt{2}(-\sin t+\cos t)=2\sin\left(\frac{\pi}{4}-t\right);$$

в) $t\in\left[0;\frac{\pi}{2}\right]$, т.е. $\sin t\geq 0,\ \cos t\geq 0$,

$$\sqrt{1-\cos 2t}+\sqrt{1+\cos 2t}=\sqrt{2}(|\sin t|+|\cos t|)=\sqrt{2}(\sin t+\cos t)=2\sin\left(t+\frac{\pi}{4}\right);$$

г) $t\in\left[\pi;\frac{3\pi}{2}\right]$, т.е. $\sin t\leq 0,\ \cos t\leq 0$,

$$\sqrt{1-\cos 2t}+\sqrt{1+\cos 2t}=\sqrt{2}(|\sin t|+|\cos t|)=-\sqrt{2}(\sin t+\cos t)=-2\sin\left(t+\frac{\pi}{4}\right).$$

519. $\cos 2x=\frac{5}{13}=2\cos^2 x-1=1-2\sin^2 x$,

откуда $\cos^2 x=\frac{1}{2}\left(1+\frac{5}{13}\right)=\frac{9}{13},\ \sin^2 x=\frac{1}{2}\left(1-\frac{5}{13}\right)=\frac{4}{13}$, а) $\sin^4 x+\cos^4 x=(\sin^2 x)^2+(\cos^2 x)^2=\left(\frac{4}{13}\right)^2+\left(\frac{9}{13}\right)^2=\frac{97}{169}$;

б) $\sin^8 x+\cos^8 x=(\sin^2 x)^4+(\cos^2 x)^4=\left(\frac{4}{13}\right)^4+\left(\frac{9}{13}\right)^4=\frac{6817}{28561}$.

520. а) $\sin^2\left(2x-\frac{\pi}{6}\right)=\frac{3}{4}$,

$\frac{1}{2}\left(1-\cos\left(4x-\frac{\pi}{3}\right)\right)=\frac{3}{4}$, $\cos\left(4x-\frac{\pi}{3}\right)=-\frac{1}{2}$,

$4x-\frac{\pi}{3}=\pm\frac{2\pi}{3}+2\pi n$, $\left[\begin{array}{l} x=\frac{\pi}{4}+\frac{\pi n}{2} \\ x=-\frac{\pi}{12}+\frac{\pi k}{2} \end{array}\right.$;

б) $\cos^2\left(x+\frac{\pi}{3}\right)=1$,

$\frac{1}{2}\left(1+\cos\left(2x-\frac{2\pi}{3}\right)\right)=1$, $\cos\left(2x+\frac{2\pi}{3}\right)=1$,

$2x+\frac{2\pi}{3}=2\pi n$, $x=-\frac{\pi}{3}+\pi n$;

в) $\sin^2\left(x+\frac{\pi}{2}\right)=\frac{1}{2}$, $\frac{1}{2}(1-\cos(2x+\pi))=\frac{1}{2}$, $\frac{1}{2}(1+\cos 2x)=\frac{1}{2}$,

$\cos 2x=-1$, $2x=\pi+2\pi n$, $x=\frac{\pi}{2}+\pi n$;

г) $\cos^2\left(3x-\frac{\pi}{4}\right)=\frac{3}{4}$, $\frac{1}{2}\left(1+\cos\left(6x-\frac{\pi}{2}\right)\right)=\frac{3}{4}$, $\frac{1}{2}(1+\sin 6x)=\frac{3}{4}$,

$\sin 6x=\frac{1}{2}$, $6x=(-1)^n\frac{\pi}{6}+\pi n$, $x=(-1)^n\frac{\pi}{36}+\frac{\pi n}{6}$.

521. а) $4\sin^2 x+\sin^2 2x=3$,
$2(1-\cos 2x)+1-\cos^2 2x=3$, $\cos^2 2x+2\cos 2x=0$,
$\cos 2x(\cos 2x+2)=0$, $\cos 2x=0$ (т.к. $\cos 2x+2>0$ для всех x),

$2x=\frac{\pi}{2}+\pi n$, $x=\frac{\pi}{4}+\frac{\pi n}{2}$;

б) $4\cos^2 2x+8\cos^2 x=7$, $4\cos^2 2x+4(1+\cos 2x)=7$,
$4\cos^2 2x+4\cos 2x-3=0$,

$\cos 2x=\frac{-2\pm\sqrt{4+12}}{4}=\frac{-2\pm 4}{4}=\left[\begin{array}{l} \frac{1}{2} \\ -\frac{3}{2} \end{array}\right.$,

$\cos 2x = \frac{1}{2}$ (т.к. $\cos 2x \neq -\frac{3}{2}$ при всех x),

$2x = \pm\frac{\pi}{3} + 2\pi n,\ \ x = \pm\frac{\pi}{6} + \pi n$.

522. а) $4\sin^2 3x < 3$,

$2(1-\cos 6x) < 3,\ \ \cos 6x > -\frac{1}{2},\ \ -\frac{2\pi}{3} + 2\pi n < 6x < \frac{2\pi}{3} + 2\pi n$,

$-\frac{\pi}{9} + \frac{\pi n}{3} < x < \frac{\pi}{9} + \frac{\pi n}{3}$;

б) $4\cos^2\frac{x}{4} > 1,\ \ 2\left(1+\cos\frac{x}{2}\right) > 1,\ \ \cos\frac{x}{2} > -\frac{1}{2}$,

$-\frac{2\pi}{3} + 2\pi n < \frac{x}{2} < \frac{2\pi}{3} + 2\pi n,\ \ -\frac{4\pi}{3} + 4\pi n < x < \frac{4\pi}{3} + 4\pi n$.

§26. Преобразование сумм тригонометрических функций в произведение

523. а) $\sin 40^\circ + \sin 16^\circ = 2\sin\frac{40^\circ+16^\circ}{2}\cos\frac{40^\circ-16^\circ}{2} = 2\sin 28^\circ\cos 12^\circ$;

б) $\sin 20^\circ - \sin 40^\circ = 2\sin\frac{20^\circ-40^\circ}{2}\cos\frac{20^\circ+40^\circ}{2} = -2\sin 10^\circ\cos 30^\circ = -\sqrt{3}\sin 10^\circ$;

в) $\sin 10^\circ + \sin 50^\circ = 2\sin\frac{10^\circ+50^\circ}{2}\cos\frac{50^\circ-10^\circ}{2} = 2\sin 30^\circ\cos 20^\circ = \cos 20^\circ$;

г) $\sin 52^\circ - \sin 36^\circ = 2\sin\frac{52^\circ-36^\circ}{2}\cos\frac{52^\circ+36^\circ}{2} = 2\sin 8^\circ\cos 44^\circ$.

524. а) $\cos 15^\circ + \cos 45^\circ = 2\cos\frac{15^\circ+45^\circ}{2}\cos\frac{15^\circ-45^\circ}{2} = 2\cos 30^\circ\cos 15^\circ = \sqrt{3}\cos 15^\circ$;

б) $\cos 46^\circ - \cos 74^\circ = 2\sin\frac{46^\circ+74^\circ}{2}\sin\frac{74^\circ-46^\circ}{2} = 2\sin 60^\circ\sin 14^\circ = \sqrt{3}\cos 14^\circ$;

в) $\cos 20^\circ + c0s40^\circ = 2\cos\frac{20^\circ+40^\circ}{2}\cos\frac{40^\circ-20^\circ}{2} = 2\cos 30^\circ\cos 10^\circ = \sqrt{3}\cos 10^\circ$;

г) $\cos 75^\circ - \cos 15^\circ = 2\sin\frac{75^\circ+15^\circ}{2}\sin\frac{15^\circ-75^\circ}{2} = -2\sin 45^\circ\sin 30^\circ = -\frac{\sqrt{2}}{2}$.

525. а) $\sin\frac{\pi}{5} - \sin\frac{\pi}{10} = 2\sin\frac{\frac{\pi}{5}-\frac{\pi}{10}}{2}\cos\frac{\frac{\pi}{5}+\frac{\pi}{10}}{2} = 2\sin\frac{\pi}{20}\cos\frac{3\pi}{20}$;

б) $\sin\frac{\pi}{3} + \sin\frac{\pi}{4} = 2\sin\frac{\frac{\pi}{3}+\frac{\pi}{4}}{2}\cos\frac{\frac{\pi}{3}-\frac{\pi}{4}}{2} = 2\sin\frac{7\pi}{24}\cos\frac{\pi}{24}$;

в) $\sin\frac{\pi}{6} + \sin\frac{\pi}{7} = 2\sin\frac{\frac{\pi}{6}+\frac{\pi}{7}}{2}\cos\frac{\frac{\pi}{6}-\frac{\pi}{7}}{2} = 2\sin\frac{13\pi}{84}\cos\frac{\pi}{84}$;

г) $\sin\frac{\pi}{3}-\sin\frac{\pi}{11}=2\sin\frac{\frac{\pi}{3}-\frac{\pi}{11}}{2}\cos\frac{\frac{\pi}{3}+\frac{\pi}{11}}{2}=2\sin\frac{4\pi}{33}\cos\frac{7\pi}{33}$.

526. а) $\cos\frac{\pi}{10}-\cos\frac{\pi}{20}=2\sin\frac{\frac{\pi}{10}+\frac{\pi}{20}}{2}\sin\frac{\frac{\pi}{20}-\frac{\pi}{10}}{2}=-2\sin\frac{3\pi}{40}\sin\frac{\pi}{40}$;

б) $\cos\frac{11\pi}{12}+\cos\frac{3\pi}{4}=2\cos\frac{\frac{11\pi}{12}+\frac{3\pi}{4}}{2}\cos\frac{\frac{11\pi}{12}-\frac{3\pi}{3}}{2}=2\cos\frac{5\pi}{6}\cos\frac{\pi}{12}=-\sqrt{3}\cos\frac{\pi}{12}$;

в) $\cos\frac{\pi}{5}-\cos\frac{\pi}{11}=2\sin\frac{\frac{\pi}{5}+\frac{\pi}{11}}{2}\sin\frac{\frac{\pi}{11}-\frac{\pi}{5}}{2}=-2\sin\frac{8\pi}{55}\sin\frac{3\pi}{55}$;

г) $\cos\frac{3\pi}{8}+\cos\frac{5\pi}{4}=2\cos\frac{\frac{3\pi}{8}+\frac{5\pi}{4}}{2}\cos\frac{\frac{3\pi}{8}-\frac{5\pi}{3}}{2}=2\cos\frac{13\pi}{16}\cos\frac{7\pi}{16}$.

527. а) $\sin 3t-\sin t=2\sin\frac{3t-t}{2}\cos\frac{3t+t}{2}=2\sin t\cos t$;

б) $\cos(\alpha-2\beta)-\cos(\alpha+2\beta)=2\sin\frac{\alpha-2\beta+\alpha+2\beta}{2}\sin\frac{\alpha+2\beta-\alpha+2\beta}{2}=2\sin\alpha\sin 2\beta$;

в) $\cos 6t+\cos 4t=2\cos\frac{6t+4t}{2}\cos\frac{6t-4t}{2}=2\cos 5t\cos t$;

г) $\sin(\alpha-2\beta)-\sin(\alpha+2\beta)=2\sin\frac{\alpha-2\beta-\alpha-2\beta}{2}\cos\frac{\alpha-2\beta+\alpha+2\beta}{2}=-2\sin 2\beta\cos\alpha$.

528. а) $\operatorname{tg}25^\circ+\operatorname{tg}35^\circ=\frac{\sin(25^\circ+35^\circ)}{\cos 25^\circ\cos 35^\circ}=\frac{\sin 60^\circ}{\cos 25^\circ\cos 35^\circ}=\frac{\sqrt{3}}{\cos 25^\circ\cos 35^\circ}$;

б) $\operatorname{tg}\frac{\pi}{5}-\operatorname{tg}\frac{\pi}{10}=\frac{\sin\left(\frac{\pi}{5}-\frac{\pi}{10}\right)}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\frac{\operatorname{tg}\frac{\pi}{10}}{\cos\frac{\pi}{5}}$;

в) $\operatorname{tg}20^\circ+\operatorname{tg}40^\circ=\frac{\sin(20^\circ+40^\circ)}{\sin 20^\circ\sin 40^\circ}=\frac{\sin 60^\circ}{\sin 20^\circ\sin 40^\circ}=\frac{\sqrt{3}}{2\sin 20^\circ\sin 40^\circ}$;

г) $\operatorname{tg}\frac{\pi}{3}-\operatorname{tg}\frac{\pi}{4}=\frac{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\cos\frac{\pi}{3}\cos\frac{\pi}{4}}=\frac{\sin\frac{\pi}{12}}{\frac{1}{2}\frac{\sqrt{2}}{2}}=2\sqrt{2}\sin\frac{\pi}{12}$.

529. а) $\frac{\cos 68^\circ-\cos 22^\circ}{\sin 68^\circ-\sin 22^\circ}=\frac{2\sin\frac{68^\circ+22^\circ}{2}\sin\frac{22^\circ-68^\circ}{2}}{2\sin\frac{68^\circ-22^\circ}{2}\sin\frac{68^\circ+22^\circ}{2}}=\frac{-\sin 45^\circ\sin 23^\circ}{2\sin 23^\circ\cos 45^\circ}=$

$=-\operatorname{tg}45^\circ=-1$;

б) $\dfrac{\sin 130^\circ + \sin 110^\circ}{\cos 130^\circ + \cos 110^\circ} = \dfrac{2\sin\dfrac{130^\circ + 110^\circ}{2}\cos\dfrac{130^\circ - 110^\circ}{2}}{2\cos\dfrac{130^\circ + 110^\circ}{2}\cos\dfrac{130^\circ - 110^\circ}{2}} = \dfrac{\sin 120^\circ \sin 10^\circ}{\cos 120^\circ \cos 10^\circ} =$

$= \dfrac{\sin 120^\circ}{\cos 120^\circ} = \mathrm{tg}\, 120^\circ = -\sqrt{3}$.

530. а) $\sin 35^\circ + \sin 25^\circ = \cos 5^\circ$,

$\sin 35^\circ + \sin 25^\circ = 2\sin\dfrac{35^\circ + 25^\circ}{2}\cos\dfrac{35^\circ - 25^\circ}{2} = 2\sin 30^\circ \cos 5^\circ = \cos 5^\circ$

б) $\sin 40^\circ + \cos 70^\circ = \cos 10^\circ$,

$\sin 40^\circ + \cos 70^\circ = \sin 40^\circ + \sin 20^\circ = 2\sin\dfrac{40^\circ + 20^\circ}{2}\cos\dfrac{40^\circ - 20^\circ}{2} =$

$= 2\sin 30^\circ \cos 10^\circ = \cos 10^\circ$;

в) $\cos 12^\circ - \cos 48^\circ = \sin 18^\circ$,

$\cos 12^\circ - \cos 48^\circ = 2\sin\dfrac{12^\circ + 48^\circ}{2}\sin\dfrac{48^\circ - 12^\circ}{2} = 2\sin 30^\circ \sin 18^\circ = \sin 18^\circ$;

г) $\cos 20^\circ - \sin 50^\circ = \sin 10^\circ$,

$\cos 20^\circ - \sin 50^\circ = \cos 20^\circ - \cos 40^\circ = 2\sin\dfrac{20^\circ + 40^\circ}{2}\sin\dfrac{40^\circ - 20^\circ}{2} =$

$= 2\sin 30^\circ \sin 10^\circ = \sin 10^\circ$.

531. а) $\dfrac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \mathrm{tg}\, 4\alpha$,

$\dfrac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \dfrac{2\sin\dfrac{2\alpha + 6\alpha}{2}\cos\dfrac{6\alpha - 2\alpha}{2}}{2\cos\dfrac{2\alpha + 6\alpha}{2}\cos\dfrac{6\alpha - 2\alpha}{2}} = \dfrac{\sin 4\alpha \cos 2\alpha}{\cos 4\alpha \cos 2\alpha} = \dfrac{\sin 4\alpha}{\cos 4\alpha} = \mathrm{tg}\, 4\alpha$;

б) $\dfrac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \mathrm{tg}\, 3\alpha\, \mathrm{tg}\, \alpha$,

$\dfrac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \dfrac{2\sin\dfrac{2\alpha + 4\alpha}{2}\cos\dfrac{4\alpha - 2\alpha}{2}}{2\cos\dfrac{2\alpha + 4\alpha}{2}\cos\dfrac{4\alpha - 2\alpha}{2}} = \dfrac{\sin 3\alpha \sin \alpha}{\cos 3\alpha \cos \alpha} = \mathrm{tg}\, 3\alpha\, \mathrm{tg}\, \alpha$.

532. а) $\cos x + \cos 3x = 0$; $2\cos\dfrac{x + 3x}{2}\cos\dfrac{3x - x}{2} = 0$, $\cos 2x \cos x = 0$,

$\left[\begin{array}{l} \cos 2x = 0 \\ \cos x = 0 \end{array}\right.$; $\left[\begin{array}{l} 2x = \dfrac{\pi}{2} + \pi n \\ x = \dfrac{\pi}{2} + \pi k \end{array}\right.$; $\left[\begin{array}{l} x = \dfrac{\pi}{4} + \dfrac{\pi n}{2} \\ x = \dfrac{\pi}{2} + \pi k \end{array}\right.$;

б) $\sin 12x + \sin 4x = 0$; $2\sin\dfrac{12x + 4x}{2}\cos\dfrac{12x - 4x}{2} = 0$, $\sin 8x \cos 4x = 0$,

$$\left[\begin{array}{l}\sin 8x=0\\ \cos 4x=0\end{array}\right.;\quad \left[\begin{array}{l}8x=\pi n\\ 4x=\frac{\pi}{2}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{8}\\ x=\frac{\pi}{8}+\frac{\pi k}{4}\end{array}\right.;\quad x=\frac{\pi n}{8};$$

в) $\cos x=\cos 5x$; $\cos x-\cos 5x=0,\ 2\sin\frac{x+5x}{2}\cos\frac{5x-x}{2}=0$,

$$\sin 3x\sin 2x=0,\quad \left[\begin{array}{l}\sin 3x=0\\ \cos 2x=0\end{array}\right.;\quad \left[\begin{array}{l}3x=\pi n\\ 2x=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{3}\\ x=\frac{\pi k}{2}\end{array}\right.;$$

г) $\sin 3x=\sin 17x$; $\sin 17x-\sin 3x=0,\ 2\sin\frac{17x-3x}{2}\cos\frac{17x+3x}{2}=0$,

$$\sin 7x\cos 10x=0,\quad \left[\begin{array}{l}\sin 7x=0\\ \cos 10x=0\end{array}\right.;\quad \left[\begin{array}{l}7x=\pi n\\ 10x=\frac{\pi}{2}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{7}\\ x=\frac{\pi}{20}+\frac{\pi k}{10}\end{array}\right..$$

533. а) $\sin x+\sin 2x+\sin 3x=0$,

$\sin 2x+2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2}=0,\ \sin 2x+2\sin 2x\cos x=0$,

$$\sin 2x(1+2\cos x)=0,\quad \left[\begin{array}{l}\sin 2x=0\\ \cos x=-\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x=\pi n\\ x=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{2}\\ x=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.;$$

б) $\cos 3x-\cos 5x=\sin 4x$,

$2\sin\frac{3x+5x}{2}\cos\frac{5x-3x}{2}=\sin 4x,\ 2\sin 4x\sin x-\sin 4x=0$,

$$\sin 4x(2\sin x-1)=0,\quad \left[\begin{array}{l}\sin 4x=0\\ \cos x=\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}4x=\pi n\\ x=(-1)^k\frac{\pi}{6}+\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{4}\\ x=(-1)^k\frac{\pi}{6}+\pi k\end{array}\right..$$

534. а) $\frac{1}{2}-\cos t=\cos\frac{\pi}{3}-\cos t=2\sin\frac{\frac{\pi}{3}+t}{2}\sin\frac{t-\frac{\pi}{3}}{2}=2\sin\left(\frac{\pi}{6}+\frac{t}{2}\right)\sin\left(\frac{t}{2}-\frac{\pi}{6}\right)$;

б) $\frac{\sqrt{3}}{2}+\sin t=\sin\frac{\pi}{3}+\sin t=2\sin\frac{\frac{\pi}{3}+t}{2}\cos\frac{\frac{\pi}{3}-t}{2}=2\cos\left(\frac{\pi}{6}+\frac{t}{2}\right)\cos\left(\frac{\pi}{6}-\frac{t}{2}\right)$;

в) $1+2\cos t=2\left(\frac{1}{2}+\cos t\right)=2\left(\cos\frac{\pi}{3}+\cos t\right)=4\cos\frac{\frac{\pi}{3}+t}{2}\cos\frac{\frac{\pi}{3}-t}{2}=$

$=4\cos\left(\frac{\pi}{6}+\frac{t}{2}\right)\cos\left(\frac{\pi}{6}-\frac{t}{2}\right)$;

г) $\cos t+\sin t=\cos t+\cos\left(\frac{\pi}{2}-t\right)=2\cos\frac{t+\frac{\pi}{2}-t}{2}\cos\frac{t-\frac{\pi}{2}+t}{2}=$

$=2\cos\frac{\pi}{4}\cos\left(t-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(t-\frac{\pi}{4}\right)$.

535. а) $\sin 5x+2\sin 6x+\sin 7x=(\sin 5x+\sin 7x)+2\sin 6x=$

$=2\sin\frac{5x+7x}{2}\cos\frac{7x-5x}{2}+2\sin 6x=2\sin 6x\cos x+2\sin 6x=$

$=2\sin 6x(1+\cos x)=4\sin 6x\cos^2\frac{x}{2}$;

б) $2\cos x+\cos 2x+\cos 4x=2\cos x+2\cos\frac{2x+4x}{2}\cos\frac{4x-2x}{2}=$

$=2\cos x+2\cos x\cos 3x=2\cos x(1+\cos 3x)=4\cos x\cos^2\frac{3x}{2}$.

536. а) $\sin t+\sin 2t+\sin 3t+\sin 4t=(\sin t+\sin 4t)+(\sin 2t+\sin 3t)=$

$=2\sin\frac{t+4t}{2}\cos\frac{4t-t}{2}+2\sin\frac{2t+3t}{2}\cos\frac{3t-2t}{2}=2\sin\frac{5t}{2}\cos\frac{3t}{2}+2\sin\frac{5t}{2}\cos\frac{t}{2}=$

$=2\sin\frac{5t}{2}\left(\cos\frac{3t}{2}+\cos\frac{t}{2}\right)=4\sin\frac{5t}{2}\cos\frac{\frac{3t}{2}+\frac{t}{2}}{2}\cos\frac{\frac{3t}{2}+\frac{t}{2}}{2}=4\sin\frac{5t}{2}\cos t\cos\frac{t}{2}$;

б) $\cos 2t-\cos 4t-\cos 6t+\cos 8t=(\cos 2t+\cos 8t)-(\cos 4t+\cos 6t)=$

$=2\cos\frac{2t+8t}{2}\cos\frac{8t-2t}{2}-2\cos\frac{4t+6t}{2}\cos\frac{6t-4t}{2}=$

$=2\cos 5t\cos 3t-2\cos 5t\cos t=2\cos 5t(\cos 3t-\cos t)=$

$=4\cos 5t\sin\frac{3t+t}{2}\sin\frac{t-3t}{2}=-4\cos 5t\sin 2t\sin t$.

537. а) $\sin 20^\circ+\sin 40^\circ-\cos 10^\circ=0$,

$\sin 20^\circ+\sin 40^\circ-\cos 10^\circ=2\sin\frac{20^\circ+40^\circ}{2}\cos\frac{40^\circ-20^\circ}{2}-\cos 10^\circ=$

$=2\sin 30^\circ\cos 10^\circ-\cos 10^\circ=2\cdot\frac{1}{2}\cdot\cos 10^\circ-\cos 10^\circ=0$;

б) $\cos 85^\circ+\cos 35^\circ-\cos 25^\circ=0$,

$\cos 85^\circ+\cos 35^\circ-\cos 25^\circ=2\cos\frac{85^\circ+35^\circ}{2}\cos\frac{85^\circ-35^\circ}{2}-\cos 25^\circ=$

$=2\cos 60^\circ\cos 25^\circ-\cos 25^\circ=2\cdot\frac{1}{2}\cdot\cos 25^\circ-\cos 25^\circ=0$.

538. а) $\sin 87^\circ-\sin 59^\circ-\sin 93^\circ+\sin 61^\circ=\sin 1^\circ$,

$\sin 87^\circ-\sin 59^\circ-\sin 93^\circ+\sin 61^\circ=(\sin 87^\circ-\sin 93^\circ)+(\sin 61^\circ-\sin 59^\circ)=$

$=2\sin\frac{87^\circ-93^\circ}{2}\cos\frac{87^\circ+93^\circ}{2}+2\sin\frac{61^\circ-59^\circ}{2}\cos\frac{61^\circ+59^\circ}{2}=$

$=-2\sin 3^\circ\cos 90^\circ+2\sin 1^\circ\cos 60^\circ=0+2\cdot\frac{1}{2}\cdot\sin 1^\circ=\sin 1^\circ$;

б) $\cos 115° - \cos 35° + \cos 65° + \cos 25° = \sin 5°$,

$\cos 115° - \cos 35° + \cos 65° + \cos 25° = (\cos 115° + \cos 65°) + (\cos 25° - \cos 35°) =$

$$= 2\cos\frac{115°+65°}{2}\cos\frac{115°-65°}{2} + 2\sin\frac{25°+35°}{2}\sin\frac{35°+25°}{2} =$$

$$= 2\sin 90°\cos 25° + 2\sin 30°\sin 5° = 0 + 2\cdot\frac{1}{2}\cdot\sin 5° = \sin 5°.$$

539. а) $$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha s\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\cos\beta} =$$

$$= \frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta} = \frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha;$$

б) $$\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta-\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta-\sin\alpha\cos\beta+\cos\alpha\cos\beta} =$$

$$= \frac{2\sin\alpha\sin\beta}{2\cos\alpha\sin\beta} = \frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha.$$

540. а) $$\sin x + \sin y + \sin(x-y) = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} + 2\sin\frac{x-y}{2}\cos\frac{x-y}{2} =$$

$$= 2\cos\frac{x-y}{2}\left(\sin\frac{x+y}{2}+\sin\frac{x-y}{2}\right) = 4\cos\frac{x-y}{2}\sin\frac{\frac{x+y}{2}+\frac{x-y}{2}}{2}\cos\frac{\frac{x+y}{2}-\frac{x-y}{2}}{2} =$$

$$= 4\sin\frac{x}{2}\cos\frac{x}{2}\cos\frac{x-y}{2};$$

б) $$\frac{\sin x+\sin 2x+\sin 3x}{\cos x+\cos 2x+\cos 3x} = \frac{\sin 2x+2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2}}{\cos 2x+2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2}} = \frac{\sin 2x+2\sin 2x\cos x}{\cos 2x+2\cos 2x\cos x} =$$

$$= \frac{\sin 2x(1+2\cos x)}{\cos 2x(1+2\cos x)} = \frac{\sin 2x}{\cos 2x} = \operatorname{tg}2x.$$

541. а) $\sin^2(\alpha+\beta) - \sin^2(\alpha-\beta) = (\sin(\alpha+\beta) - \sin(\alpha-\beta))(\sin(\alpha+\beta) + \sin(\alpha-\beta)) =$
$= (2\cos\alpha\sin\beta)(2\sin\alpha\cos\beta) = (2\sin\alpha\cos\alpha)(2\sin\beta\cos\beta) = \sin 2\alpha\sin 2\beta$;

б) $\cos^2(\alpha-\beta) - \cos^2(\alpha+\beta) = (\cos(\alpha-\beta) - \cos(\alpha+\beta))(\cos(\alpha-\beta) + \cos(\alpha+\beta)) =$
$= (2\sin\alpha\sin\beta)(2\cos\alpha\cos\beta) = (2\sin\alpha\cos\alpha)(2\sin\beta\cos\beta) = \sin 2\alpha\sin 2\beta$.

542. $$\frac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha} = \frac{(\sin\alpha+\sin 7\alpha)+(\sin 3\alpha+\sin 5\alpha)}{(\cos\alpha+\cos 7\alpha)+(\cos 3\alpha+\cos 5\alpha)} =$$

$$= \frac{2\sin 4\alpha\cos 3\alpha+2\sin 4\alpha\cos\alpha}{2\cos 4\alpha\cos 3\alpha+2\cos 4\alpha\cos\alpha} = \frac{2\sin 4\alpha(\cos 3\alpha+\cos\alpha)}{2\cos 4\alpha(\cos 3\alpha+\cos\alpha)} = \frac{\sin 4\alpha}{\cos 4\alpha} = \frac{1}{\operatorname{ctg}4\alpha} = \frac{1}{0{,}2} = 5.$$

543. а) $\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma = \operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}(\pi-\alpha-\beta) = (\operatorname{tg}\alpha + \operatorname{tg}\beta) - \operatorname{tg}(\alpha+\beta) =$

$$= \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} - \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \sin(\alpha+\beta)\left(\frac{1}{\cos\alpha\cos\beta} - \frac{1}{\cos(\alpha+\beta)}\right) =$$

$$= \sin(\pi-\alpha-\beta)\left(\frac{1}{\cos\alpha\cos\beta} - \frac{1}{-\cos(\pi-\alpha-\beta)}\right) = \sin\gamma\left(\frac{1}{\cos\alpha\cos\beta} + \frac{1}{\cos\gamma}\right) =$$

$$=\sin\gamma\frac{\cos\gamma+\cos\alpha\cos\beta}{\cos\alpha\cos\beta\cos\gamma}=\sin\gamma\frac{-\cos(\alpha+\beta)+\cos\alpha\cos\beta}{\cos\alpha\cos\beta\cos\gamma}=\frac{\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}=\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma;$$

б) $\sin\alpha+\sin\beta+\sin\gamma=\sin\alpha+\sin\beta+\sin(\pi-\alpha-\beta)=(\sin\alpha+\sin\beta)+\sin(\alpha+\beta)=$

$$=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}=2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right)=$$

$$=4\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=4\cos\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}.$$

544. а) $\sin^2 10^\circ+\sin^2 130^\circ+\sin^2 110^\circ=\frac{1}{2}\left(1-\cos 20^\circ+1-\cos 260^\circ+1-\cos 220^\circ\right)=$

$$=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\frac{1}{2}\cos 80^\circ+\frac{1}{2}\cos 40^\circ=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\cos\frac{80^\circ+40^\circ}{2}\cos\frac{80^\circ-40^\circ}{2}=$$

$$=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\cos 60^\circ\cos 20^\circ=\frac{3}{2}-\frac{1}{2}\cos 20^\circ+\frac{1}{2}\cos 20^\circ=\frac{3}{2};$$

б) $\cos^2 35^\circ+\cos^2 25^\circ+\cos^2 5^\circ=\frac{1}{2}\left(1+\cos 70^\circ+1+\cos 50^\circ-1-\cos 10^\circ\right)=$

$$=\frac{1}{2}\left(1+\cos 70^\circ+\cos 50^\circ-\cos 10^\circ\right)=\frac{1}{2}+\cos\frac{70^\circ+50^\circ}{2}\cos\frac{70^\circ-50^\circ}{2}-\frac{1}{2}\cos 10^\circ=$$

$$=\frac{1}{2}+\cos 60^\circ\cos 10^\circ-\frac{1}{2}\cos 10^\circ=\frac{1}{2}+\frac{1}{2}\cos 10^\circ-\frac{1}{2}\cos 10^\circ=\frac{1}{2}.$$

545. а) $\sin 3x=\cos 2x$; $\sin 3x-\sin\left(\frac{\pi}{2}-2x\right)=0$; $2\sin\frac{3x-\frac{\pi}{2}+2x}{2}\cos\frac{3x-\frac{\pi}{2}-2x}{2}=0$;

$$\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)=0;\ \left[\begin{array}{l}\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)=0\\ \cos\left(\frac{x}{2}+\frac{\pi}{4}\right)=0\end{array}\right.;\ \left[\begin{array}{l}\frac{5x}{2}-\frac{\pi}{4}=\pi n\\ \frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{10}+\frac{2\pi n}{5}\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.;\ x=\frac{\pi}{10}+\frac{2\pi n}{5};$$

б) $\sin(5x-x\_=\cos(2x+7\pi)$; $\sin x=-\cos 2x$; $\cos 2x+\cos\left(\frac{\pi}{2}-x\right)=0$;

$$2\cos\frac{2x+\frac{\pi}{2}-x}{2}\cos\frac{2x+\frac{\pi}{2}+x}{2}=0;\ \cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0;$$

$$\left[\begin{array}{l}\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)=0\\ \cos\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0\end{array}\right.;\ \left[\begin{array}{l}\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+\pi n\\ \frac{3x}{2}-\frac{\pi}{4}=\frac{\pi}{2}+\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{2}+2\pi n\\ x=\frac{\pi}{2}+\frac{2\pi k}{3}\end{array}\right.;\ x=\frac{\pi}{2}+\frac{2\pi k}{3};$$

в) $\cos 5x=\sin 15x$; $\cos 5x-\cos\left(\frac{\pi}{2}-15x\right)=0$; $2\sin\frac{5x+\frac{\pi}{2}-15x}{2}\sin\frac{\frac{\pi}{2}-15x-5x}{2}=0$;

$$\sin\left(\frac{\pi}{4}-5x\right)\sin\left(\frac{\pi}{4}-10x\right)=0;\ \sin\left(5x-\frac{\pi}{4}\right)\sin\left(10x-\frac{\pi}{4}\right)=0;$$

$\left[\begin{array}{l}\sin\left(5x-\frac{\pi}{4}\right)=0\\ \cos\left(10x-\frac{\pi}{4}\right)\end{array}\right.$; $\left[\begin{array}{l}5x-\frac{\pi}{4}=\pi n\\ 10x-\frac{\pi}{4}=\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{10}+\frac{2\pi n}{5}\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.$;

г) $\sin(7\pi+x)=\cos(9\pi+2x)$; $-\sin x=-\cos 2x$; $\cos 2x-\sin x=0$; $\cos 2x-\cos\left(\frac{\pi}{2}-x\right)=0$;

$2\sin\frac{2x+\frac{\pi}{2}-x}{2}\sin\frac{\frac{\pi}{2}-x-2}{2}=0$; $\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}-\frac{3x}{2}\right)=0$; $\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{3x}{2}-\frac{\pi}{4}\right)=0$;

$\left[\begin{array}{l}\sin\left(\frac{x}{2}-\frac{\pi}{4}\right)=0\\ \sin\left(\frac{3x}{2}-\frac{\pi}{4}\right)\end{array}\right.$; $\left[\begin{array}{l}\frac{x}{2}+\frac{\pi}{4}=,\pi n\\ \frac{3x}{2}-\frac{\pi}{4}=\pi k\end{array}\right.$; $\left[\begin{array}{l}x=-\frac{\pi}{2}+2\pi n\\ x=\frac{\pi}{6}+\frac{2\pi k}{3}\end{array}\right.$; $x=\frac{\pi}{6}+\frac{2\pi k}{3}$.

546. а) $1+\cos 6x=2\sin^2 5x$; $1+\cos 6x=1-\cos 10x$; $\cos 6x+\cos 10x=0$;

$2\cos\frac{6x+10x}{2}\cos\frac{10x-6x}{2}=0$; $\cos 8x\cos 2x=0$; $\left[\begin{array}{l}\cos 8x=0\\ \cos 2x=0\end{array}\right.$; $\left[\begin{array}{l}8x=\frac{\pi}{2}+\pi k\\ 2x=\frac{\pi}{2}+\pi n\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{16}+\frac{\pi k}{8}\\ x=\frac{\pi}{4}+\frac{\pi n}{2}\end{array}\right.$;

б) $\cos^2 2x=\cos^2 4x$; $\frac{1}{2}(1+\cos 4x)=\frac{1}{2}(1+\cos 8x)$; $\cos 4x-\cos 8x=0$;

$2\sin\frac{4x+8x}{2}\cos\frac{8x-4x}{2}=0$; $\sin 6x\sin 2x=0$; $\left[\begin{array}{l}\sin 6x=0\\ \sin 2x=0\end{array}\right.$; $\left[\begin{array}{l}6x=\pi n\\ 2x=\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi n}{6}\\ x=\frac{\pi k}{2}\end{array}\right.$; $x=\frac{\pi n}{6}$;

в) $\sin^2\frac{x}{2}=\cos^2\frac{7x}{2}$; $\frac{1}{2}(1-\cos x)=\frac{1}{2}(1+\cos 7x)$; $\cos 7x+\cos x=0$;

$2\cos\frac{7x+x}{2}\cos\frac{7x-x}{2}=0$; $\cos 4x\cos 3x=0$; $\left[\begin{array}{l}\cos 4x=0\\ \cos 3x=0\end{array}\right.$; $\left[\begin{array}{l}4x=\frac{\pi}{2}+\pi n\\ 3x=\frac{\pi}{2}+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{8}+\frac{\pi n}{4}\\ x=\frac{\pi}{6}+\frac{\pi k}{3}\end{array}\right.$;

г) $\sin^2 x+\sin^2 3x=1$; $\frac{1}{2}(1-\cos 2x)=\frac{1}{2}(1-\cos 6x)$; $\cos 2x-\cos 6x=0$;

$2\cos\frac{2x+6x}{2}\cos\frac{6x-2x}{2}=0$; $\cos 4x\cos 2x=0$; $\left[\begin{array}{l}\cos 4x=0\\ \cos 2x=0\end{array}\right.$; $\left[\begin{array}{l}4x=\frac{\pi}{2}+\pi n\\ 2x=\frac{\pi}{2}+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{8}+\frac{\pi n}{4}\\ x=\frac{\pi}{4}+\frac{\pi k}{2}\end{array}\right.$.

547. а) $2\sin 2x+\cos 5x=1$; $1-\cos 2x+5x=1$; $\cos 5x-\cos 2x=0$;

$\left[\begin{array}{l}\sin\frac{7x}{2}=0\\ \sin\frac{3x}{2}=0\end{array}\right.$; $\left[\begin{array}{l}\frac{7x}{2}=\pi n\\ \frac{3x}{2}=\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{2\pi n}{7}\\ x=\frac{2\pi k}{3}\end{array}\right.$;

б) $2\sin^2 3x - 1 = \cos^2 4x - \sin^2 4x$; $-\cos 6x = \cos 8x$; $\cos 6x + \cos 8x = 0$;

$$2\cos\frac{6x+8x}{2}\cos\frac{8x-6}{2}=0;\ \cos 7x\cos x=0;\ \left[\begin{array}{l}\cos 7x=0\\ \cos x=0\end{array}\right.;\ \left[\begin{array}{l}7x=\frac{\pi}{2}+\pi n\\ x=\frac{\pi}{2}+\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{14}+\frac{\pi n}{7}\\ x=\frac{\pi}{6}+\frac{\pi k}{3}\end{array}\right.;\ x=\frac{\pi}{14}+\frac{\pi n}{7}.$$

548. а) $\operatorname{tg} x + \operatorname{tg} 5x = 0$; $\dfrac{\sin(x+5x)}{\cos x\cos 5x} = 0$; $\dfrac{\sin 6x}{\cos x\cos 5x} = 0$;

$$\begin{cases}\sin 6x = 0\\ \cos x \ne 0\\ \cos 5x \ne 0\end{cases};\ \begin{cases}6x = \pi n\\ x \ne \frac{\pi}{2} + \pi k\\ 5x \ne \frac{\pi}{2} + \pi l\end{cases};\ \begin{cases}x = \frac{\pi n}{6}\\ x \ne \frac{\pi}{2} + \pi k\\ x \ne \frac{\pi}{10} + \frac{\pi l}{5}\end{cases};\ \begin{cases}x = \frac{\pi n}{6}\\ n \ne 3 + 6k\\ 5n \ne 3 + 6l\end{cases};$$

б) $\operatorname{tg} 3x = \operatorname{ctg} x$; $\dfrac{\sin 3x}{\cos 3x} = \dfrac{\cos x}{\sin x}$; $\begin{cases}\sin 3x\sin x = \cos 3x\cos x\\ \cos 3x \ne 0\\ \sin x \ne 0\end{cases}$; $\begin{cases}\sin 3x\sin x - \cos 3x\cos x = 0\\ \cos 3x \ne 0\\ \sin x \ne 0\end{cases}$;

$$\begin{cases}\cos(3x+x) = 0\\ \cos 3x \ne 0\\ \sin x \ne 0\end{cases};\ \begin{cases}\cos 4x = 0\\ \cos 3x \ne 0;\\ \sin x \ne 0\end{cases}\ \begin{cases}4x = \frac{\pi}{2} + \pi n\\ 3x \ne \frac{\pi}{2} + \pi k;\\ x \ne \pi l\end{cases}\ \begin{cases}x = \frac{\pi}{8} + \frac{\pi n}{4}\\ x \ne \frac{\pi}{6} + \frac{\pi k}{3};\\ x \ne \pi l\end{cases}\ x = \frac{\pi}{8} + \frac{\pi n}{4};$$

в) $\operatorname{tg} 2x + \operatorname{tg} 4x = 0$; $\dfrac{\sin 2x}{\cos 2x} = \dfrac{\sin 4x}{\cos 4x}$; $\begin{cases}\sin 2x\cos 4x = \sin 4x\cos 2x\\ \cos 2x \ne 0\\ \cos 4x \ne 0\end{cases}$;

$$\begin{cases}\sin(4x-2) = 0\\ \cos 2x \ne 0\\ \cos 4x \ne 0\end{cases};\ \begin{cases}\sin 2x = 0\\ \cos 2x \ne 0;\\ \cos 4x \ne 0\end{cases}\ \begin{cases}2x = \pi n\\ 2x \ne \frac{\pi}{2} + \pi k;\\ 4x \ne \frac{\pi}{2} + \pi l\end{cases}\ \begin{cases}x = \frac{\pi n}{2}\\ x \ne \frac{\pi}{4} + \frac{\pi k}{2};\\ x \ne \frac{\pi}{8} + \frac{\pi l}{4}\end{cases}\ x = \frac{\pi n}{2};$$

г) $\operatorname{ctg}\dfrac{x}{2} + \operatorname{ctg}\dfrac{3x}{2} = 0$; $\dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}} + \dfrac{\cos\frac{3x}{2}}{\sin\frac{3x}{2}} = 0$; $\begin{cases}\cos\frac{x}{2}\sin\frac{3x}{2} + \cos\frac{3x}{2}\sin\frac{x}{2} = 0\\ \sin\frac{x}{2} \ne 0\\ \sin\frac{3x}{2} \ne 0\end{cases}$;

$$\begin{cases}\sin 2x = 0\\ \sin\dfrac{x}{2}\neq 0\\ \sin\dfrac{3x}{2}\neq 0\end{cases};\ \begin{cases}2x=\pi n\\ \dfrac{x}{2}\neq \pi k\\ \dfrac{3x}{2}\neq \pi l\end{cases};\ \begin{cases}x=\dfrac{\pi n}{2}\\ x\neq 2\pi k;\\ x\neq\dfrac{2\pi l}{3}\end{cases}\begin{cases}x=\dfrac{\pi n}{2}\\ n\neq 4k\\ 3n\neq 4l\end{cases}.$$

549. а) $\sin x + \sin 3x + \cos x + \cos 3x = 0$; $2\sin\dfrac{x+3x}{2}\cos\dfrac{3x-x}{2}+2\cos\dfrac{x+3x}{2}\cos\dfrac{3x-x}{2}=0$;

$\sin 2x\cos x + \cos 2x\cos x = 0$; $\cos x(\sin 2x + \cos 2x) = 0$;

$$\sqrt{2}\cos x\sin\left(\frac{\pi}{4}+2x\right)=0;\ \left[\begin{array}{l}\cos x = 0\\ \sin\left(\dfrac{\pi}{4}+2x\right)=0\end{array}\right.;\ \left[\begin{array}{l}x=\dfrac{\pi}{2}+\pi n\\ \dfrac{\pi}{4}+2x=\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\dfrac{\pi}{2}+\pi n\\ x=-\dfrac{\pi}{8}+\dfrac{\pi k}{2}\end{array}\right.;$$

б) $\sin 5x + \sin x + 2\sin^2 x = 1$; $\sin 5x + \sin x - \cos 2x = 0$;

$2\sin\dfrac{5x+x}{2}\cos\dfrac{5x-x}{2}-\cos 2x = 0$; $2\sin 3x\cos 2x - \cos 2x = 0$;

$$\cos 2x(2\sin 3x - 1) = 0;\ \left[\begin{array}{l}\cos 2x = 0\\ \sin 3x=\dfrac{1}{2}\end{array}\right.;\ \left[\begin{array}{l}2x=\dfrac{\pi}{2}+\pi n\\ 3x=(-1)^k\dfrac{\pi}{6}+\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\dfrac{\pi}{4}+\dfrac{\pi n}{2}\\ x=(-1)^k\dfrac{\pi}{18}+\dfrac{\pi k}{3}\end{array}\right..$$

550. а) $\sin 2x + \sin 6x = \cos 2x$; $2\sin\dfrac{2x+6x}{2}\cos\dfrac{6x-2x}{2}-\cos 2x = 0$;

$2\sin 4x\cos 2x - \cos 2x = 0$; $\cos 2x(2\sin 4x - 1) = 0$;

$$\left[\begin{array}{l}\cos 2x = 0\\ \sin 4x=\dfrac{1}{2}\end{array}\right.;\ \left[\begin{array}{l}2x=\dfrac{\pi}{2}+\pi n\\ 4x=(-1)^k\dfrac{\pi}{6}+\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\dfrac{\pi}{4}+\dfrac{\pi n}{2}\\ x=\dfrac{\pi}{24}+2\pi k\\ x=\dfrac{5\pi}{24}+2\pi k\end{array}\right..$$

На отрезке $\left[0,\dfrac{\pi}{2}\right]$ данное уравнение имеет три корня: $x=\dfrac{\pi}{4}, x=\dfrac{\pi}{24}, x=\dfrac{5\pi}{24}$;

б) $2\cos^2 x - 1 = \sin 3x$; $\cos 2x = \sin 3x$; $\cos 2x - \cos\left(\dfrac{\pi}{2}-3x\right)=0$;

$$2\sin\frac{2x+\frac{\pi}{2}-3x}{2}\sin\frac{\frac{\pi}{2}-3x-2x}{2}=0;\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)=0;$$

$$\left[\begin{array}{l}\sin\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)=0\\ \cos\left(\dfrac{5x}{2}-\dfrac{\pi}{4}\right)=0\end{array}\right.;\ \left[\begin{array}{l}\dfrac{x}{2}-\dfrac{\pi}{4}=\pi n\\ \dfrac{5x}{2}-\dfrac{\pi}{4}=\pi k\end{array}\right.\left[\begin{array}{l}x=\dfrac{\pi}{2}+2\pi n\\ x=\dfrac{\pi}{10}+\dfrac{2\pi k}{5}\end{array}\right..$$

На отрезке $\left[0,\dfrac{\pi}{2}\right]$ данное уравнение имеет два корня: $x=\dfrac{\pi}{10}, x=\dfrac{\pi}{2}$.

551. а) $\cos 6x+\cos 8x=\cos 10x+\cos 12x$; $2\cos\frac{6x+8x}{2}\cos\frac{8x-6x}{2}=2\cos\frac{10x+12x}{2}\cos\frac{12x-10x}{2}$;

$\cos 7x\cos x=\cos 11x\cos x$; $\cos x\,(\cos 7x-\cos 11x)=0$;

$2\cos x\sin\frac{7x+11x}{2}\sin\frac{11x-7x}{2}=0$; $\cos x\sin 9x\sin 2x=0$;

$\left[\begin{array}{l}\cos 8x=0\\ \cos 2x=0\end{array}\right.$; $\left[\begin{array}{l}8x=\frac{\pi}{2}+\pi k\\ 2x=\frac{\pi}{2}+\pi n\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{16}+\frac{\pi k}{8}\\ x=\frac{\pi}{4}+\frac{\pi n}{2}\end{array}\right.$.

На отрезке $[0,\pi]$ данное уравнение имеет девять корней: $x=\frac{\pi k}{9}(k=1,...,8)$ и $x=\frac{\pi}{2}$;

б) $\sin 2x+5\sin 4x+\sin 6x=0$; $(\sin 2x+\sin 6x)+5\sin 4x=0$;
$2\sin 4x\cos 2x+5\sin 4x=0$; $\sin 4x\,(2\cos 2x+5)=0$; $\sin 4x=0$ (т.к. $2\cos 2x+5>0$ при любом x); $4x=\pi$; $x=\frac{\pi n}{4}$. На отрезке $[0,\pi]$ данное уравнение имеет три корня: $x=\frac{\pi n}{4}\ (n=1,2,3)$.

552. Числа a, b, c образуют арифметическую прогрессию, если $b-a=c-d$;
а) $\cos 2x-\cos 7x=\cos 11x-\cos 2x$; $2\cos 2x-(\cos 7x+\cos 11x)=0$;
$2\cos 2x-2\cos 9x\cos 2x=0$; $\cos 2x\,(1-\cos 9x)=0$;

$\left[\begin{array}{l}\cos 2x=0\\ \cos 9x=1\end{array}\right.$; $\left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 9x=2\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\frac{2\pi k}{9}\end{array}\right.$;

б) $\cos x-\sin 3x=\sin 5x-\cos x$; $2\cos x-(\sin 3x+\sin 5x)=0$;
$2\cos x-2\sin 4x\cos x=0$; $\cos x\,(1-\sin 4x)=0$;

$\left[\begin{array}{l}\cos x=0\\ \sin 4x=1\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ 4x=\frac{\pi}{2}+2\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ x=\frac{\pi}{8}+\frac{\pi k}{2}\end{array}\right.$.

§27. Преобразование произведений тригонометрических функций в сумму

553. а) $\sin 23°\sin 32°=\frac{1}{2}(\cos(32°-23°)-\cos(32°+23°))=\frac{1}{2}(\cos 9°-\cos 55°)$;

б) $\cos\frac{\pi}{12}\cos\frac{\pi}{8}=\frac{1}{2}\left(\cos\left(\frac{\pi}{8}-\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{8}+\frac{\pi}{12}\right)\right)=\frac{1}{2}\left(\cos\frac{\pi}{24}+\cos\frac{5\pi}{24}\right)$;

в) $\sin 14°\sin 16°=\frac{1}{2}(\cos(16°-14°)-\cos(16°+14°)=\frac{1}{2}(\cos 2°-\cos 30°)=$
$=\frac{1}{2}\left(\cos 2°-\frac{\sqrt{3}}{2}\right)$;

г) $2\sin\frac{\pi}{8}\cos\frac{\pi}{5}=\sin\left(\frac{\pi}{8}+\frac{\pi}{5}\right)+\sin\left(\frac{\pi}{8}-\frac{\pi}{5}\right)=\sin\frac{13\pi}{40}-\sin\frac{3\pi}{40}$.

554. а) $\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta=\cos^2\beta-\cos^2\alpha$;
б) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha=\cos^2\alpha-\sin^2\beta$;
в) $\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)=\cos^2\frac{\beta}{2}-\sin^2\frac{\alpha}{2}=\cos^2\frac{\alpha}{2}-\sin^2\frac{\beta}{2}$;
г) $\cos\left(\alpha+\frac{\pi}{4}\right)\cos\left(\alpha-\frac{\pi}{4}\right)=\cos^2\frac{\pi}{4}-\sin^2\alpha=\frac{1}{2}-\sin^2\alpha$.

555. а) $\cos\alpha\sin(\alpha+\beta)=\frac{1}{2}(\sin(2\alpha+\beta)+\sin(\alpha+\beta-\alpha))=\frac{1}{2}(\sin(2\alpha+\beta)+\sin\beta))$;

б) $\sin(60°+\alpha)\sin(60°-\alpha)=\sin^2 60°-\sin^2\alpha=\frac{3}{2}\sin^2\alpha$;

в) $\sin\beta\cos(\alpha+\beta)=\frac{1}{2}(\sin(\alpha+2\beta)+\sin(\beta-\alpha-\beta)=\frac{1}{2}(\sin(\alpha+2\beta)-\sin\alpha))$;

г) $\cos\left(\alpha+\frac{\pi}{4}\right)\cos\left(\alpha-\frac{\pi}{4}\right)=\cos^2\frac{\pi}{4}-\sin^2\alpha=\frac{1}{2}-\sin^2\alpha$.

556. а) $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)-0{,}25=0;\ \frac{1}{2}\left(\cos\left(x+\frac{\pi}{3}-x+\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}+x-\frac{\pi}{3}\right)\right)=\frac{1}{4}$;

$\frac{1}{2}\left(\cos^2\frac{\pi}{3}+\cos 2x\right)=\frac{1}{4};\ \frac{1}{2}\left(-\frac{1}{2}+\cos 2x\right)=\frac{1}{4}$; $\cos 2x=1$; $2x=2\pi n$; $x=\pi n$;

б) $\sin\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{6}\right)=1;\ \frac{1}{2}\left(\sin\left(x+\frac{\pi}{3}-x-\frac{\pi}{6}\right)+\sin\left(x+\frac{\pi}{3}-x+\frac{\pi}{6}\right)\right)=1$;

$\frac{1}{2}\left(\sin\left(2x+\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right)=1;\ \frac{1}{2}\sin\left(2x+\frac{\pi}{6}\right)+\frac{1}{2}=1;\ \sin\left(2x+\frac{\pi}{6}\right)=1$;

$2x+\frac{\pi}{6}=\frac{\pi}{2}+2\pi n;\ x=\frac{\pi}{6}+\pi n$.

557. а) $2\sin x\cos 3x+\sin 4x=0$; $\sin(x+3x)+\sin(x-3x)+\sin 4x=0$;
$2\sin 4x-\sin 2x=0$; $4\sin 2x\cos 2x-\sin 2x=0$; $\sin 2x(4\cos 2x-1)=0$;

$$\left[\begin{array}{l}\sin 2x=0\\ \cos 2x=\frac{1}{4}\end{array}\right.;\ \left[\begin{array}{l}2x=\pi n\\ 2x=\pm\arccos\frac{1}{4}+2\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi n}{2}\\ x=\pm\frac{1}{2}\arccos\frac{1}{4}+\pi k\end{array}\right..$$

б) $\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2};\ \frac{1}{2}\left(\cos\left(\frac{x}{2}-\frac{3x}{2}\right)-\cos\left(\frac{x}{2}+\frac{3x}{2}\right)\right)=\frac{1}{2}$; $\cos x-\cos 2x=1$;

$2\cos^2 x-\cos x=0$; $\cos x(2\cos x-1)=0$; $\left[\begin{array}{l}\cos x=0\\ \cos x=\frac{1}{2}\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ x=\pm\frac{\pi}{3}+2\pi k\end{array}\right..$

558. а) $\sin 10° \cos 8° \cos 6° = \frac{1}{2}\sin 10° (\cos (8° - 6°) + \cos (8° + 6°)) =$

$= \frac{1}{2}\sin 10° \cos 2° + \frac{1}{2}\sin 10° = \frac{1}{4}(\sin 12° + \sin 8° + \sin 24° - \sin 4°)$;

б) $4 \sin 25° \cos 15° \sin 5° = 2 \sin 25° (\sin 20° + \sin (-10°)) = 2 (\sin 25° \sin 20° -$

$- \sin 25° \sin 10°) = \cos 5° - \cos 45° - \cos 15° + \cos 35° = \cos 5° - \cos 15° + \cos 35° - \frac{1}{\sqrt{2}}$.

559. а) $2 \sin t \sin 2t + \cos 3t = \cos (2t - 1) - \cos (2t + 1) + \cos 3t = \cos t$.

б) $\sin\alpha - 2\sin\left(\frac{\alpha}{2} - 15°\right)\cos\left(\frac{\alpha}{2} + 15°\right) = \sin\alpha - \sin\left(\frac{\alpha}{2} - 15° + \frac{\alpha}{2} - 15°\right) -$

$-\sin\left(\frac{\alpha}{2} - 15° - \frac{\alpha}{2} - 15°\right) = \sin\alpha - \sin\alpha - \sin(-30°) = \sin 30° = \frac{1}{2}$.

560. а) $\cos^2 3° + \cos^2 1° - \cos 4° \cos 2° = \cos^2 3° + \cos^2 1° - \cos (3° + 1°) \cos (3° - 1°) =$
$= \cos^2 3° + \cos^2 1° - \cos^2 1° + \sin^2 3° = \cos^2 3° + \sin^2 3° = 1$;

б) $\sin^2 10° + \cos 50° \cos 70° = \sin^2 10° = \cos (60° - 10°) \cos (60° + 10°) =$

$= \sin^2 10° + \cos^2 60° - \sin^2 10° = \cos^2 60° = \frac{1}{4}$.

561. а) $\frac{1}{2\sin 10°} - 2\sin 70° = \frac{1 - 4\sin 10° \sin 70°}{2\sin 10°} = \frac{1 - 2\cos(70° - 10°) + 2\cos(70° + 10°)}{2\sin 10°} =$

$= \frac{1 - 2\cos 60° + 2\cos 80°}{2\sin 10°} = \frac{1 - 2\cdot\frac{1}{2} + 2\sin 10°}{2\sin 10°} = 1$;

б) $\frac{\operatorname{tg} 60°}{\sin 40°} + 4\cos 100° = \frac{\operatorname{tg} 60° + 4\sin 40° \cos 100°}{\sin 40°} = \frac{\operatorname{tg} 60° + 2\sin 140° + 2\sin(-60°)}{\sin 40°} =$

$= \frac{\sqrt{3} + 2\sin 40° - 2\cdot\frac{\sqrt{3}}{2}}{\sin 40°} = \frac{2\sin 40°}{\sin 40°} = 2$.

562. а) $\sin 3x \cos x = \sin\frac{5x}{2}\cos\frac{3x}{2}$.

$\frac{1}{2}(\sin(3x + x) + \sin(3x - x)) = \frac{1}{2}\sin\left(\sin\left(\frac{5x}{2} + \frac{3x}{2}\right) + \sin\left(\frac{5x}{2} + \frac{3x}{2}\right)\right)$;

$\sin 4x + \sin 2x = \sin 4x + \sin x$; $\sin 2x - \sin x = 0$; $2\sin\frac{2x - x}{2}\cos\frac{2x + x}{2} = 0$;

$\sin\frac{x}{2}\cos\frac{3x}{2} = 0$; $\left[\begin{array}{l} \sin\frac{x}{2} = 0 \\ \cos\frac{3x}{2} \end{array}\right.$; $\left[\begin{array}{l} \frac{x}{2} = \pi n \\ \frac{3x}{2} = \frac{\pi}{2} + \pi k \end{array}\right.$; $\left[\begin{array}{l} x = 2\pi n \\ x = \frac{\pi}{3} + \frac{2\pi k}{3} \end{array}\right.$

б) $2\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) + \sin^2 x = 0$.

$2\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)+\sin^2 x=0$; $1-\sin^2 x=0$; $-\frac{1}{2}(1-\cos 2x)+1=0$;

$\cos 2x=-1$; $2x=\pi+2\pi n$; $x=\frac{\pi}{2}+\pi n$.

в) $\sin 2x\cos x=\sin x\cos 2x$.

$\frac{1}{2}(\sin 3x+\sin x)=\frac{1}{2}(\sin 3x-\sin x)$; $\sin x=0$; $x=\pi n$.

г) $\cos 2x\cos x=\cos 2{,}5x\cos 0{,}5x$.

$\frac{1}{2}(\cos x\ 9\cos 3x)=\frac{1}{2}(\cos 2x+\cos 3x)$; $\cos x=\cos 2x$; $\cos x-\cos 2x=0$;

$$2\sin\frac{x+2x}{2}\sin\frac{2x-x}{2}=0;\ \sin\frac{3x}{2}\sin\frac{x}{2}=0;\left[\begin{array}{l}\sin\frac{3x}{2}=0\\ \sin\frac{x}{2}\end{array}\right.;\left[\begin{array}{l}\frac{3x}{2}=\pi n\\ \frac{x}{2}=\pi k\end{array}\right.;\left[\begin{array}{l}x=\frac{2\pi n}{3}\\ x=2\pi k\end{array}\right..$$

563. **а)** $\sin x\sin 3x=0{,}5$; $\frac{1}{2}(\cos 2x-\cos 4x)=\frac{1}{2}$; $\cos 2x=1+\cos 4x$; $\cos 2x=2\cos^2 2x$;

$$\cos 2x\,(2\cos 2x-1)=0;\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=\frac{1}{2}\end{array}\right.;\left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.;\left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{6}+\pi k\end{array}\right..$$

б) $\cos x\cos 3x+0{,}5$; $\frac{1}{2}(\cos 2x+\cos 4x)=\frac{1}{2}$; $\cos 2x+(1+\cos 4x)=0$; $\cos 2x+$

$$2\cos^2 2x=0;\ \cos 2x\,(2\cos 2x+1)=0;\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=-\frac{1}{2}\end{array}\right.;\left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.;\left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{3}+\pi k\end{array}\right..$$

564. **а)** $f(x)=\sin\left(x+\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{24}\right)=\frac{1}{2}\left(\sin\left(x+\frac{\pi}{8}+x-\frac{\pi}{24}\right)+\sin\left(x+\frac{\pi}{8}+x+\frac{\pi}{24}\right)\right)=$

$=\frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{2}\sin\frac{\pi}{6}=\frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{4}$.

Поскольку наибольшее и наименьшее значения функции $y=\frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)$ равны 1 и -1 соответственно, то наибольшее и наименьшее значения функции $f(x)$ равны $\frac{3}{4}$ и $-\frac{1}{4}$ соответственно.

б) $f(x)=\sin\left(x-\frac{\pi}{3}\right)\cos\left(x+\frac{\pi}{3}\right)=\frac{1}{2}\left(\sin\left(x+\frac{\pi}{8}+x-\frac{\pi}{24}\right)+\sin\left(x+\frac{\pi}{8}-x+\frac{\pi}{24}\right)\right)=$

$=\frac{1}{2}\left(\cos\frac{2\pi}{3}-\cos 2x\right)=-\frac{1}{4}-\frac{1}{2}\cos 2x$.

Поскольку наибольшее и наименьшее значения функции $y=\cos 2x$ равны 1 и

-1 соответственно, то наибольшее и наименьшее значения функции $f(x)$ равны $\frac{1}{4}$ и $-\frac{3}{4}$ соответственно.

565. $\cos^2(45° - \alpha) - \cos^2(60° + \alpha) - \cos 75° \sin(75° - 2\alpha) = \cos^2(45° - \alpha) -$

$- \cos^2(60° + \alpha) - \frac{1}{2}(\sin(75° - 2\alpha + 75°) + \sin(75° - 2\alpha - 75°)) =$

$= \frac{1}{2}(1 + \cos(90° - 2\alpha)) - \frac{1}{2}(1 + \cos(120° + 2\alpha)) - \sin(150° - 2\alpha) + \sin 2\alpha) =$

$= \sin 2\alpha - \frac{1}{2}(\cos 120° \cos 2\alpha - \sin 120° \sin 2\alpha) - \frac{1}{2}(\sin 150° \cos 2\alpha - \sin 2\alpha \cos 150°) =$

$= \sin 2\alpha - \frac{1}{2}(-\frac{1}{2}\cos 2\alpha - \frac{\sqrt{3}}{2}\sin 2\alpha + \frac{1}{2}\cos 2\alpha + \frac{\sqrt{3}}{2}\sin 2\alpha) = \sin 2\alpha.$

566. Числа a, b, c образуют геометрическую прогрессию, если $a \neq 0, b \neq 0, c \neq 0$ и $\frac{b}{a} = \frac{c}{b}$.

а) $\begin{cases} \frac{\cos 4x}{\cos 6x} = \frac{\cos 2x}{\cos 4x} \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}$; $\begin{cases} \cos^2 4x = \cos 2x \cos 6x \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}$; $\begin{cases} \frac{1}{2}(1 + \cos 8x) = \frac{1}{2}(\cos 8x + \cos 4x) \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}$;

$\begin{cases} \cos 4x = 1 \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}$; $\begin{cases} 4x = 2\pi n \\ \cos 2x \neq 0; \\ \cos 6x \neq 0 \end{cases}$ $\begin{cases} x = \frac{\pi n}{2} \\ 2x \neq \frac{\pi}{2} + \pi k; \\ 6x \neq \frac{\pi}{2} + \pi \ell \end{cases}$ $\begin{cases} x = \frac{\pi n}{2} \\ x \neq \frac{\pi}{4} + \frac{\pi k}{2}; \\ x \neq \frac{\pi}{12} + \frac{\pi \ell}{6} \end{cases}$ $x = \frac{\pi n}{2}$.

б) $\begin{cases} \frac{\sin 3x}{\sin 2x} = \frac{\sin 4x}{\sin 3x} \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}$; $\begin{cases} \sin^2 3x = \sin 2x \sin 4x \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}$; $\begin{cases} \frac{1}{2}(1 - \cos 6x) = \frac{1}{2}(\cos 2x - \cos 6x) \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}$;

$\begin{cases} \cos 2x = 1 \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}$; $\begin{cases} 2x = 2\pi n \\ 2x \neq \pi k \\ 4x \neq \pi \ell \\ 6x \neq \pi m \end{cases}$ $\Rightarrow a, b, c$ не образуют геометрическую прогрессию.

§ 28. Преобразование выражения $A \sin x + B \cos x$ к виду $C \sin(x + t)$

567. а) $\sqrt{3}\sin x + \cos x = 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) = 2\left(\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right) = 2\sin\left(x + \frac{\pi}{6}\right)$;

б) $\sin x+\sqrt{3}\cos x=2\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)=2\left(\cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}\right)=2\cos\left(x-\frac{\pi}{6}\right)$;

в) $\sin x-\cos x=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)=\sqrt{2}\left(\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}\right)=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$;

г) $2\sin x-\sqrt{12}\cos x=4\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)=4\left(\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}\right)=4\sin\left(x-\frac{\pi}{3}\right)$.

568. а) $3\sin x+4\cos x=5\left(\frac{3}{5}\sin x+\frac{4}{5}\cos x\right)=$

$=5\left(\sin x\cos\left(\arcsin\frac{4}{5}\right)+\cos x\sin\left(\arcsin\frac{4}{5}\right)\right)=5\sin\left(x+\arcsin\frac{4}{5}\right)$;

б) $5\cos x-12\sin x=13\left(\frac{5}{13}\cos x-\frac{12}{13}\sin x\right)=$

$=13\left(\cos x\cos\left(\arcsin\frac{12}{13}\right)-\sin x\sin\left(\arcsin\frac{12}{13}\right)\right)=13\cos\left(x+\arcsin\frac{12}{13}\right)$;

в) $7\sin x-24\cos x=25\left(\frac{7}{25}\sin x-\frac{24}{25}\cos x\right)=$

$=25\left(\sin x\cos\left(\arcsin\frac{24}{25}\right)-\cos x\sin\left(\arcsin\frac{24}{25}\right)\right)=25\sin\left(x-\arcsin\frac{24}{25}\right)$;

г) $8\cos x+15\sin x=17\left(\frac{8}{17}\cos x+\frac{15}{17}\sin x\right)=$

$=17\left(\cos x\cos\left(\arcsin\frac{15}{17}\right)+\sin x\sin\left(\arcsin\frac{15}{17}\right)\right)=17\cos\left(x-\arcsin\frac{15}{17}\right)$.

569. а) $\sqrt{3}\sin x+\cos x=1.$ $2\left(\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right)=1$; $\sin x\sin\frac{\pi}{3}+\cos x\cos\frac{\pi}{3}=\frac{1}{2}$;

$\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$; $\left[\begin{array}{l}x-\frac{\pi}{3}=\frac{\pi}{3}+2\pi k\\ x-\frac{\pi}{3}=-\frac{\pi}{3}+2\pi n\end{array}\right.$; $\left[\begin{array}{l}x=\frac{2\pi}{3}+2\pi k\\ x=2\pi n\end{array}\right.$.

б) $\sin x+\cos x=\sqrt{2}.\frac{1}{\sqrt{2}\sin x}+\frac{1}{\sqrt{2}}\cos x=1$; $\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}=1$;

$\sin\left(x+\frac{\pi}{4}\right)=1$; $x+\frac{\pi}{4}=\frac{\pi}{2}+2\pi n$; $x=\frac{\pi}{4}+2\pi n$.

в) $\sin x-\sqrt{3}\cos x=\sqrt{3}.$ $\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{3}}{2}$; $\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$;

$\sin\left(x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$; $\left[\begin{array}{l}x-\frac{\pi}{3}=\frac{\pi}{3}+2\pi n\\ x-\frac{\pi}{3}=\frac{2\pi}{3}+2\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{2\pi}{3}+2\pi n\\ x=\pi+2\pi k\end{array}\right.$.

г) $\sin x-\cos x=1$. $\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$; $\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$;

$$\sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}};\ \left[\begin{array}{l}x-\frac{\pi}{4}=\frac{\pi}{4}+2\pi n\\ x-\frac{\pi}{4}=\frac{3\pi}{4}+2\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{2}+2\pi n\\ x=\pi+2\pi k\end{array}\right..$$

570. **а)** $y=\sqrt{3}\sin x+\cos x=2\left(\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right)=2\left(\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}\right)=2\sin\left(x+\frac{\pi}{6}\right)$.

Наибольшее значение 2, наименьшее –2.

б) $y=\sin x-\sqrt{3}\cos x=2\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)=2\left(\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}\right)=2\sin\left(x-\frac{\pi}{3}\right)$.

Наибольшее значение 2, наименьшее –2.

в) $y=\sin x-\cos x=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$.

Наибольшее значение $\sqrt{2}$, наименьшее $-\sqrt{2}$.

г) $y=\sqrt{6}\sin x-\sqrt{2}\cos x=2\sqrt{2}\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)=2\sqrt{2}\left(\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}\right)=$

$=2\sqrt{2}\sin\left(x-\frac{\pi}{6}\right)$. Наибольшее значение $2\sqrt{2}$, наименьшее $-2\sqrt{2}$.

571. **а)** $y=3\sin 2x-4\cos 2x=5\left(\frac{3}{5}\sin 2x-\frac{4}{5}\cos 2x\right)=$

$=5\left(\sin 2x\cos\left(\arcsin\frac{4}{5}\right)+\cos 2x\sin\left(\arcsin\frac{4}{5}\right)\right)=5\sin\left(2x-\arcsin\frac{4}{5}\right)$.

Область значений [–5; 5].

б) $y=5\cos 3x+12\sin 3x=13\left(\frac{5}{13}\cos 3x+\frac{12}{13}\sin 3x\right)=$

$=13\left(\cos 3x\cos\left(\arcsin\frac{12}{13}\right)+\sin 3x\sin\left(\arcsin\frac{12}{13}\right)\right)=13\cos\left(3x-\arcsin\frac{12}{13}\right)$.

Область значений [–13; 13].

в) $y=7\sin\frac{x}{2}+24\cos\frac{x}{2}=25\left(\frac{7}{25}\sin\frac{x}{2}+\frac{24}{25}\cos\frac{x}{2}\right)=$

$=25\left(\sin\frac{x}{2}\sin\left(\arcsin\frac{7}{25}\right)+\cos\frac{x}{2}\cos\left(\arcsin\frac{7}{25}\right)\right)=25\cos\left(\frac{x}{2}-\arcsin\frac{7}{25}\right)$.

Область значений [–25; 25].

г) $y=8\cos\frac{x}{3}-15\sin\frac{x}{3}=17\left(\frac{8}{17}\cos\frac{x}{3}-\frac{15}{17}\sin\frac{x}{3}\right)=$

$=17\left(\cos\frac{x}{3}\cos\left(\arcsin\frac{15}{17}\right)-\sin\frac{x}{3}\sin\left(\arcsin\frac{15}{17}\right)\right)=17\cos\left(\frac{x}{3}+\arcsin\frac{15}{17}\right)$.

Область значений [–17; 17].

572. а) $\sin 5x + \cos 5x = 1{,}5.\ \sqrt{2}\sin\left(5x+\frac{\pi}{4}\right)=\frac{3}{2};\ \sin\left(5x+\frac{\pi}{4}\right)=\frac{3}{2\sqrt{2}}=\sqrt{\frac{9}{8}}$.

Это равенство не выполняется ни при одном значении x, т.к.

$\sin\left(5x+\frac{\pi}{4}\right)\le 1$ при всех x, а $\sqrt{\frac{9}{8}}>1$.

б) $3\sin 2x - 4\cos 2x = \sqrt{26}.\ 5\left(\frac{3}{5}\sin 2x - \frac{4}{5}\cos 2x\right)=\sqrt{26};$

$5\left(\sin 2x\cos\left(\arcsin\frac{4}{5}\right)-\cos 2x\sin\left(\arcsin\frac{4}{5}\right)\right)=\sqrt{26};\ 5\sin\left(2x-\arcsin\frac{4}{5}\right)=\sqrt{26};$

$\sin\left(2x-\arcsin\frac{4}{5}\right)=\frac{\sqrt{26}}{5}$. Это равенство не выполняется ни при одном значении x, т.к. $\sin\left(2x-\arcsin\frac{4}{5}\right)\le 1$ при всех x, а $\frac{\sqrt{26}}{5}>1$.

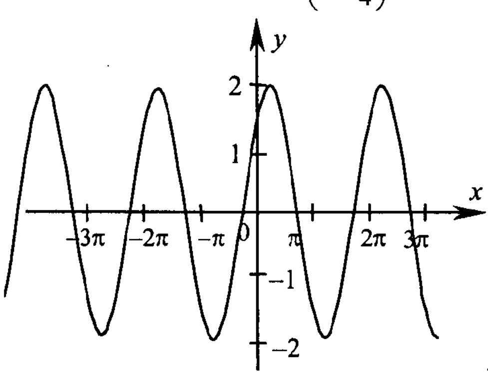
в) $\sin 7x - \sqrt{3}\cos 7x = \frac{\pi}{2}.\ \frac{1}{2}\sin 7x - \frac{\sqrt{3}}{2}\cos 7x = \frac{\pi}{4};\ \sin 7x\cos\frac{\pi}{3} - \cos 7x\sin\frac{\pi}{3}=\frac{\pi}{4};$

$\sin\left(7x-\frac{\pi}{3}\right)=\frac{\pi}{4}$. Значения x, при которых выполняется это равенство, существуют, т.к. область значений функции $y=\sin\left(7x-\frac{\pi}{3}\right)$ – это отрезок $[-1;\ 1]$ и $-1<\frac{\pi}{4}<1$.

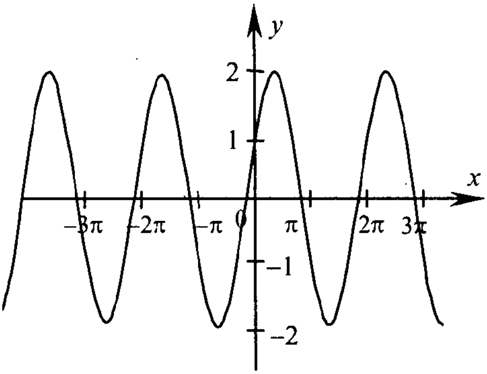
г) $5\sin x + 12\cos x = \sqrt{170}.\ \frac{5}{13}\sin x + \frac{12}{13}\cos x = \frac{\sqrt{170}}{13};\ \sin x\cos\left(\arcsin\frac{12}{13}\right)+$

$+\cos x\sin\left(\arcsin\frac{12}{13}\right)=\frac{\sqrt{170}}{13};\ \sin\left(x+\arcsin\frac{12}{13}\right)=\frac{\sqrt{170}}{13}$. Это равенство не выполняется ни при одном значении x, т.к. $\sin\left(x+\arcsin\frac{12}{13}\right)\le 1$ при всех x, а $\frac{\sqrt{170}}{13}>1$.

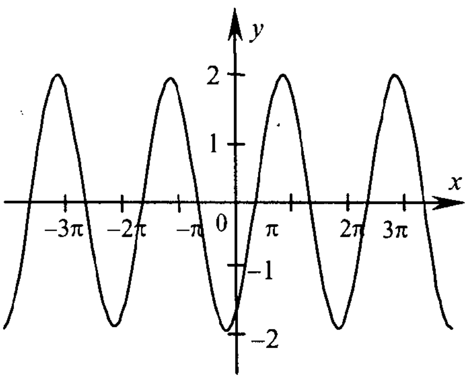
573. а) $y=\sqrt{2}(\sin x+\cos x)=2\sin\left(x+\frac{\pi}{4}\right)$.



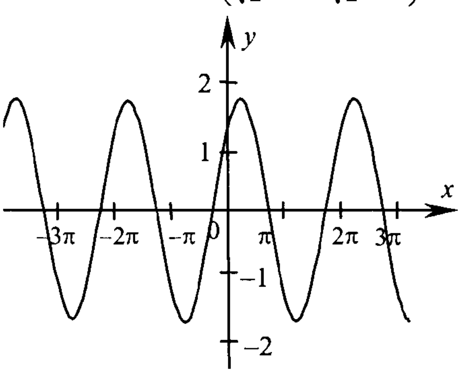
б) $y=\sqrt{3}\sin x+\cos x=2\left(\sin x\cdot\frac{\sqrt{3}}{2}+\cos x\cdot\frac{1}{2}\right)=2\left(\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}\right)=2\sin\left(x+\frac{\pi}{6}\right)$.



в) $y=\sin x-\sqrt{3}\cos x=2\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)=2\left(\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}\right)=2\sin\left(x-\frac{\pi}{3}\right)$.



г) $y=\sin x-\cos x=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)=\sqrt{2}\left(\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}\right)=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$.



574. а) $\cos 2x+\sqrt{3}\sin 2x=\sqrt{2}$; $\frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x=\frac{1}{\sqrt{2}}$; $\cos 2x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\sin 2x=\frac{1}{\sqrt{2}}$;

$$\cos\left(2x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}};\ \left[\begin{array}{l}2x-\frac{\pi}{3}=\frac{\pi}{4}+2\pi n\\ 2x-\frac{\pi}{3}=-\frac{\pi}{4}+2\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{7\pi}{24}+\pi n\\ x=\frac{\pi}{24}+\pi k\end{array}\right..$$

б) $\sin 5x-\cos 5x=\frac{\sqrt{6}}{2}$; $\frac{1}{\sqrt{2}}\sin 5x-\frac{1}{\sqrt{2}}\cos 5x=\frac{\sqrt{3}}{2}$; $\sin 5x\cos\frac{\pi}{4}-\cos 5x\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}$;

$$\sin\left(5x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2};\ \left[\begin{array}{l}5x-\frac{\pi}{4}=\frac{\pi}{3}+2\pi n\\ 5x-\frac{\pi}{4}=\frac{2\pi}{3}+2\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{7\pi}{60}+\frac{2\pi n}{5}\\ x=\frac{3\pi}{20}+\frac{2\pi k}{5}\end{array}\right..$$

в) $\cos\frac{x}{2}-\sqrt{3}\sin\frac{x}{2}+1=0$; $\frac{1}{2}\cos\frac{x}{2}-\frac{\sqrt{3}}{2}\sin\frac{x}{2}=-\frac{1}{2}$; $\sin\frac{x}{2}\cos\frac{\pi}{6}-\cos\frac{x}{2}\sin\frac{\pi}{6}=\frac{1}{2}$;

$$\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)=\frac{1}{2};\ \left[\begin{array}{l}\frac{x}{2}-\frac{\pi}{6}=\frac{\pi}{6}+2\pi n\\ \frac{x}{2}-\frac{\pi}{6}=\frac{5\pi}{6}+2\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{2\pi}{3}+4\pi n\\ x=2\pi+2\pi k\end{array}\right..$$

г) $\sin\frac{x}{3}+\cos\frac{x}{3}=1$; $\frac{1}{\sqrt{2}}\sin\frac{x}{3}+\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$; $\sin\frac{x}{3}\cos\frac{\pi}{4}+\cos\frac{x}{3}\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$;

$$\sin\left(\frac{x}{3}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}};\ \left[\begin{array}{l}\frac{x}{3}+\frac{\pi}{4}=\frac{\pi}{4}+2\pi k\\ \frac{x}{3}+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi n\end{array}\right.;\ \left[\begin{array}{l}x=6\pi k\\ x=\frac{3\pi}{2}+6\pi n\end{array}\right..$$

575. а) $4\sin x-3\cos x=5$; $\frac{4}{5}\sin x-\frac{3}{5}\cos x=1$; $\sin x\cos\left(\arcsin\frac{3}{5}\right)-\cos x\sin\left(\arcsin\frac{3}{5}\right)=1$;

$\sin\left(x-\arcsin\frac{3}{5}\right)=1$; $x-\arcsin\frac{3}{5}=\frac{\pi}{2}+2\pi n$; $x=\frac{\pi}{2}+\arcsin\frac{3}{5}+2\pi n$.

б) $3\sin 2x+4\cos 2x=2{,}5$; $\frac{3}{5}\sin 2x+\frac{4}{5}\cos x=\frac{1}{2}$; $\sin 2x\cos\left(\arcsin\frac{4}{5}\right)+$

$+\cos 2x\sin\left(\arcsin\frac{4}{5}\right)=\frac{1}{2}$; $\sin\left(2x+\arcsin\frac{4}{5}\right)=\frac{1}{2}$; $2x+\arcsin\frac{4}{5}+(-1)^n\frac{\pi}{6}+\pi n$;

$x=(-1)^n\frac{\pi}{12}-\frac{1}{2}\arcsin\frac{4}{5}+\frac{\pi n}{2}$.

в) $12\sin x+5\cos x+13=0$; $\frac{12}{13}\sin x+\frac{5}{13}\cos x=-1$; $\sin x\cos\left(\arcsin\frac{5}{13}\right)+$

$+\cos x\sin\left(\arcsin\frac{5}{13}\right)=-1$; $\sin\left(x+\arcsin\frac{5}{13}\right)=-1$; $x+\arcsin\frac{5}{13}=-\frac{\pi}{2}+2\pi n$;

$x=-\frac{\pi}{2}-\arcsin\frac{5}{13}+2\pi n$.

г) $5\cos\frac{x}{2}-12\sin\frac{x}{2}=6{,}5$; $\frac{5}{13}\cos\frac{x}{2}-\frac{12}{13}\sin\frac{x}{2}=\frac{1}{2}$; $\cos\frac{x}{2}\cos\left(\arcsin\frac{12}{13}\right)-\sin\left(\arcsin\frac{12}{13}\right)=\frac{1}{2}$;

$\cos\left(\frac{x}{2}+\arcsin\frac{12}{13}\right)=\frac{1}{2}$; $\frac{x}{2}+\arcsin\frac{12}{13}=\pm\frac{\pi}{3}+2\pi n$; $x=\pm\frac{2\pi}{3}-2\arcsin\frac{12}{13}+4\pi n$.

576. а) $\sin x+\cos x+\sqrt{2}=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)+\sqrt{2}=\sqrt{2}\left(1+\cos\left(x-\frac{\pi}{4}\right)\right)=2\sqrt{2}\cos^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$;

б) $\cos 2x-\sin 2x=\sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)-\sqrt{2}=-\sqrt{2}\left(1-\cos\left(2x+\frac{\pi}{4}\right)\right)=$

$$=-2\sqrt{2}\sin^2\left(\frac{2x+\frac{\pi}{4}}{2}\right)=-2\sqrt{2}\sin^2\left(x+\frac{\pi}{8}\right).$$

577. а) $2\sin 17x+\sqrt{3}\cos 5x+\sin 5x=0$; $2\sin 17x+2\left(\frac{\sqrt{3}}{2}\cos 5x+\frac{1}{2}\sin 5x\right)=0$;

$2\sin 17x+2\sin\left(5x+\frac{\pi}{3}\right)=0$; $\sin 17x+\sin\left(5x+\frac{\pi}{3}\right)=0$; $2\sin\frac{17x+5x+\frac{\pi}{3}}{2}\cos\frac{17x-5x-\frac{\pi}{3}}{2}$;

$\sin\left(11x+\frac{\pi}{6}\right)\cos\left(6x-\frac{\pi}{6}\right)=0$; $\left[\begin{array}{l}\sin\left(11x+\frac{\pi}{6}\right)=0\\ \cos\left(6x-\frac{\pi}{6}\right)=0\end{array}\right.$; $\left[\begin{array}{l}11x+\frac{\pi}{6}=\pi n\\ 6x-\frac{\pi}{6}=\frac{\pi}{2}+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=-\frac{\pi}{66}+\frac{\pi n}{11}\\ x=\frac{\pi}{9}+\frac{\pi k}{6}\end{array}\right.$.

б) $5\sin x-12\cos x+13\sin 3x=0$; $\frac{5}{13}\sin x-\frac{12}{13}\cos x+\sin 3x=0$;

$\sin x\cos\left(\arcsin\frac{12}{13}\right)-\cos x\sin\left(\arcsin\frac{12}{13}\right)+\sin 3x=0$; $\sin\left(x-\arcsin\frac{12}{13}\right)+\sin 3x=0$;

$2\sin\frac{x-\arcsin\frac{12}{13}+3x}{2}\cos\frac{x-\arcsin\frac{12}{13}-3x}{2}=0$;

$\sin\left(2x-\frac{1}{2}\arcsin\frac{12}{13}\right)\cos\left(2x+\frac{1}{2}\arcsin\frac{12}{13}\right)=0$; $\left[\begin{array}{l}\sin\left(2x-\frac{1}{2}\arcsin\frac{12}{13}\right)=0\\ \cos\left(x+\frac{1}{2}\arcsin\frac{12}{13}\right)=0\end{array}\right.$;

$\left[\begin{array}{l}2x-\frac{1}{2}\arcsin\frac{12}{13}=\pi n\\ x+\frac{1}{2}\arcsin\frac{12}{13}=\frac{\pi}{2}+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=\frac{1}{4}\arcsin\frac{12}{13}+\frac{\pi n}{2}\\ x=\frac{\pi}{2}-\frac{1}{2}\arcsin\frac{12}{13}+\pi k\end{array}\right.$.

578. а) $\left(\sin x+\sqrt{3}\cos x\right)^2-5=\cos\left(\frac{\pi}{6}-x\right).\ 4\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)^2-5=\cos\left(\frac{\pi}{6}-x\right);$

$4\left(\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right)^2-5=\cos\left(\frac{\pi}{6}-x\right);\ 4\sin^2\left(x+\frac{\pi}{3}\right)-5=\cos\left(\frac{\pi}{6}-x\right);$

$4\cos^2 x\left(\frac{\pi}{6}-x\right)-5=\cos\left(\frac{\pi}{6}-x\right);\ 4\cos^2\left(\frac{\pi}{6}-x\right)-\cos\left(\frac{\pi}{6}-x\right)-5=0;$

$\cos\left(\frac{\pi}{6}-x\right)=\frac{1\pm\sqrt{1+80}}{8}=\frac{1\pm 9}{8}=\left[\begin{array}{l}\frac{5}{4}\\ -1\end{array}\right.$; т.к. $\cos\left(\frac{\pi}{6}-x\right)\le 1$, то $\cos\left(\frac{\pi}{6}-x\right)\ne\frac{5}{4}$.

$\cos\left(\frac{\pi}{6}-x\right)=-1;\ x-\frac{\pi}{6}=\pi+2\pi n;\ x=\frac{7\pi}{6}+2\pi n.$

б) $\left(\sqrt{3}\sin x-\cos x\right)^2+1=4\cos\left(x+\frac{\pi}{3}\right);\ 4\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)^2+1=4\cos\left(x+\frac{\pi}{3}\right);$

$4\cos^2\left(x+\frac{\pi}{3}\right)+1-4\cos\left(x+\frac{\pi}{3}\right)=0;\ \left(2\cos\left(x+\frac{\pi}{3}\right)-1\right)^2=0;$

$\cos\left(x+\frac{\pi}{3}\right)=\frac{1}{2};\ \left[\begin{array}{l}x+\frac{\pi}{3}=\frac{\pi}{3}+2\pi k\\ x+\frac{\pi}{3}=-\frac{\pi}{3}+2\pi n\end{array}\right.;\ \left[\begin{array}{l}x=2\pi k\\ x=-\frac{2\pi}{3}+2\pi n\end{array}\right.$

579. а) $\sqrt{3}\sin x+\cos x+2=\frac{12}{\pi}x.\ \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+1=\frac{6}{\pi}x;\ \sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}+1=\frac{6}{\pi}x;$

$\sin\left(x+\frac{\pi}{6}\right)=\frac{6}{\pi}x-1;\ x=\frac{\pi}{3}.$

б) $\sqrt{2}\left(\cos x-\sin x\right)=2x-\frac{\pi}{2};\ 2\left(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}\right)=2x-\frac{\pi}{2};$

$\cos\left(x+\frac{\pi}{4}\right)=x-\frac{\pi}{4};\ x=\frac{\pi}{4}.$

580. а) $\sqrt{3}\sin x+\cos x>1;\ \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x>\frac{1}{2};\ \sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}>\frac{1}{2};$

$\sin\left(x+\frac{\pi}{6}\right)>\frac{1}{2};\ \frac{\pi}{6}+2\pi n<x+\frac{\pi}{6}<\frac{5\pi}{6}+2\pi n;\ 2\pi n<x<\frac{2\pi}{3}+2\pi n.$

б) $3\sin x-4\cos x<2{,}5;\ \frac{3}{5}\sin x-\frac{4}{5}\cos x<\frac{1}{2};\ \sin x\cos\left(\arcsin\frac{4}{5}\right)-\cos x\sin\left(\arcsin\frac{4}{5}\right)<\frac{1}{2};$

$\sin\left(x-\arcsin\frac{4}{5}\right)<\frac{1}{2};\ -\frac{7\pi}{6}+2\pi n<x-\arcsin\frac{4}{5}<\frac{\pi}{6}+2\pi n;$

$-\frac{7\pi}{6}+2\pi n+\arcsin\frac{4}{5}<x<\frac{\pi}{6}+2\pi n+\arcsin\frac{4}{5}.$

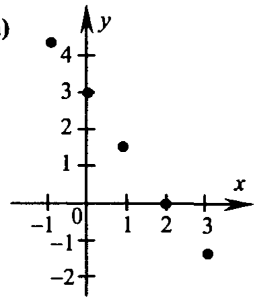
Глава 4. Производная

§29. Числовые последовательности

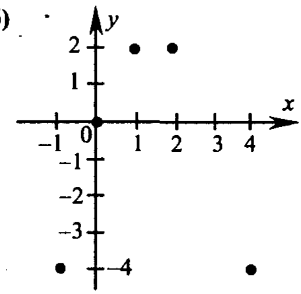
581. **а)** $y = 3x^2 + 5$, $x \in Z$, является; **б)** $y = \sin x$, $x \in [0; 2\pi]$, не является;

в) $y = 7 - x^2$, $x \in Q$, не является; **г)** $y = \cos\dfrac{x}{2}$, $x \in N$, является.

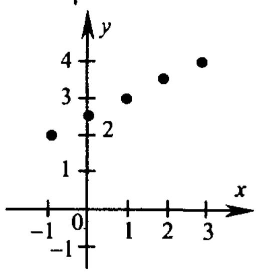
582. **а)**



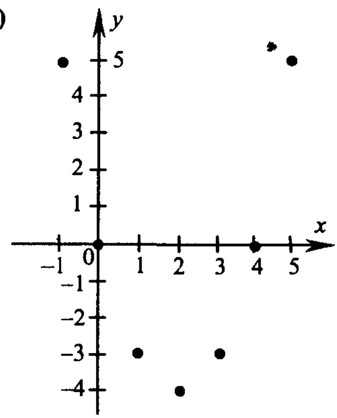
б)



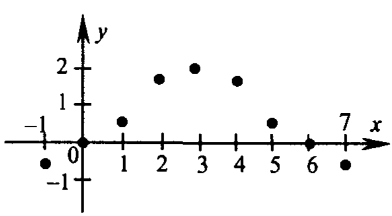
в)



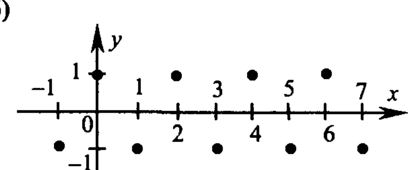
г)



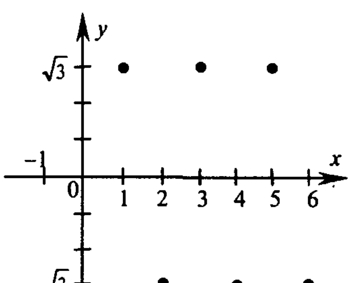
583. **а)**



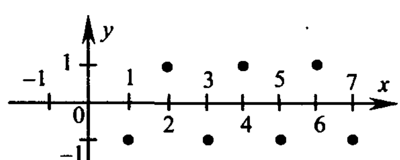
б)



в)



г)



584. **а)** $y_n = 2^n$; **б)** 1, 2, 4, 8, 16…; **в)** $b_1 = 2, b_{n+1} = b_n \cdot 2$.

585. **а)** $y_n = -n, n \in N, y_1 = -1, y_2 = -2, y_3 = -3, y_4 = -4, y_5 = -5$;

б) $y_n = \sqrt{n}, n \in N, y_1 = 1, y_2 = \sqrt{2}, y_3 = \sqrt{3}, y_4 = 2, y_5 = \sqrt{5}$;

в) $y_n = -5, y_1 = -5, y_2 = -5, y_3 = -5, y_4 = -5, y_5 = -5$;

г) $y_n = \frac{n^2}{2}, n \in N, y_1 = \frac{1}{2}, y_2 = 2, y_3 = \frac{9}{2}, y_4 = 8, y_5 = \frac{25}{2}$.

586. **а)** $y_7 = 42, y_9 = 54, y_{17} = 72, 6 \cdot n = y_n$; **б)** $y_6 = 42, y_{10} = 70, y_{31} = 217, 7 \cdot n = y_n$.

587. **а)** $y_1 = 2, y_2 = 7, y_3 = 12, y_4 = 17, y_5 = 22$; **б)** $S = 3 + 7 + 11 + 15 + 19 + 23 = 78$.

588. **а)** $y_n = n^5, n \in N, y_1 = 1, y_2 = 32, y_n = n^5, y_{n+1} = (n+1)^5$;

б) $y_n = 3^5, y_5 = 3^5 = 243, y_7 = 3^7 = 2187, y_{21} = 3^{21}, y_{2n} = 3^{2n} \cdot y_{2n+5} = 3^{2n+5}$.

589. **а)** y_{732} и $y_{745}, y_{733}, y_{734}, \ldots, y_{744}$; **б)** y_{n-1} и y_{n+2}, y_n, y_{n+1};

в) y_{998} и $y_{1003}, y_{999}, y_{1000}, y_{1001}, y_{1002}$; **г)** $y_{2n-2}, y_{2n-1}, y_{2n}, y_{2n+1}, y_{2n+2}$.

590. **а)** $y_n = 3 - 2n, y_1 = 1, y_2 = -1, y_3 = -3, y_4 = -5, y_5 = -7$;

б) $y_n = 2n^2 - n, y_1 = 1, y_2 = 6, y_3 = 15, y_4 = 28, y_5 = 45$;

в) $y_n = n^3 - 1, y_1 = 0, y_2 = 7, y_3 = 26, y_4 = 63, y_5 = 124$;

г) $y_n = \frac{3n-1}{2n} = \frac{3}{2} - \frac{1}{2n}, y_1 = 1, y_2 = \frac{5}{4} = 1\frac{1}{4}, y_3 = \frac{8}{6} = 1\frac{1}{3}, y_4 = \frac{11}{8} = 1\frac{3}{8}, y_5 = \frac{7}{5} = 1\frac{2}{5}$.

591. **а)** $y_n = (-1)^n, y_1 = -1, y_2 = 1, y_3 = -1, y_4 = 1, y_5 = -1$;

б) $y_n = \frac{(-2)^n}{n^2+1}, y_1 = -1, y_2 = \frac{4}{5}, y_3 = -\frac{8}{10} = -\frac{4}{5}, y_4 = \frac{16}{17}, y_5 = -\frac{32}{26} = -\frac{16}{13} = -1\frac{3}{13}$;

в) $y_n = (-1)^n \frac{1}{10^n}, y_1 = -\frac{1}{10}, y_2 = \frac{1}{100}, y_3 = -\frac{1}{1000}, y_4 = \frac{1}{10000}, y_5 = -\frac{1}{100000}$;

г) $y_n = \frac{(-1)^n + 2}{3n-2}, y_1 = 1, y_2 = \frac{3}{4}, y_3 = \frac{1}{7}, y_4 = \frac{3}{10}, y_5 = \frac{1}{13}$.

593. **а)** $y_n = 3\cos\frac{2\pi}{n}, y_1 = 3, y_2 = -3, y_3 = -\frac{3}{2}, y_4 = 0, y_5 = 3\cos\frac{2\pi}{5}$;

б) $y_n = \operatorname{tg}\left((-1)^n \frac{\pi}{4}\right), y_1 = -1, y_2 = 1, y_3 = -1, y_4 = 1, y_5 = -1$;

в) $y_n = 1 - \cos^2\frac{\pi}{n}, y_1 = 0, y_2 = 1, y_3 = \frac{3}{4}, y_4 = \frac{1}{2}, y_5 = \sin^2\frac{\pi}{5}$;

г) $y_n = \sin \pi n - \cos \pi n = -\cos \pi n, y_1 = 1, y_2 = -1, y_3 = 1, y_4 = -1, y_5 = 1$.

593. $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 = 4 + 9 + 25 + 79 + 121 + 169 + 289 + 361 = 1027$.

594. **а)** $y_n = n, n = 1, 2, 3\ldots$; **б)** $y_n = -n, n \in N$; **в)** $y_n = n + 4, n \in N$; **г)** $y_n = 11 - n, n \in N$.

595. **а)** $y_n = 5n, n \in N$; **б)** $y_n = 6n, n \in N$; **в)** $y_n = 4n, n \in N$; **г)** $y_n = 3n, n \in N$.

596. **а)** $y_n = 3^n, n \in N$; **б)** $y_n = (n+2)^2, n \in N$; **в)** $y_n = n^3, n \in N$; **г)** $y_n = n^3 + 1, n \in N$.

597. **а)** $y_n = \frac{2}{2^n} = \frac{1}{2^{n-1}}$; **б)** $y_n = \frac{2n+1}{2n+2}$; **в)** $y_n = \frac{1}{n^3}$; **г)** $y_n = \frac{1}{(2n+1)(2n+3)}$.

598. **а)** $y_n = 1{,}5n, y_1 = 1{,}5, y_2 = 3, y_3 = 4{,}5$; **б)** $y_n = (-1)^n, y_1 = -1, y_2 = 1, y_3 = -1$;

в) $y_n = \frac{8}{n}, y_1 = 8, y_2 = 4, y_3 = \frac{8}{3} \ldots$; **г)** $y_n = (-1)^{n+1} n, y_1 = 1, y_2 = -2, y_3 = 3, y_4 = -4$.

599. а) 1; 1,4; 1,41; 1,414; 1,4142. б) 2; 1,5; 1,42; 1,415; 1,4143.

600. $y_n = \dfrac{2-n}{5n+1}$

а) $y_n = 0$, $n = 2$; б) $y_n = -\dfrac{3}{26}$, $n = 5$;

в) $y_n = -\dfrac{1}{6}$, $\dfrac{2-n}{5n+1} = -\dfrac{1}{6}$, $6n - 12 = 5n + 1$, $n = 13$;

г) $-\dfrac{43}{226} = y_n$, $\dfrac{2-n}{5n+1} = -\dfrac{43}{226}$, $226n - 452 = 215n + 43$, $11n = 495$, $n = 45$.

601. $a_n = (2n - 1)(3n + 2)$

а) $a_n = 0$. нет, т.к. n не может быть равным $\dfrac{1}{2}$, $-\dfrac{2}{3}$.

б) $a_n = 24$. $6n^2 + n - 2 = 24$. $6n^2 + n - 26 = 0$.

$n = \dfrac{-1+25}{12} = 2$, *или* $n = -\dfrac{26}{12}$. (не подходит, т.к. $n \in N$).

Ответ: является.

в) $a_n = 153$. $6n^2 + n - 155 = 0$. $n = \dfrac{-1+61}{12} = 5$. $n = -\dfrac{62}{12}$ (не подходит).

Ответ: является.

г) $a_n = -2$. $6n^2 + n = 0$

$n(6n + 1) = 0$. решение в натуральных числах нет $\Rightarrow$ не является.

602. а) 1; $\dfrac{1}{2}$; $\dfrac{1}{3}$; $\dfrac{1}{4}$… ограничена снизу и сверху.

б) –1; 2; –3; 4; –5… не ограничена. в) $\dfrac{1}{2}$; $\dfrac{2}{3}$; $\dfrac{3}{4}$; $\dfrac{4}{5}$… снизу ограничена.

г) 5; 4; 3; 2; 1; 0; –1… не ограничена снизу.

603. а) –3; –2; –1; 0; 1… не ограничена сверху.

б) 1; –1; 1; –2; 1; –3… ограничена сверху.

в) $\dfrac{1}{2}$; $\dfrac{1}{3}$; $\dfrac{1}{4}$; $\dfrac{1}{5}$ … ограничена сверху.

г) $\dfrac{1}{2}$; $\dfrac{2}{3}$; $\dfrac{3}{4}$; $\dfrac{4}{5}$… ограничена сверху.

604. а) $\dfrac{1}{2}$; $\dfrac{1}{3}$; $\dfrac{1}{4}$;… $\dfrac{1}{n}$ … ограничена, т.к. $0 < y_n < 1$.

б) $\dfrac{1}{2}$; $\dfrac{3}{4}$; $\dfrac{5}{6}$;… $\dfrac{2n-1}{2n}$ … ограничена, т.к. $0 < y_n < 1$.

в) 5; –5; 5; –5; $(-1)^{n-1}5$ ограничена, т.к. $-10 < y_n < 10$.

г) –2; 3; –4; 5; $(-1)^n(n + 1)$ не ограничена.

605. а) $x_n = 3 \cdot n + 2$; возьмем $n_2 > n_1$

$x_{n_2} = 3 \cdot 2n_2 + 2 > 3 \cdot n_1 + 2 \Rightarrow$ последов. возрастающая.

б) $x_n = \dfrac{5}{n+3}$ убывающая, т.к. при $n_2 > n_1$, $x_{n_2} < x_{n_1}$

в) $x_n = n^3$ возрастающая, т.к. при $n_2 > n_1$, $x_{n_2} > x_{n_1}$

г) $x_n = (-1)^{n-1}$ ни возрастающая, ни убывающая.

606. а) $x_n = \left(\frac{1}{3}\right)^{n+1}$ – убывающая. б) $x_n = 7^{n-5}$ – возрастающая.

в) $x_n = 6^{1-n}$ – убывающая. г) $x_n = \left(-\frac{1}{5}\right)^{2n-1}$ – возрастает, т.к. степень нечетная.

607. а) $y_{n+1} - y_n > 0$; $y_{n+1} > y_n \Rightarrow$ по определению она возрастающая.

б) $\frac{y_{n+1}}{y_n} < 1$, $y_n > 0$; $y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_{n+1} - y_n < 0$; $y_{n+1} < y_n \Rightarrow$ убывающая.

г) $\frac{y_{n+1}}{y_n} < 1$, $y_n < 0$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

608. а) $y_n = 2n - 1$; $y_{n+1} = 2n + 1$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

б) $y_n = 5^{-n}$; $y_{n+1} = 5^{-n-1} = \frac{5^{-n}}{5}$; $y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_n = n^2 + 8$, $y_{n+1} = n^2 + 2n + 9$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

г) $y_n = \frac{2}{3n+1}$, $y_{n+1} = \frac{2}{3n+4}$, $y_{n+1} < y_n \Rightarrow$ убывающая.

609. а) $x_n = (-2)^n$, $x_1 = -2$, $x_2 = 4$, $x_3 = -8$

$\Rightarrow$ последовательность не является монотонной.

б) $y_n = \cos\frac{\pi}{n+5}$; $y_1 = \cos\frac{\pi}{6}$, $y_2 = \cos\frac{\pi}{7}$, $y_3 = \cos\frac{\pi}{8}$. $y_1 < y_2 < y_3 \ldots \Rightarrow$ возрастает.

в) $y_n = n^3 - 5$; $y_1 = -4$, $y_2 = 3$, $y_3 = 22$. $\Rightarrow$ возрастает.

г) $y_n = \sqrt{n+8}$, $y_1 = 3$, $y_2 = \sqrt{10}$, $y_3 = \sqrt{11}$.

610. а) $y_n = 2^n$; б) $y_n = 2^n$; в) $y_n = \left(\frac{1}{2}\right)^n$; г) $y_n = -n$.

611. а) $y_n = \sin\frac{\pi n}{2} - \mathrm{ctg}\frac{\pi}{4}(2n+1)$; $y_1 = 2$, $y_2 = -1$, $y_3 = 0$, $y_4 = 1$, $y_5 = 2$.

б) $y_n = \cos\frac{\pi n}{2} + \mathrm{tg}\frac{\pi}{4}(2n+1)$; $y_1 = -1$, $y_2 = 0$, $y_3 = -1$, $y_4 = 2$, $y_5 = -1$.

в) $y_n = n\sin\frac{\pi n}{2} + n^2\cos\frac{\pi n}{2}$; $y_1 = 1$, $y_2 = -4$, $y_3 = -3$, $y_4 = 16$, $y_5 = 5$.

г) $y_n = \sin\frac{\pi n}{4} - n\cos\frac{\pi n}{4}$; $y_1 = 0$, $y_2 = 1$, $y_3 = 2\sqrt{2}$, $y_4 = 4$, $y_5 = 2\sqrt{2}$.

612. а) $y_n = \frac{1\cdot2\cdot3\cdot4\cdots n}{n^3+1}$; $y_1 = \frac{1}{2}$, $y_2 = \frac{2}{9}$, $y_3 = \frac{6}{28} = \frac{3}{14}$, $y_4 = \frac{24}{65}$, $y_5 = \frac{120}{126} = \frac{20}{21}$.

б) $y_n = \frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{2\cdot4\cdot6\cdot\ldots\cdot2n}$;

$$y_1 = \frac{1}{2},\ y_2 = \frac{3}{8},\ y_3 = \frac{3\cdot 5}{2\cdot 4\cdot 2\cdot 3} = \frac{5}{16},\ y_4 = \frac{5\cdot 7}{16\cdot 8} = \frac{35}{128},\ y_5 = \frac{35\cdot 9}{128\cdot 10} = \frac{7\cdot 9}{126\cdot 5} = \frac{63}{256}.$$

613. а) $x_1 = 2,\ x_2 = 5 - 2 = 3,\ x_3 = 2,\ x_4 = 3,\ x_5 = 2$.

б) $x_1 = 2,\ x_2 = 12,\ x_3 = 22,\ x_4 = 32,\ x_5 = 42$.

в) $x_1 = -1,\ x_2 = 1,\ x_3 = 3,\ x_4 = 5,\ x_5 = 7$. **г)** $x_1 = 4,\ x_2 = 1,\ x_3 = -2,\ x_4 = -5,\ x_5 = -8$.

614. а) $x_1 = 2,\ x_2 = 4,\ x_3 = 12,\ x_4 = 48,\ x_5 = 240$.

б) $x_1 = -5,\ x_2 = \frac{5}{2},\ x_3 = -\frac{5}{4},\ x_4 = \frac{5}{8},\ x_5 = -\frac{5}{16}$.

в) $x_2 = -2,\ x_2 = 2,\ x_3 = -2,\ x_4 = 2,\ x_5 = -2$.

г) $x_1 = 1,\ x_2 = 10,\ x_3 = 100,\ x_4 = 1000,\ x_5 = 10\,000$.

615. а) $-\frac{1}{2}; \frac{3}{4}; -\frac{5}{6}; \frac{7}{8}; -\frac{9}{10} \ldots; \frac{(-1)^n\cdot(2n-1)}{2n} = y_n$;

б) $\frac{2}{\sqrt{3}}; \frac{4}{3}; \frac{6}{3\sqrt{3}}; \frac{8}{9}; \frac{10}{9\sqrt{3}} \ldots; y_n = \frac{2n}{\left(\sqrt{3}\right)^n}$;

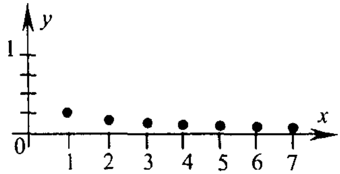
в) $\frac{3}{4}; \frac{9}{16}; \frac{27}{64}; \frac{81}{256} \ldots; y_n = \left(\frac{3}{4}\right)^n$;

г) $\frac{1}{\sqrt{2}}; \frac{3}{2}; \frac{5}{2\sqrt{2}}; \frac{7}{4}; \frac{9}{4\sqrt{2}} \ldots; y_n = \frac{2n-1}{\left(\sqrt{2}\right)^n}$.

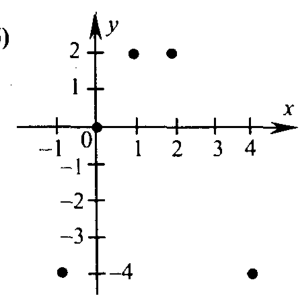
616. а) $y_n = (-1)^{n+1}\frac{n^2}{\sqrt{n(n+1)}}$; **б)** $y_n = (-1)^{n+1}\frac{2n+3}{(n+1)^2(n+2)^2}$;

в) $y_n = (-1)^{n+1}\frac{5n-1}{n(n+1)(n+2)}$; **г)** $y_n = \frac{(-1)^n + 1}{1\cdot 2\cdot 3\cdot 4 \ldots n}$.

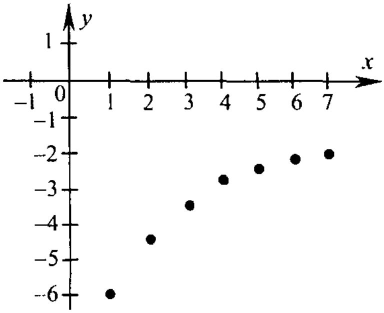
617. а)



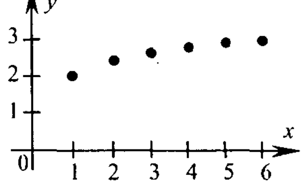
б)



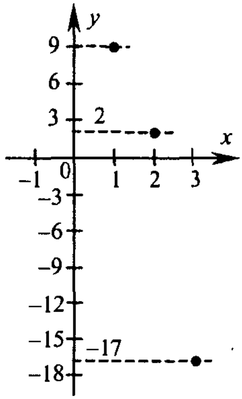
в)



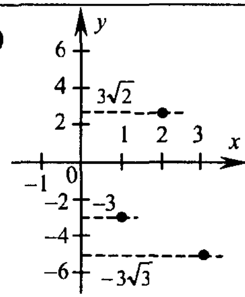
г)



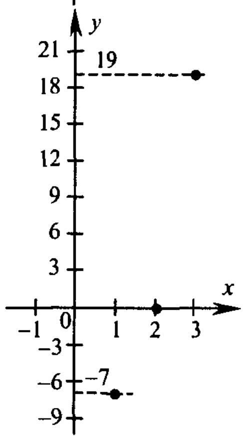
618. а)



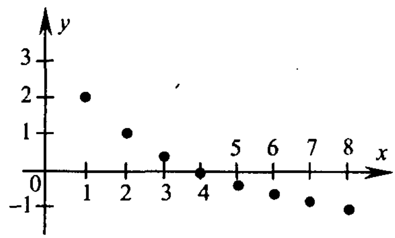
б)



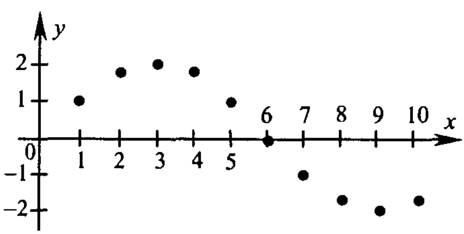
в)



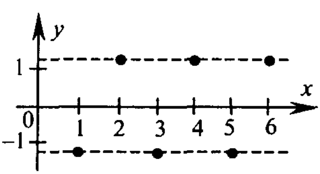
г)



619. а)



б)



620. $x_1 = 5, x_2 = -3, x_3 = 1, x_4 = 1, x_5 = 0,\ x_6 = -\frac{1}{2}; x_n = \frac{x_{n-2} + x_{n-1}}{2}$.

621. $a_1 = 1, S_1 = 1, P_1 = 4; a_2 = \sqrt{2}\ , S_2 = 2, P_2 = 4\sqrt{2}\ ; a_3 = 2, S_1 = 4, P_1 = 8; a_3 = 2\sqrt{2}\ ,$
$S_3 = 8, P_3 = 8\sqrt{2}\ ; a_4 = 4, S_4 = 16, P_5 = 16;$
$a_n\left(\sqrt{2}\right)^{n-1}; S_n = 2^{n-1}; P_n = 4\cdot\left(\sqrt{2}\right)^{n-1}; a_{11} = \left(\sqrt{2}\right)^{10} = 2^5 = 32\ ; S_{17} = 2^{16}.$

622. $y_n = 2n^2\ \ 7n + 5;\ \begin{cases} 2n^2 - 7n + 5 \le 5. \\ 2n^2 - 7n + 5 \ge 2. \end{cases} \begin{cases} n(2n-7) \le 0. \\ 2n^2 - 7n + 3 \ge 0. \end{cases} (x-3)(2x-1) \ge 0\ ;$

$$\begin{cases} n \in \left[0; \frac{7}{2}\right] \\ n \in \left(-\infty; \frac{1}{2}\right] \cup [3; +\infty) \end{cases} \Rightarrow n \in \left[0; \frac{1}{2}\right] \cup \left[3; \frac{7}{2}\right]$$, т.к. $x \in N$, то $n = 3$, $y_3 \in [2; 5]$.

623. а) $x_n = 3n - 2$ $A = 15$. $3n - 2 > 15$, $n > \frac{17}{3}$. $n \geq 6$. Начиная с x_6.

б) $x_n = 5^{n-1}$ $A = 125$. $5^{n-1} > 125$; $5^{n-1} > 5^3$. $n > 4$. Начиная с x_5.

в) $x_n = n^2 - 17A = -2$. $n^2 - 17 > -2$. $n^2 > 15$. Начиная с x_4.

г) $x_n = 3^{n-5}$ $A = 27$. $3^{n-5} > 3^3$. $n > 8$. Начиная с x_9.

624. а) $x_1 = -14$ $x_n = x_{n-1} + 7$ $A = 25$; $x_1 = -14$, $x_2 = -7$, $x_3 = 0$.

$x_n = -21 + 7n$. $-21 + 7n > 25$. $n > \frac{46}{7}$. Начиная с x_7.

б) $x_1 = 3$, $x_n = 6x_{n-1}$, $A = 168$. $x_1 = 3$, $x_2 = 18$, $x_3 = 108$.
$x_n = 3 \cdot 6^{n-1}$. $3 \cdot 6^{n-1} > 168$. $6^{n-1} > 56$. Начиная с x_4.

в) $x_1 = 0$, $x_n = x_{n-1} + 3$, $A = 28$. $x_n = 3(n-1)$. $3(n-1) > 28$. $n > \frac{28}{3} + 1$. Начиная с x_{11}.

г) $x_1 = 3$, $x_n = 7x_{n-1}$, $A = 285$. $x_n = 7^{n-1}$. $7^{n-1} > 285$. Начиная с x_4.

625. а) $\frac{1}{3125}; \frac{1}{625}; \frac{1}{125}...$; $x_n = \frac{1}{3125} \cdot 5^{(n-1)} \cdot 5^{-5} \cdot 5^{n-1} \leq 1$. $n - 1 \leq 5$. $n \leq 6$. шесть членов.

б) $\frac{6}{377}; \frac{11}{379}; \frac{16}{381}...$ $\frac{6 + 5(n-1)}{377 + 2(n-1)} = x_n$. $\frac{5n+1}{375+2n} \leq 1$.

$3n \leq 374$. $n \leq \frac{374}{3}$. 124 члена.

в) $\frac{2}{729}; \frac{2}{243}; \frac{2}{81}...$; $\frac{2 \cdot 3^{n-1}}{729} = x_n$. $2 \cdot 3^{n-1} \leq 729$. $2 \cdot 3^{n-1} \leq 3^6$. $n < 7$ шесть членов.

г) $\frac{2}{219}; \frac{9}{222}; \frac{16}{225}...$ $\frac{2 + 7(n-1)}{219 + 3(n-1)} = x_n$

$\frac{7n-5}{3n+216} \leq 1$; $7n - 5 \leq 3n + 216$; $4n \leq 221$; $n \leq \frac{221}{4}$ 55 членов.

626. а) $y_n = \frac{n^2}{n+1} = n + 1 - \frac{2n+1}{n+1} = n + 1 - 2 + \frac{1}{n+1} = n - 1 + \frac{1}{n+1}$.

Ограничена снизу.

б) $y_n = \frac{(-1)^n + 1}{2n}$. $y_1 = 0$ $y_2 = \frac{1}{2}$ $y_3 = 0$ $y_4 = \frac{1}{4}$

Ограничена снизу.

в) $y_1 = 0$, $y_2 = 8$, $y_3 = 0$, $y_4 = 32$; ограничена снизу;

г) $y_1 = 0$, $y_2 = -\frac{3}{4}$, $y_3 = -\frac{8}{27}$; ограничена снизу.

627. а) $x_1 = \frac{1}{2}$, $x_2 = \frac{2}{3}$, $x_3 = \frac{3}{4}$; ограничена сверху;

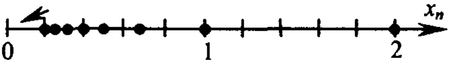
б) $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = \frac{1}{2}$; ограничена сверху;

в) $x_1 = 0, x_2 = 4, x_3 = 0, x_4 = 16$; не ограничена сверху;

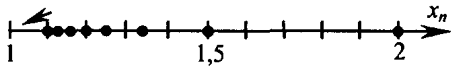
г) $x_1 = 0, x_2 = \frac{3}{6}, x_3 = \frac{8}{11}, x_4 = \frac{15}{18}$; ограничена сверху.

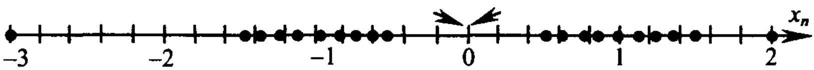
628. а) Ограничена; б) ограничена; в) ограничена; г) не ограничена.

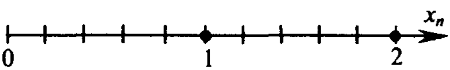
629. а) Ограничена; б) ограничена; в) ограничена; г) ограничена.

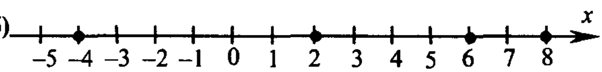
630. а) 

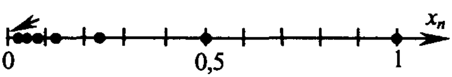
б) 

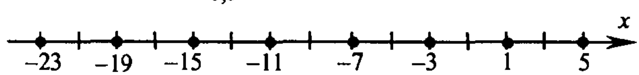
в) 

г) 

631. а) 

б) 

в) 

г) 

632. а) $6 \le a_n \le 7$; б) $2 \le b_n \le 3$; в) $1 \le p_n \le 2$; г) $0 \le q_n \le 1$.

§30. Предел числовой последовательности

633. а) $a = 0$, $r = 0{,}1$. $(-0{,}1;\ 0{,}1)$; б) $a = -3$, $r = 0{,}5$. $(-3{,}5;\ -2{,}5)$
в) $a = 2$, $r = 1$. $(1;\ 3)$; г) $a = 0{,}2$, $r = 0{,}3$. $(-0{,}1;\ 0{,}5)$.

634. а) $(1;\ 3)$, $a = 2$, $r = 1$. б) $(-0{,}2;\ 0{,}2)$, $a = 0$, $r = 0{,}2$.
в) $(2{,}1;\ 2{,}3)$, $a = 2{,}2$, $r = 0{,}1$. г) $(-7;\ -5)$, $a = -6$, $r = 1$.

635. а) $x_1 = 1$, $a = 2$, $r = 0{,}5$ $1 \notin (1{,}5;\ 2{,}5)$; б) $x_1 = 1{,}1$, $a = 1$, $r = 0{,}2$. $1{,}1 \in (0{,}8;\ 1{,}2)$;
в) $x_1 = -0{,}2$, $a = 0$, $r = 0{,}3$. $-0{,}2 \in (-0{,}3;\ 0{,}3)$;
г) $x_1 = 2{,}75$, $a = 2{,}5$, $r = 0{,}3$. $2{,}75 \in (2{,}2;\ 2{,}8)$.

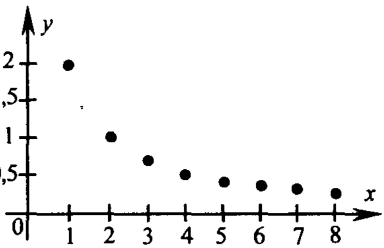
636. а) $x_n = \frac{1}{n^2}$, $a = 0$, $r = 0{,}1$. $\frac{1}{n^2} \in (-0{,}1; 0{,}1)$. При $n_0 \ge 4$.

б) $x_n = \frac{1}{n^2}$, $a = 1$, $r = 0{,}1$. Такого n_0 не существует.

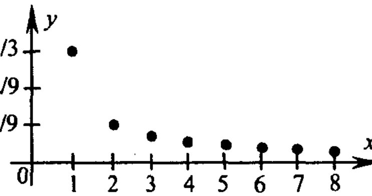
в) $x_n = \frac{n}{n+1}$, $a = 0$, $r = 0{,}1$. Такого n_0 не существует.

г) $x_n = \dfrac{n}{n+1}$, $a = 1$, $r = 0{,}1$. $x_n \in (0{,}9;\ 1{,}1)$ при $n_0 \geq 10$.

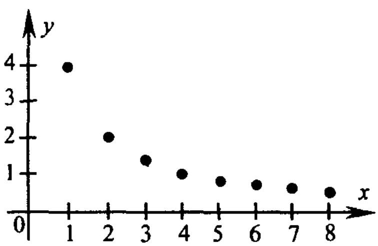
637. а) $y_n = \dfrac{2}{n}$, $y = 0$.



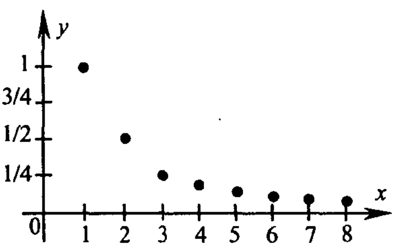
б) $y_n = \left(\dfrac{1}{3}\right)^n$, $y = 0$.



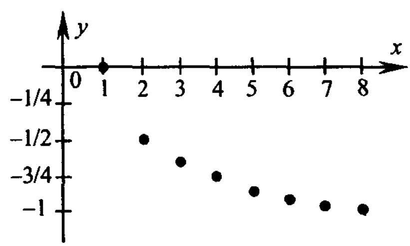
в) $y_n = \dfrac{4}{n}$, $y = 0$.



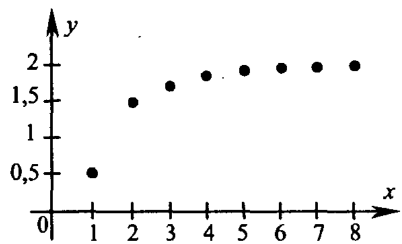
г) $y_n = \left(\dfrac{1}{2}\right)^{n-1}$, $y = 0$.



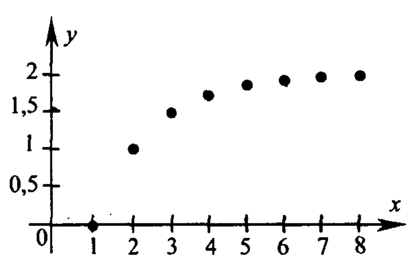
638. а) $y_n = -1 + \dfrac{1}{n}$, $y = -1$.



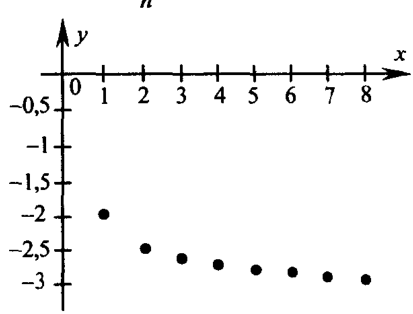
б) $y_n = 2 - \dfrac{1}{n^2}$, $y = 2$.



в) $y_n = 2 - \dfrac{2}{n}$, $y = 2$.



г) $y_n = -3 + \dfrac{1}{n^2}$, $y = -3$.



639. **а)** $\lim\limits_{n\to\infty}\frac{5}{n^2}=0$, **б)** $\lim\limits_{n\to\infty}\left(-\frac{17}{n^3}\right)=0$, **в)** $\lim\limits_{n\to\infty}\left(-\frac{15}{n^2}\right)=0$, **г)** $\lim\limits_{n\to\infty}\frac{3}{\sqrt{n}}=0$.

640. **а)** $\lim\limits_{n\to\infty}\left(\frac{7}{n}+\frac{8}{\sqrt{n}}+\frac{9}{n^3}\right)=0+0+0=0$;

б) $\lim\limits_{n\to\infty}\left(6-\frac{7}{n^2}-\frac{3}{n}-\frac{3}{\sqrt{n}}\right)=6-0-0-0=6$;

в) $\lim\limits_{n\to\infty}\left(\frac{3}{n}+\frac{7}{n^2}-\frac{5}{n^3}+\frac{13}{n^4}\right)=0+0-0+0=0$;

г) $\lim\limits_{n\to\infty}\left(\frac{1}{n}+\frac{3}{\sqrt{n}}-4+\frac{7}{n^2}\right)=0+0-4+0=-4$;

641. **а)** $\lim\limits_{n\to\infty}\frac{5n=3}{n+1}=\lim\limits_{n\to\infty}\frac{5+\frac{3}{n}}{1+\frac{1}{n}}=5$; **б)** $\lim\limits_{n\to\infty}\frac{7n-5}{n+2}=\lim\limits_{n\to\infty}\frac{7-\frac{5}{n}}{1+\frac{2}{n}}=7$;

в) $\lim\limits_{n\to\infty}\frac{3n+1}{n+2}=\lim\limits_{n\to\infty}\frac{3+\frac{1}{n}}{1+\frac{2}{n}}=3$; **г)** $\lim\limits_{n\to\infty}\frac{2n+1}{3n-1}=\lim\limits_{n\to\infty}\frac{2+\frac{1}{n}}{3-\frac{1}{n}}=\frac{2}{3}$.

642. **а)** $\lim\limits_{n\to\infty}\frac{5}{2^n}=0$; **б)** $\lim\limits_{n\to\infty}\frac{1}{2}\cdot 5^{-n}=0$; **в)** $\lim\limits_{n\to\infty}7\cdot 3^{-n}=0$; **г)** $\lim\limits_{n\to\infty}\frac{4}{3^{n+1}}=0$.

643. **а)** $\lim\limits_{n\to\infty}\frac{2n^2-1}{n^2}=\lim\limits_{n\to\infty}\left(2-\frac{1}{n^2}\right)=2$; **б)** $\lim\limits_{n\to\infty}\frac{\left(1+2n+n^2\right)}{n^2}=\lim\limits_{n\to\infty}\left(\frac{\frac{1}{n^2}+\frac{2}{n}+1}{1}\right)=1$;

в) $\lim\limits_{n\to\infty}\frac{3-n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{3}{n^2}-1\right)=-1$; **г)** $\lim\limits_{n\to\infty}\frac{3n-4-2n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{\frac{3}{n}-\frac{4}{n^2}-2}{1}\right)=2$.

644. **а)** $b_1=3$ $q=\frac{1}{3}$ $S_n=\frac{3}{1-\frac{1}{3}}=4\frac{1}{2}$. **б)** $b_1=-5$ $q=-0{,}1$ $S_n=\frac{-5}{1+0{,}1}=-\frac{50}{11}=-4\frac{6}{11}$.

в) $b_1=-1$ $q=0{,}2$ $S_n=\frac{-1}{0{,}8}=-\frac{5}{4}=-1\frac{1}{4}$.

г) $b_1=2$ $q=-\frac{1}{3}$ $S_n=\frac{2}{1+\frac{1}{3}}=\frac{6}{4}=\frac{3}{2}$.

645. **а)** 32, 16, 8, 4 …; $b_1=32$ $q=\frac{1}{2}$ $S_n=\frac{32}{1-\frac{1}{2}}=64$.

б) $24, -8, \frac{8}{3}, -\frac{8}{9}...; b_1 = 24\ q = -\frac{1}{3}\ S_n = \frac{24}{1+\frac{1}{3}} = 36$.

в) $27, 9, 3, 1, \frac{1}{3}\ldots; b_1 = 27\ q = \frac{1}{3}\ S_n = \frac{27}{\frac{2}{3}} = 40{,}5$.

г) $18, -6, 2, -\frac{1}{3}\ldots; b_1 = 18\ q = -\frac{1}{3}\ S_n = \frac{18}{1+\frac{1}{3}} = \frac{27}{2}$.

646. а) $2+1+\frac{1}{2}+\frac{1}{4}\ldots; b_1 = 2\ q = \frac{1}{2}\ S_n = \frac{2}{1-\frac{1}{2}} = 4$.

б) $49+7+1+\frac{1}{7}\ldots; b_1 = 49\ q = \frac{1}{7}\ S_n = \frac{49}{1-\frac{1}{7}} = 57\frac{1}{6}$.

в) $\frac{3}{2}-1+\frac{2}{3}-\frac{4}{9}\ldots; b_1 = \frac{3}{2}\ q = -\frac{2}{3}\ S_n = \frac{\frac{3}{2}}{1+\frac{2}{3}} = \frac{3}{2}\cdot\frac{3}{5} = \frac{9}{10}$.

г) $125+25+5+1\ldots; b_1 = 125\ q = \frac{1}{5}\ S_n = \frac{125}{\frac{4}{5}} = \frac{625}{4} = 156{,}25$.

647. а) $-6+\frac{2}{3}-\frac{2}{27}+\frac{2}{243}\ldots; b_1 = -6\ q = -\frac{1}{9}\ S_n = \frac{-6}{1+\frac{1}{9}} = -\frac{54}{10} = -\frac{27}{5}$.

б) $3+\sqrt{3}+1+\frac{1}{\sqrt{3}}\ldots; b_1 = 3\ q = \frac{1}{\sqrt{3}}\ S_n = \frac{3}{1-\frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3}-1} = \frac{3\sqrt{3}\left(\sqrt{3}+1\right)}{2}$.

в) $4-14+4-\frac{8}{7}\ldots; b_1 = 49\ q = -\frac{2}{7}\ S_n = \frac{49}{1+\frac{2}{7}} = \frac{343}{9} = 38\frac{1}{9}$.

г) $4+2\sqrt{2}+2+\sqrt{2}\ldots; b_1 = 4\ q = \frac{1}{\sqrt{2}}\ S_n = \frac{4}{1-\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1} = 4\sqrt{2}\left(\sqrt{2}+1\right)$.

648. а) $b_1 = -2\ b_2 = 1\ q = -\frac{1}{2}\ S_n = \frac{-2}{1+\frac{1}{2}} = -\frac{4}{3} = -1\frac{1}{3}$.

б) $b_1 = 3\ b_2 = \frac{1}{3}\ q = \frac{1}{9}\ S_n = \frac{3}{1-\frac{1}{9}} = \frac{27}{8} = 3\frac{3}{8}$.

в) $b_1 = 7\ b_2 = -1\ q = -\frac{1}{7}\ S_n = \frac{7}{1+\frac{1}{7}} = \frac{49}{8} = 6\frac{1}{8}$.

г) $b_1=-20\ b_2=4\ q=-\frac{1}{5}\ \ S_n=\frac{-20}{1+\frac{1}{5}}=-\frac{100}{6}=-\frac{50}{3}=-16\frac{2}{3}$.

649. а) $S_n=2\ b_1=3;\ S_n=\frac{b_1}{1-q}\ \ 1-q=\frac{b_1}{S_n}\ \ q=1-\frac{b_1}{S_n}=1-\frac{1}{2}=\frac{1}{2}$.

б) $S_n=-10\ b_1=-5\ q=1-\frac{5}{10}=\frac{1}{2}$.

в) $S_n=-\frac{9}{4}\ b_1=-3\ q=1-\frac{3}{9/4}=1-\frac{4}{3}=-\frac{1}{3}$.

г) $S_n=1{,}5\ b_1=2\ q=1-\frac{2}{3/2}=1-\frac{4}{3}=-\frac{1}{3}$.

650. а) $S=10,\ q=\frac{1}{10},\ S=\frac{b_1}{1-q},\ b_1=S(1-q)=10\cdot\frac{9}{10}=9$.

б) $S=-3,\ q=-\frac{1}{3},\ b_1=-3\cdot\left(1+\frac{1}{3}\right)=-4$.

в) $S=6,\ q=-\frac{1}{2},\ b_1=6\cdot\frac{3}{2}=9$. г) $S=-21,\ q=\frac{1}{7},\ b_1=-21\cdot\left(1-\frac{1}{7}\right)=-18$.

651. а) $S=15,\ q=-\frac{1}{3},n=3,\ b_1=15\cdot\frac{4}{3}=20,\ b_1=20\cdot\left(-\frac{1}{3}\right)^2=\frac{20}{9}=2\frac{2}{9}$.

б) $S=-20,b_1=-16,n=4,\ q=1-\frac{16}{20}=\frac{1}{5},\ b_4=-16\cdot\left(\frac{1}{5}\right)^3=-\frac{16}{125}$.

в) $S=20,b_1=22,n=4,\ q=1-\frac{22}{20}=-\frac{1}{10},\ b_4=22\cdot\left(-\frac{1}{10}\right)^3=-\frac{11}{500}$.

г) $S=21,\ q=\frac{2}{3},n=3,\ b_1=21\cdot\left(1-\frac{2}{3}\right)=7,\ b_3=7\cdot\left(\frac{2}{3}\right)^2=\frac{28}{9}=3\frac{1}{9}$.

652. а) $x_n=\frac{1}{2n},a=0,r=0{,}1.\ \frac{1}{2n}<0{,}1,\ 2n>10,n>5$ начиная с 6-ого.

б) $x_n=3+\frac{1}{n^2},a=3,r=0{,}2.\ 3+\frac{1}{n^2}<3{,}2,\ n^2>5,n>\sqrt{5}$ начиная с 3-его.

в) $x_n=1+\frac{2}{n^2},a=1,r=0{,}01.\ 1+\frac{2}{n^2}<1{,}01,\ \frac{2}{n^2}<0{,}01,\ \frac{1}{n^2}<\frac{1}{200},n>14$, начиная с 15-ого.

г) $x_n=-\frac{3}{n},a=0,r=0{,}1.\ -\frac{3}{n}>-\frac{1}{10},\ \frac{3}{n}<\frac{1}{10},n>30$, начиная с 31-ого.

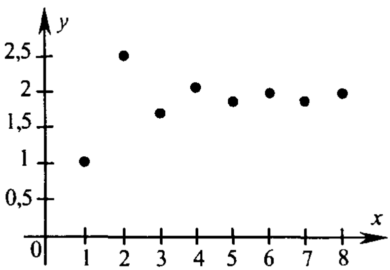
653. а) $x_n=\left(\frac{1}{3}\right)^n,a=0,\ r=\frac{1}{27};\ \left(\frac{1}{3}\right)^n<\frac{1}{27},\ \left(\frac{1}{3}\right)^n<\left(\frac{1}{3}\right)^3,n>3$ начиная с 4-ого.

б) $x_n = (-1)^n \frac{1}{2^n}$, $a = 0$, $r = \frac{1}{64}$; $\frac{1}{2^n} < \frac{1}{64}$, $\left(\frac{1}{2}\right)^n < \left(\frac{1}{2}\right)^6$, $n > 6$, начиная с 7-ого.

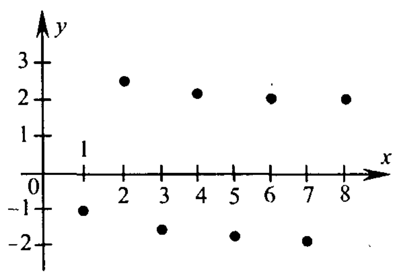
в) $x_n = 2 + \left(\frac{1}{2}\right)^n$, $a = 2$, $r = \frac{1}{128}$; $2 + \left(\frac{1}{2}\right)^n < 2\frac{1}{128}$, $\left(\frac{1}{2}\right)^n < \frac{1}{128} = \left(\frac{1}{2}\right)^7$, $n > 7$, начиная с 8-ого.

г) $x_n = 3 - \left(\frac{1}{3}\right)^n$, $a = 3$, $r = \frac{1}{81}$; $3 - \left(\frac{1}{3}\right)^n > 2\frac{80}{81}$, $\left(\frac{1}{3}\right)^n < \frac{1}{81} = \left(\frac{1}{3}\right)^4$, начиная с 5-ого.

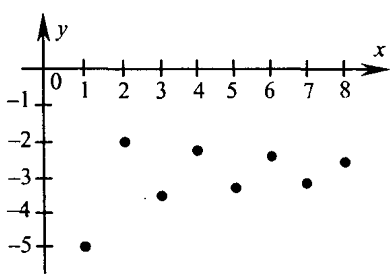
654. а) $y_n = 2 + (-1)^n \frac{1}{n}$, $y = 2$.



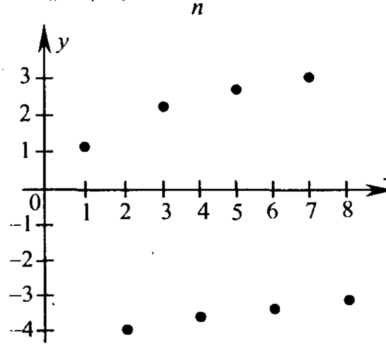
б) $y_n = (-1)^n 2 + \frac{1}{n}$.



в) $y_n = -3 + (-1)^n \frac{2}{n}$, $y = -3$.



г) $y_n = (-1)^{n+1} 3 - \frac{2}{n}$.



655. а) Нет, $y_n = \left(-\frac{1}{n}\right)^n$. **б)** нет, $y_n = n$. **в)** да, $y_n = \frac{1}{n}$. **г)** нет, $y_n = \left(-\frac{1}{n}\right)^n$.

656. а) $\operatorname{Lim}_{n\to\infty} \frac{(2n+1)(n-3)}{n^2} = \operatorname{Lim}_{n\to\infty} \frac{2n^2 - 5n - 3}{n^2} = \operatorname{Lim}_{n\to\infty}\left(2 - \frac{5}{n} - \frac{3}{n^2}\right) = 2$;

б) $\operatorname{Lim}_{n\to\infty} \frac{(3n+1)(4n-1)}{(n-1)^2} = \operatorname{Lim}_{n\to\infty}\left(\frac{12n^2 + n - 1}{n^2 - 2n + 1}\right) = \operatorname{Lim}_{n\to\infty} \frac{12 + \frac{1}{n} - \frac{1}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^2}} = 12$;

в) $\operatorname{Lim}_{n\to\infty} \frac{(3n-2)(2n+3)}{n^2} = \operatorname{Lim}_{n\to\infty} \frac{6n^2 + 5n - 6}{n^2} = \operatorname{Lim}_{n\to\infty}\left(6 + \frac{5}{n} - \frac{6}{n^2}\right) = 6$;

г) $\lim\limits_{n\to\infty}\dfrac{(1-2n)(1+n)}{(n+2)^2}=\lim\limits_{n\to\infty}\dfrac{1-n-2n^2}{n^2+4n+4}=\lim\limits_{n\to\infty}\dfrac{\frac{1}{n^2}-\frac{1}{n}-2}{1+\frac{4}{n}+\frac{4}{n^2}}=-2$.

657. а) $\lim\limits_{n\to\infty}\dfrac{(2n+1)(3n-4)-6n^2+12n}{n+5}=\lim\limits_{n\to\infty}\dfrac{7n-4}{n+5}=7$;

б) $\lim\limits_{n\to\infty}\dfrac{n^2(2n+5)-2n^3+5n^2-13}{n(n+1)(n-7)+1-n}=\lim\limits_{n\to\infty}\dfrac{10n^2-13}{n^3-6n^2-8n+1}=$

$=\lim\limits_{n\to\infty}\dfrac{10/n-13/n^3}{1-6/n-8/n^2-1/n^3}=0$;

в) $\lim\limits_{n\to\infty}\dfrac{(1-n)(n^2+1)+n^3}{n^2+2n}=\lim\limits_{n\to\infty}\dfrac{n^2+1-n}{n^2+2n}=1$;

г) $\lim\limits_{n\to\infty}\dfrac{n(7-n^2)+n^3-3n-1}{(n+1)(n+2)+2n^2+1}=\lim\limits_{n\to\infty}\dfrac{7n-n^3+n^3-3n-1}{n^2+3n+2+2n^2+1}=$

$=\lim\limits_{n\to\infty}\dfrac{4n-1}{3n^2+3n+3}=\lim\limits_{n\to\infty}\dfrac{4/n-1/n^2}{3+3/n+3/n^2}=0$.

658. а) $b_n=\dfrac{25}{3^n}$, $b_1=\dfrac{25}{3}$, $b_2=\dfrac{25}{9}$, $q=\dfrac{1}{3}$, $S_n=\dfrac{25/3}{2/3}=12{,}5$;

б) $b_n=(-1)^n\dfrac{13}{2^{n-1}}$, $b_1=-13$, $b_2=\dfrac{13}{2}$, $q=-\dfrac{1}{2}$, $S_n=\dfrac{-13}{3/2}=-\dfrac{26}{3}=-8\dfrac{2}{3}$;

в) $b_n=\dfrac{45}{3^n}$, $b_1=\dfrac{45}{3}$, $b_2=\dfrac{45}{9}$, $q=\dfrac{1}{3}$, $S_n=\dfrac{45/3}{2/3}=\dfrac{45}{2}=22{,}5$;

г) $b_n=(-1)^n\dfrac{7}{6^{n-2}}$, $b_1=-42$, $b_2=7$, $q=-\dfrac{1}{6}$, $S_n=\dfrac{-42}{1+1/6}=-36$.

659. $\begin{cases}b_1+b_3=29\\ b_2+b_4=11{,}6\end{cases}$, $\begin{cases}b_1+b_1q^2=29\\ b_1q+b_1q^3=11{,}6\end{cases}$, $\begin{cases}b_1(1+q^2)=29\\ b_1q(1+q^2)=11{,}6\end{cases}$, $q=\dfrac{2}{5}$.

$b_1=\dfrac{29}{1+\frac{4}{25}}=25$, $S_n=\dfrac{25}{1-\frac{2}{5}}=\dfrac{25\cdot 5}{3}=41\dfrac{2}{3}$.

660. а) $S_n=24,\ S_3=21.$ $\begin{cases}\dfrac{b_1}{1-q}=24\\ \dfrac{b_1(q^3-1)}{q-1}=21\end{cases}$ $q^3-1=-\dfrac{7}{8}$ $q^3=\dfrac{1}{8}$ $q=\dfrac{1}{2}$ $\begin{cases}\dfrac{b_1}{1/2}=24\\ q=\dfrac{1}{2}\end{cases}$ $b_1=12$

б) $\begin{cases}\dfrac{b_1}{1-q}=31{,}25\\ \dfrac{b_1(q^3-1)}{q-1}=31\end{cases}$ $-(q^3-1)=\dfrac{31\cdot 4}{125}=\dfrac{124}{125}$ $q=\dfrac{1}{5}$

$\frac{b_1}{1-1/5}=\frac{125}{4}$, $\frac{5b_1}{4}=\frac{125}{4}$, $b_1=25$; $q=\frac{1}{5}$, $b_1=25$, $b_7=25\cdot\left(\frac{1}{5}\right)^6=\frac{1}{625}$.

661. $\begin{cases} S_n=18 \\ b_1^2+b_1^2q^2+b_1^2q^4\ldots=162 \end{cases}$

$\begin{cases} \frac{b_1}{q-1}=-18 \\ \frac{b_1^2}{1-q^2}=162 \end{cases}$, $\begin{cases} \frac{b_1}{1-q}=18 \\ \frac{b_1^2}{1-q^2}=162 \end{cases}$, $\begin{cases} b_1=18(1-q) \\ 324\left(1-2q+q^2\right)=162-162q^2 \end{cases}$

$2q^2-4q+2=1-q^2$, $3q^2-4q+1=0$; $q=\frac{2+1}{3}=1$, $b_1=0$ не может быть;

$q=\frac{1}{3}$, $b_1=12$.

662. а) $2+4+6+\ldots+20+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots=10\frac{20+2}{2}+\frac{1/2}{1-1/2}=110+1=111$;

б) $1+3+5+\ldots+99+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\ldots=$

$=\frac{1+99}{2}\cdot\frac{99-1+2}{2}+\frac{2/5}{1-2/5}=2500+\frac{2}{3}=2500\frac{2}{3}$;

в) $21+24+27+\ldots+51+\frac{1}{3}-\frac{1}{9}+\frac{1}{27}\ldots=$

$=\frac{21+51}{2}\cdot\frac{51-21+3}{3}+\frac{1/3}{1+1/3}=12\cdot33+\frac{1}{4}=396\frac{1}{4}$;

г) $1+4+7+\ldots+100+0{,}1+0{,}01+0{,}001+\ldots=$

$=\frac{1+100}{2}\cdot\frac{100-1+3}{3}+\frac{0{,}1}{1-0{,}1}=1717+\frac{1}{9}=1717\frac{1}{9}$.

663. а) $\sin x+\sin^2 x+\sin^3 x+\ldots=\frac{\sin x}{1-\sin x}$; **б)** $\cos x-\cos^2 x+\cos^3 x+\ldots=\frac{\cos x}{1+\cos x}$;

в) $\cos^2 x+\cos^4 x+\cos^6 x+\ldots=\frac{\cos^2 x}{1-\cos^2 x}=\operatorname{ctg}^2 x$;

г) $1-\sin^3 x+\sin^6 x-\sin^9 x+\ldots=\frac{1}{1+\sin^3 x}$.

664. а) $x+x^2+x^3+\ldots=4$; $\frac{x}{1-x}=4$, $x=4-4x$, $x=\frac{4}{5}$;

б) $2x-4x^2+8x^3-16x^4+\ldots=\frac{3}{8}$; $\frac{2x}{1+2x}=\frac{3}{8}$, $2x=\frac{3}{8}+\frac{3}{4}x$, $10x=3$, $x=\frac{3}{10}$.

665. а) $\frac{1}{x}+1+x+x^2+\ldots=\frac{9}{2}$; $\frac{\frac{1}{x}}{1-x}=\frac{9}{2}\cdot2=9x-9x^2$, $9x^2-9x+2=0$, $x=\frac{1}{3}$ или $x=\frac{2}{3}$;

б) $2x+1+x^2-x^3+x^4-...=\frac{13}{6}$, $2x+1+\frac{x^2}{1+x}=\frac{13}{6}$

$2x+1+2x^2+x+x^2-\frac{13}{6}x-\frac{13}{6}=0$, $18x^2+18x-13x-7=0$, $18x^2+5x-7=0$,

$x=\frac{-5+23}{36}=\frac{1}{2}$, $x=\frac{-5-23}{36}=-\frac{7}{9}$.

666. а) $\sin x+\sin^2 x+\sin^3 x+...=5$; $\frac{\sin x}{1-\sin x}=5$, $6\sin x=5$, $x=(-1)^n \arcsin\frac{5}{6}+\pi n$.

б) $\cos x-\cos^2 x+\cos^3 x+...=2$

$\frac{\cos x}{1+\cos x}=2$, $\cos x=2+2\cos x$, $\cos x=-2$ – решений нет

в) $1+\sin^2 x+\sin^4 x+...=\frac{4}{3}$; $\frac{1}{1-\sin^2 x}=\frac{4}{3}\cdot\cos^2 x=\frac{3}{4}$, $x=\pm\frac{\pi}{6}+2\pi n$,

$x=\pm\frac{5\pi}{6}+2\pi n$;

г) $7\cos^3 x+7\cos^6 x+...=1$; $\frac{\cos^3 x}{1-\cos^3 x}=\frac{1}{7}$, $7\cos^3 x=1-\cos^3 x$;

$\cos^3 x=\frac{1}{8}$, $\cos x=\frac{1}{2}$, $x=\pm\frac{\pi}{3}+2\pi n$.

§31. Предел функции

667. а) при $x\to+\infty$ рис. 23, 25 учебника. б) при $x\to-\infty$ рис. 24, 25 учебника.
в) при $x\to\infty$ рис. 25 учебника.

668. а) $y=3$ – горизонт. асимптота на луче $(-\infty; 4]$ $\lim\limits_{x\to\infty} f(x)=3$,

$\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$ не существуют

б) $y=-2$ – горизонт. асимптота на луче $[-6; +\infty)$ $\lim\limits_{x\to\infty} f(x)$

$\lim\limits_{x\to-\infty} f(x)$ не существуют, $\lim\limits_{x\to+\infty} f(x)=-2$

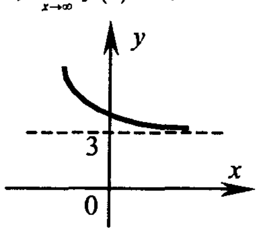
в) $y=-5$ – горизонт. асимптота на луче $(-\infty; 3]$ $\lim\limits_{x\to-\infty} f(x)=-5$,

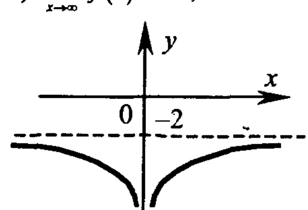
$\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to\infty} f(x)$ не существуют

г) $y=5$ – горизонт. асимптота на луче $[4; +\infty)$ $\lim\limits_{x\to\infty} f(x)$

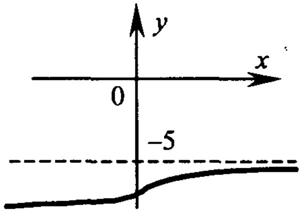
$\lim\limits_{x\to-\infty} f(x)$ не существуют, $\lim\limits_{x\to+\infty} f(x)=5$

669. а) $\lim\limits_{x\to\infty} f(x)=3$; б) $\lim\limits_{x\to\infty} f(x)=-2$;

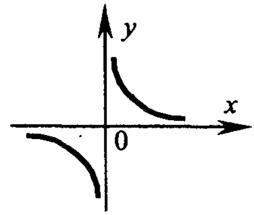




в) $\lim\limits_{x\to\infty} f(x) = -5$;



г) $\lim\limits_{x\to\infty} f(x) = 0$.



670. $\lim\limits_{x\to+\infty} f(x) = -3$;

а) $\lim\limits_{x\to\infty} 6f(x) = -18$; б) $\lim\limits_{x\to-\infty} \frac{f(x)}{3} = -1$; в) $\lim\limits_{x\to-\infty} 8f(x) = -24$; г) $\lim\limits_{x\to\infty} 0{,}4f(x) = -\frac{6}{5}$.

671. $\lim\limits_{x\to\infty} f(x) = 2$, $\lim\limits_{x\to\infty} g(x) = -3$, $\lim\limits_{x\to\infty} h(x) = 9$.

а) $\lim\limits_{x\to\infty}(f(x)+g(x)) = \lim\limits_{x\to\infty} f(x) + \lim\limits_{x\to\infty} g(x) = 2-3=-1$;

б) $\lim\limits_{x\to\infty}(f(x)-h(x)) = 2-9=-7$; в) $\lim\limits_{x\to\infty}(g(x)+h(x)) = -3+9=6$;

г) $\lim\limits_{x\to\infty}(f(x)+g(x)-h(x)) = 2-3-9=-10$.

672. $\lim\limits_{x\to\infty} f(x) = -2$, $\lim\limits_{x\to\infty} g(x) = 7$, $\lim\limits_{x\to\infty} h(x) = -2$.

а) $\lim\limits_{x\to\infty}(f(x)\cdot g(x)) = -14$; б) $\lim\limits_{x\to\infty}(f(x))^2 = 4$;

в) $\lim\limits_{x\to\infty}\left(g(x)\cdot(h(x))^2\right) = 7\cdot 4 = 28$; г) $\lim\limits_{x\to\infty}(f(x)\cdot g(x)\cdot h(x)) = 7\cdot 4 = 28$.

673. $\lim\limits_{x\to\infty} f(x) = 6$, $\lim\limits_{x\to\infty} g(x) = -10$, $\lim\limits_{x\to\infty} L(x) = 25$.

а) $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = \frac{6}{-10} = -\frac{3}{5}$; б) $\lim\limits_{x\to\infty} \frac{(g(x))^2}{L(x)} = \frac{100}{25} = 4$;

в) $\lim\limits_{x\to\infty} \frac{f(x)g(x)}{L(x)} = \frac{6\cdot(-10)}{25} = -\frac{12}{5} = -2{,}4$; г) $\lim\limits_{x\to\infty} \frac{2L(x)}{3g(x)} = \frac{50}{-30} = -\frac{5}{3}$.

674. а) $\lim\limits_{x\to\infty}\left(\frac{1}{x^2}+\frac{3}{x^3}\right) = 0$; б) $\lim\limits_{x\to\infty}\left(\frac{7}{x^5}-\frac{2}{x^3}\right) = 0$;

в) $\lim\limits_{x\to\infty}\left(\frac{2}{x^2}+\frac{8}{x^3}\right) = 0$; г) $\lim\limits_{x\to\infty}\left(\frac{9}{x^3}-\frac{5}{x^7}\right) = 0$.

675. а) $\lim\limits_{x\to\infty}\left(\frac{2}{x^9}+1\right) = 1$; б) $\lim\limits_{x\to\infty}\left(\frac{4}{x^3}-\frac{7}{x}-21\right) = -21$;

в) $\lim\limits_{x\to\infty}\left(\frac{6}{x^5}+\frac{4}{x^2}+9\right) = 9$; г) $\lim\limits_{x\to\infty}\left(\frac{7}{x^2}-7\right) = -7$.

676. а) $\lim\limits_{x\to\infty}\left(12-\frac{1}{x^2}\right)\frac{16}{x^7} = \lim\limits_{x\to\infty}\left(\frac{12\cdot 16}{x^7}-\frac{16}{x^9}\right) = 0$;

б) $\lim\limits_{x\to\infty}\left(\frac{5}{x^3}+1\right)\left(-\frac{8}{x^2}-2\right) = 1\cdot(-2) = -2$;

685. а) $\lim\limits_{x\to 0}\frac{x^2}{x^2-x}=\lim\limits_{x\to 0}\frac{x}{x-1}=0$; б) $\lim\limits_{x\to -1}\frac{x+1}{x^2+x}=\lim\limits_{x\to -1}\frac{1}{x}=-1$;

в) $\lim\limits_{x\to 3}\frac{x^2-3x}{x-3}=\lim\limits_{x\to 3}x=3$; г) $\lim\limits_{x\to 5}\frac{x+5}{x^2+5x}=\lim\limits_{x\to 5}\frac{1}{x}=\frac{1}{5}$.

686. а) $\lim\limits_{x\to 1}\frac{x^2-1}{x-1}=\lim\limits_{x\to 1}(x-1)=0$; б) $\lim\limits_{x\to -2}\frac{x^2-4}{2+x}=\lim\limits_{x\to -2}(x-2)=-4$;

в) $\lim\limits_{x\to 5}\frac{x^2-25}{x-5}=\lim\limits_{x\to 5}(x+5)=10$; г) $\lim\limits_{x\to -3}\frac{3+x}{x^2-9}=\lim\limits_{x\to -3}\frac{1}{x-3}=-\frac{1}{6}$.

687. $y=2x-3,\ x_0=3,\ y(x_0)=3$

а) $x_1=3{,}2,\ y(x_1)=3{,}4,\ \Delta y=0{,}4$; б) $x_1=2{,}9,\ y(x_1)=2{,}8,\ \Delta y=-0{,}2$;

в) $x_1=3{,}5,\ y(x_1)=4,\ \Delta y=1$; г) $x_1=2{,}5,\ y(x_1)=2,\ \Delta y=-1$.

688. $y=x^2+2x,\ x_0=-2,\ y(x_0)=0$

а) $x_1=-1{,}9,\ y(x_1)=-0{,}19,\ \Delta y=-0{,}19$; б) $x_1=-2{,}1,\ y(x_1)=0{,}21,\ \Delta y=0{,}21$;

в) $x_1=-1{,}5,\ y(x_1)=-0{,}75,\ \Delta y=-0{,}75$; г) $x_1=-2{,}5,\ y(x_1)=1{,}25,\ \Delta y=1{,}25$.

689. $y=\sin x,\ x_0=0,\ y(x_0)=0$

а) $x_1=\frac{\pi}{6},\ y(x_1)=\frac{1}{2},\ \Delta y=\frac{1}{2}$; б) $x_1=-\frac{\pi}{6},\ y(x_1)=-\frac{1}{2},\ \Delta y=-\frac{1}{2}$;

в) $x_1=\frac{\pi}{4},\ y(x_1)=\frac{\sqrt{2}}{2},\ \Delta y=\frac{\sqrt{2}}{2}$; г) $x_1=-\frac{\pi}{3},\ y(x_1)=-\frac{\sqrt{3}}{2},\ \Delta y=-\frac{\sqrt{3}}{2}$.

690. $y=2\sin x\cos x=\sin 2x,\ x_0=0,\ y(x_0)=0$

а) $x_1=-\frac{\pi}{8},\ y(x_1)=-\frac{\sqrt{2}}{2},\ \Delta y=-\frac{\sqrt{2}}{2}$; б) $x_1=\frac{\pi}{12},\ y(x_1)=\frac{1}{2},\ \Delta y=\frac{1}{2}$;

в) $x_1=\frac{\pi}{8},\ y(x_1)=\frac{\sqrt{2}}{2},\ \Delta y=\frac{\sqrt{2}}{2}$; г) $x_1=-\frac{\pi}{12},\ y(x_1)=-\frac{1}{2},\ \Delta y=-\frac{1}{2}$.

691. $y=\sqrt{x},\ x_0=1$

а) $\Delta x=0{,}44,\ y(x+\Delta x)-y(x)=1{,}2-1=0{,}2$;

б) $\Delta x=-0{,}19,\ y(x+\Delta x)-y(x)=0{,}9-1=-0{,}1$;

в) $\Delta x=0{,}21,\ y(x+\Delta x)-y(x)=1{,}1-1=0{,}1$;

г) $\Delta x=0{,}1025,\ y(x+\Delta x)-y(x)=1{,}05-1=0{,}05$.

692. а) $f(x_1)-f(x_0)=1{,}4-2=-0{,}6$; б) $f(x_1)-f(x_0)=1-6=-5$.

693. $y=4x^2-x$.

а) $x=0,\ \Delta x=0{,}5,\ y(x+\Delta x)-y(x)=\frac{1}{2}$;

б) $x=1,\ \Delta x=-0{,}1,\ y(x+\Delta x)-y(x)=2{,}34-3=-0{,}66$;

в) $x=0,\ \Delta x=-\frac{1}{2},\ y(x+\Delta x)-y(x)=\frac{3}{2}$;

г) $x=1,\ \Delta x=0{,}1,\ y(x+\Delta x)-y(x)=3{,}74-3=0{,}74$.

694. а) $f(x)=3x+5,\ f(x+\Delta x)=3x+3\Delta x+5,\ f(x+\Delta x)-f(x)=3\Delta x$;

б) $f(x)=-x^2,\ f(x+\Delta x)=-x^2-2x\Delta x-(\Delta x)^2,\ f(x+\Delta x)-f(x)=-2x\Delta x-(\Delta x)^2$;

в) $f(x)=4-2x,\ f(x+\Delta x)=4-2x-2\Delta x,\ f(x+\Delta x)-f(x)=-2\Delta x$;

г) $f(x) = 2x^2, f(x + \Delta x) = 2x^2 + 4\Delta xx + 2\Delta x, f(x + \Delta x) - f(x) = 4\Delta xx + 2\Delta x.$

695. $y = x^2 - 4x + 1, x_0 = 2, y(x_0) = -3.$

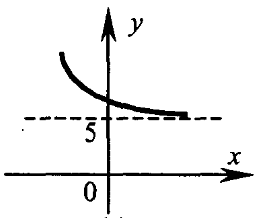
а) $x = 2{,}1, y(x) = -2{,}29, y(x) - y(x_0) = 0{,}01, \dfrac{\Delta y}{\Delta x} = \dfrac{0{,}01}{0{,}1} = 0{,}1$;

б) $x = 1{,}9, y(x) = -2{,}29, y(x) - y(x_0) = 0{,}1, \dfrac{\Delta y}{\Delta x} = -\dfrac{0{,}01}{0{,}1} = -0{,}1$;

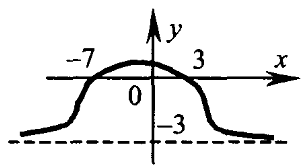
в) $x = 2{,}5, y(x) = -2{,}75, y(x) - y(x_0) = 0{,}25, \dfrac{\Delta y}{\Delta x} = \dfrac{0{,}25}{0{,}5} = 0{,}5$;

г) $x = 1{,}5, y(x) = -2{,}75, y(x) - y(x_0) = 0{,}25, \dfrac{\Delta y}{\Delta x} = -\dfrac{0{,}25}{0{,}5} = -0{,}5$.

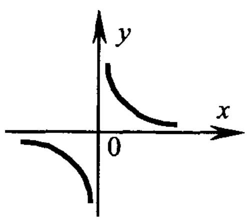
696. а) $\lim\limits_{x \to +\infty} f(x) = 5, f(x) > 0, x \in R$;



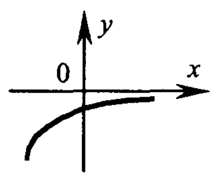
б) $\lim\limits_{x \to -\infty} f(x) = -3, f(x) \geq 0, x \in [-7; 3]$;



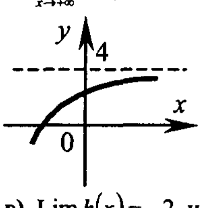
в) $\lim\limits_{x \to +\infty} f(x) = 0, f(x) > 0, x \in [0; +\infty)$;



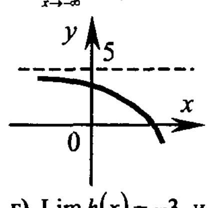
г) $\lim\limits_{x \to -\infty} f(x) = 0, f(x) < 0, x \in R$.



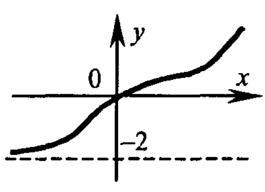
697. а) $\lim\limits_{x \to +\infty} h(x) = 4$ и функция возрастает;



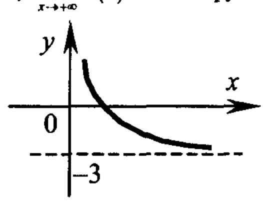
б) $\lim\limits_{x \to -\infty} h(x) = 5$ и функция убывает;



в) $\lim\limits_{x \to -\infty} h(x) = -2$ и функция возрастает;

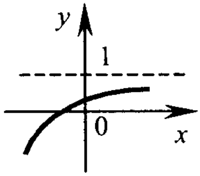


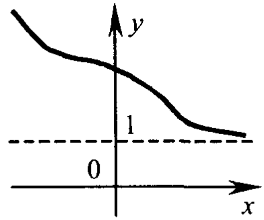
г) $\lim\limits_{x \to +\infty} h(x) = -3$ и функция убывает.



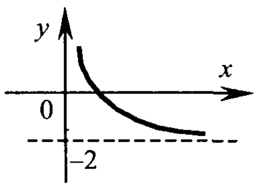
698. а) $\lim\limits_{x \to -\infty} h(x) = 1$ и функция ограничена сверху;

б) $\lim\limits_{x \to +\infty} h(x) = 1$ и функция ограничена снизу;

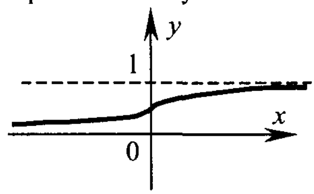




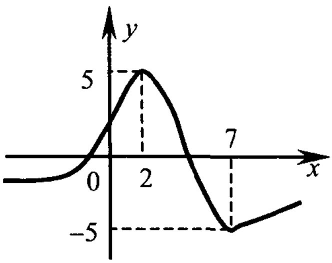
в) $\lim\limits_{x\to+\infty} h(x)=1$ и функция ограничена сверху;



г) $\lim\limits_{x\to-\infty} h(x)=1$ и функция ограничсна снизу.



699.



700. **а)** $\lim\limits_{x\to\infty}\dfrac{4x^2+9}{x^2+2}=\lim\limits_{x\to\infty}\dfrac{4+\frac{9}{x^2}}{1+\frac{2}{x^2}}=4$; **б)** $\lim\limits_{x\to\infty}\dfrac{12x^2+5x+2}{6x^2+5x-3}=\lim\limits_{x\to\infty}\dfrac{12+\frac{5}{x}+\frac{2}{x^2}}{6+\frac{5}{x}-\frac{3}{x^2}}=2$;

в) $\lim\limits_{x\to\infty}\dfrac{3x^2-8}{x^2-1}=\lim\limits_{x\to\infty}\dfrac{3-\frac{8}{x^2}}{1-\frac{1}{x^2}}=3$; **г)** $\lim\limits_{x\to\infty}\dfrac{10x^2+4x-3}{5x^2+2x+1}=\lim\limits_{x\to\infty}\dfrac{10+\frac{4}{x}-\frac{3}{x^2}}{5+\frac{2}{x}+\frac{1}{x^2}}=2$.

701. **а)** $\lim\limits_{x\to\infty}\dfrac{3x-1}{x^2+7x+5}=\lim\limits_{x\to\infty}\dfrac{\frac{3}{x}-\frac{1}{x^2}}{1+\frac{7}{x}+\frac{5}{x^2}}=0$; **б)** $\lim\limits_{x\to\infty}\dfrac{5-5x}{2x^2-9x}=\lim\limits_{x\to\infty}\dfrac{\frac{5}{x^2}-\frac{5}{x}}{2-\frac{9}{x}}=0$;

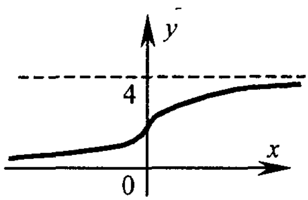
в) $\lim\limits_{x\to\infty}\dfrac{-2x-1}{3x^2-4x+1}=\lim\limits_{x\to\infty}\dfrac{-\frac{2}{x}-\frac{1}{x}}{3-\frac{4}{x}+\frac{1}{x^2}}=0$; **г)** $\lim\limits_{x\to\infty}\dfrac{4x+3}{12x^2-6x}=\lim\limits_{x\to\infty}\dfrac{\frac{4}{x}+\frac{3}{x^2}}{12-\frac{6}{x}}=0$.

702. **а)** $\lim\limits_{x\to\infty}\dfrac{4x-x^2+1}{5x^2-2x}=\lim\limits_{x\to\infty}\dfrac{\frac{4}{x}-1+\frac{1}{x^2}}{5-\frac{2}{x}}=-\dfrac{1}{5}$; **б)** $\lim\limits_{x\to\infty}\dfrac{x^3-8}{x^3+18}=\lim\limits_{x\to\infty}\dfrac{1-\frac{8}{x^3}}{1+\frac{18}{x^3}}=1$;

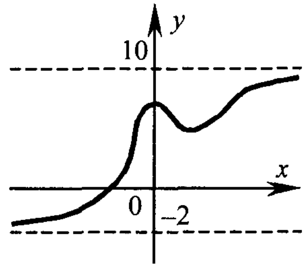
в) $\lim\limits_{x\to\infty}\dfrac{3x-2x^2+4}{3x^2+2x}=\lim\limits_{x\to\infty}\dfrac{\frac{3}{x}-2+\frac{4}{x^2}}{3+\frac{2}{x}}=-\dfrac{2}{3}$;

г) $\lim\limits_{x\to\infty}\dfrac{x^3-3x^2}{x^4+2x+1}=\lim\limits_{x\to\infty}\dfrac{1/x-3/x^2}{1+2/x^3+1/x^4}=0$.

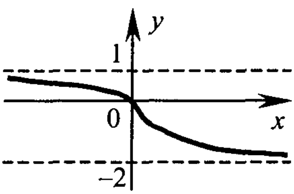
703. а) $\lim\limits_{x\to+\infty}f(x)=4$ и $\lim\limits_{x\to-\infty}f(x)=0$;



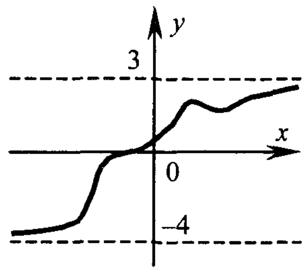
б) $\lim\limits_{x\to+\infty}f(x)=10$ и $\lim\limits_{x\to-\infty}f(x)=-2$;



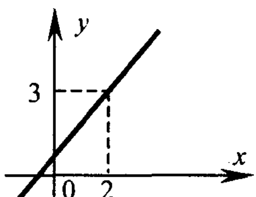
в) $\lim\limits_{x\to+\infty}f(x)=-2$ и $\lim\limits_{x\to-\infty}f(x)=1$;



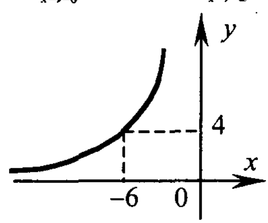
г) $\lim\limits_{x\to+\infty}f(x)=3$ и $\lim\limits_{x\to-\infty}f(x)=-4$.



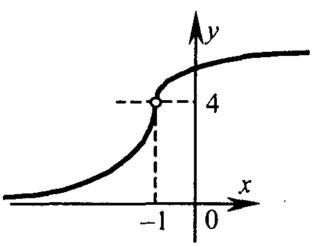
704. а) $\lim\limits_{x\to2}f(x)=3$ и $f(2)=-3$;



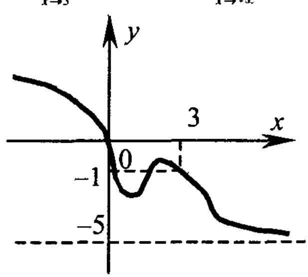
б) $\lim\limits_{x\to-6}f(x)=4$ и $\lim\limits_{x\to-\infty}f(x)=0$;



в) $\lim\limits_{x\to-1}f(x)=4$ и $f(-1)$ – не существует;



г) $\lim\limits_{x\to3}f(x)=-1$ и $\lim\limits_{x\to+\infty}f(x)=-5$.



705. а) $\lim\limits_{x\to1}\dfrac{x^2+2x-3}{x-1}=\lim\limits_{x\to1}\dfrac{(x-1)(x+3)}{x-1}=1+3=4$;

б) $\lim\limits_{x\to-2}\frac{x+2}{2x^2-x-6}=\lim\limits_{x\to-2}\frac{x+2}{(x+2)(2x-3)}=-\frac{1}{7}$;

в) $\lim\limits_{x\to-1}\frac{x+1}{x^2-2x-3}=\lim\limits_{x\to-1}\frac{x+1}{(x+1)(x-3)}=\frac{1}{-1-3}=-\frac{1}{4}$;

г) $\lim\limits_{x\to9}\frac{x^2-11x+18}{x-9}=\lim\limits_{x\to9}\frac{(x-9)(x-2)}{x-9}=7$.

706. а) $\lim\limits_{x\to-2}\frac{x+2}{x^3+8}=\lim\limits_{x\to-2}\frac{1}{x^2-2x+4}=\frac{1}{4+4+4}=\frac{1}{12}$;

б) $\lim\limits_{x\to-1}\frac{1+x^3}{1-x^2}=\lim\limits_{x\to-1}\frac{1-x+x^2}{1-x}=\frac{3}{2}$;

в) $\lim\limits_{x\to3}\frac{x-3}{x^3-27}=\lim\limits_{x\to3}\frac{x-3}{(x-3)(x^2+3x+9)}=\frac{1}{9+9+9}=\frac{1}{27}$;

г) $\lim\limits_{x\to4}\frac{16-x^2}{64-x^3}=\lim\limits_{x\to4}\frac{(4-x)(4+x)}{(4-x)(16+4x+x^2)}=\frac{8}{16+16+16}=\frac{1}{6}$.

707. а) $\lim\limits_{x\to0}\frac{\sin x}{\operatorname{tg} x}=\lim\limits_{x\to0}\cos x=1$;

б) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin 3x-\sin x}{\cos 3x+\cos x}=\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin x\cos 2x}{\cos 2x\cos x}=\lim\limits_{x\to\frac{\pi}{2}}\operatorname{tg} x$ – предела не существует;

в) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\cos x}{\operatorname{ctg} x}=\lim\limits_{x\to\frac{\pi}{2}}\sin x=1$; г) $\lim\limits_{x\to0}\frac{\cos 5x-\cos 3x}{\sin 5x+\sin 3x}=\lim\limits_{x\to0}-\frac{\sin 4x\sin x}{\sin 4x\cos x}=0$.

708. а) $f(x)=Kx+m, f(x+\Delta x)=Kx+m+K\Delta x, f(x+\Delta x)-f(x)=K\Delta x$;

б) $f(x)=ax^2, f(x+\Delta x)=ax^2+2a\Delta xx+a\Delta x^2, f(x+\Delta x)-f(x)=2a\Delta xx+a\Delta x^2$;

в) $f(x)=\frac{1}{x}, f(x+\Delta x)=\frac{1}{x+\Delta x}, f(x+\Delta x)-f(x)=\frac{x-x-\Delta x}{(x+\Delta x)x}=-\frac{\Delta x}{x^2+x\Delta x}$;

г) $f(x)=\sqrt{x}, f(x+\Delta x)=\sqrt{x+\Delta x}, f(x+\Delta x)-f(x)=\sqrt{x+\Delta x}-\sqrt{x}$.

709. а) $f(x)=Kx+m$, $\Delta f=K\Delta x$, $\frac{\Delta f}{\Delta x}=K$;

б) $f(x)=ax^2$, $\Delta f=\Delta x(2ax+a\Delta x)$, $\frac{\Delta f}{\Delta x}=2ax+a\Delta x$;

в) $f(x)=\frac{1}{x}$, $\Delta f=-\frac{\Delta x}{x^2+x\Delta x}$, $\frac{\Delta f}{\Delta x}=-\frac{1}{x^2+x\Delta x}$;

г) $f(x)=\sqrt{x}$, $\Delta f=\frac{\Delta x}{\sqrt{x+\Delta x}+\sqrt{x}}$, $\frac{\Delta f}{\Delta x}=\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}$.

710. а) $f(x)=Kx+m$, $\frac{\Delta f}{\Delta x}=K$, $\lim\limits_{\Delta x\to0}\frac{\Delta f}{\Delta x}=K$;

б) $f(x)=ax^2$, $\lim\limits_{\Delta x\to0}\frac{\Delta f}{\Delta x}=\lim\limits_{\Delta x\to0}(2ax+a\Delta x)=2ax$;

в) $f(x)=\frac{1}{x}$, $\frac{\Delta f}{\Delta x}=-\frac{1}{x^2+x\Delta x}$, $\lim\limits_{\Delta x\to0}\frac{\Delta f}{\Delta x}=-\frac{1}{x^2}$;

г) $f(x)=\sqrt{x}$, $\frac{\Delta f}{\Delta x}=\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}$, $\lim\limits_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$.

711. а) $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3}{x^2-3x}=\lim\limits_{x\to 3}\frac{x+6-9}{x(x-3)\left(\sqrt{x+6}+3\right)}=\lim\limits_{x\to 3}\frac{1}{x\left(\sqrt{x+6}+3\right)}=\frac{1}{3(3+3)}=\frac{1}{18}$;

б) $\lim\limits_{x\to\infty}\left(\sqrt{2x+3}-\sqrt{2x-7}\right)=\lim\limits_{x\to\infty}\frac{10}{\sqrt{2x+3}+\sqrt{2x-7}}=0$.

712. а) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\frac{1-\cos^2 x}{x^2(1-\cos x)}=\lim\limits_{x\to 0}\frac{\sin^2 x}{x^2(1-\cos x)}=\lim\limits_{x\to 0}\frac{1}{1-\cos x}=\frac{1}{2}$;

б) $\lim\limits_{x\to 0}\frac{\sin 7x-\sin 3x}{\sin 8x-\sin 2x}=\lim\limits_{x\to 0}\frac{\sin 2x\cos 5x}{\sin 3x\cos 5x}=\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 3x}=$

$=\lim\limits_{x\to 0}\frac{2\sin x\cos x}{3\sin x-4\sin^3 x}=\lim\limits_{x\to 0}\frac{2\cos x}{3-2\sin^2 x}=\frac{2}{3}$.

§32. Определение производной

713. а) $\Delta S=S(3)-S(2)=7-5=2$. $\Delta t=3-2=1$. $v_{ср}=\frac{\Delta S}{\Delta t}=\frac{2}{1}=2$ м/с.

б) $\Delta S=S(2{,}5)-S(2)=6-5=1$. $\Delta t=2{,}5-2=0{,}5$. $v_{ср}=\frac{\Delta S}{\Delta t}=\frac{1}{0{,}5}=2$ м/с.

в) $\Delta S=S(2{,}1)-S(2)=5{,}2-5=0{,}2$. $\Delta t=2{,}1-2=0{,}1$. $v_{ср}=\frac{\Delta S}{\Delta t}=\frac{0{,}2}{0{,}1}=2$ м/с.

г) $\Delta S=S(2{,}05)-S(2)=5{,}1-5=0{,}1$. $\Delta t=2{,}05-2=0{,}05$. $v_{ср}=\frac{\Delta S}{\Delta t}=\frac{0{,}1}{0{,}05}=2$ м/с.

В момент времени $t=2$ с мгновенная скорость $v_{мгн}=2$ м/с.

714. а) $\Delta S=S(0{,}1)-S(0)=0{,}01$. $v_{ср}=\frac{\Delta S}{\Delta t}=0{,}1$ м/с.

б) $\Delta S=S(0{,}01)-S(0)=0{,}0001$. $v_{ср}=\frac{\Delta S}{\Delta t}=0{,}01$ м/с.

в) $\Delta S=S(0{,}2)-S(0)=0{,}04$. $v_{ср}=\frac{\Delta S}{\Delta t}=0{,}2$ м/с.

г) $\Delta S=S(0{,}02)-S(0)=0{,}0004$. $v_{ср}=\frac{\Delta S}{\Delta t}=0{,}02$ м/с.

В момент времени $t=1$ с мгновенная скорость $v_{мгн}=2$ м/с.

715. а) $v_{ср}=4$ м/с; б) $v_{ср}=6$ м/с; в) $v_{ср}=3$ м/с; г) $v_{ср}=5$ м/с.

716. а) $y_1=\sqrt{3}x+K$; $y_1'(x_1)=\sqrt{3}\Rightarrow f'(x)=\sqrt{3}$; $y_2=x+K_2$; $y'_2(x_2)=1\Rightarrow f'(x_2)=1$;

б) $f'(x_1)=0$; $y=\frac{\sqrt{3}}{3}x+K$; $y'(x_2)=\frac{\sqrt{3}}{3}\Rightarrow f'(x_2)=\frac{\sqrt{3}}{3}$;

в) $f'(x_1)=0$; $f'(x_2)=-\frac{\sqrt{3}}{3}$; г) $f'(x_1)=0$; $f'(x_2)=0$.

717. а) $y = 9{,}5x - 3$. $v = 9{,}5$ м/с; б) $y = -16x + 3$. $v = -16$ м/с;
в) $y = 6{,}7x - 13$. $v = 6{,}7$ м/с; г) $y = -9x + 4$. $v = -9$ м/с.

718. а) $f(x) = x^2$. $x_0 = 2$. $f'(x_0) = 2x_0 = 4$. б) $f(x) = x^2$. $x_0 = -1$. $f'(x_0) = -2$.
в) $f(x) = x^2$. $x_0 = -2$. $f'(x_0) = -4$. г) $f(x) = x^2$. $x_0 = 9$. $f'(x_0) = 18$.

719. а) $f(x) = \frac{1}{x}$. $x_0 = 2$. $f'(x_0) = \frac{1}{4}$. б) $f(x) = \frac{1}{x}$. $x_0 = -1$. $f'(x_0) = 1$.
в) $f(x) = \frac{1}{x}$. $x_0 = 5$. $f'(x_0) = -\frac{1}{25}$. г) $f(x) = \frac{1}{x}$. $x_0 = -0{,}5$. $f'(x_0) = -4$.

720. $S(t) = t^2$. $S'(t) = 2t$. $S''(t) = 2$.
а) $t = 1$. $v = 2$ (м/с). $a = 2$ (м/с^2). б) $t = 2{,}1$. $v = 4{,}2$ (м/с), $a = 2$ (м/с^2).
в) $t = 2$. $v = 4$ (м/с). $a = 2$ (м/с^2). г) $t = 3{,}5$. $v = 7$ (м/с). $a = 2$ (м/с^2).

721. $y = x^2$. $y' = 2x$. а) $y' > 0$, при $x > 0$. б) $y' < 0$, при $x < 0$.

722. $S(t) = 2t^2 + 1$. $t_1 = 0$.
а) $t_2 = 0{,}6$. $S(t_2) - S(t_1) = 1{,}32$. $\frac{\Delta S}{\Delta t} = \frac{1{,}33}{0{,}6} = 2{,}2$ (м/с);
б) $t_2 = 0{,}2$. $S(t_2) - S(t_1) = 0{,}28$. $\frac{\Delta S}{\Delta t} = \frac{0{,}28}{0{,}2} = 1{,}4$ (м/с);
в) $t_2 = 0{,}5$. $S(t_2) - S(t_1) = 1$. $\frac{\Delta S}{\Delta t} = \frac{1}{0{,}5} = 2$ (м/с);
г) $t_2 = 0{,}1$. $S(t_2) - S(t_1) = 0{,}12$. $\frac{\Delta S}{\Delta t} = \frac{0{,}12}{0{,}1} = 1{,}2$ (м/с).

723. а) $S(t) = t^2 + 3$. $S'(t)=2t$. $v_{\text{мгн}} = 2t$ (м/с); б) $S(t) = t^2 - t$. $S'(t) = 2t - 1$. $v_{\text{мгн}} = 2t - 1$ (м/с);
в) $S(t) = t^2 + 4$. $S'(t) = 2t$. $v_{\text{мгн}} = 2t$ (м/с); г) $S(t) = t^2 - 2t$. $S'(t) = 2t - 2$. $v_{\text{мгн}} = 2t - 2$ (м/с).

724. а) $f'(-7) < f'(-2)$. б) $f'(-4) < f'(2)$. в) $f'(-9) < f'(0)$. г) $f'(-1) > f'(5)$.

725. а) $f'(x_1) > 0, f'(x_2) > 0$. $x_1 = 0, x_2 = 1$. б) $f'(x_1) < 0, f'(x_2) > 0$. $x_1 = -6, x_2 = 0$.
в) $f'(x_1) < 0, f'(x_2) < 0$. $x_1 = -5, x_2 = -4$. г) $f'(x_1) > 0, f'(x_2) < 0$. $x_1 = 2, x_2 = 4$.

726. а) $\varphi'(x) > 0$; $x = -7, -6, -5$. б) $\varphi'(x) < 0$ *и* $x > 0$; $x = 4{,}5$.
в) $\varphi'(x) < 0$; $x = -3, -2$. г) $\varphi'(x) > 0$ *и* $x < 0$; $x = -5, -6$.

727. $S(t) = t^2 + 4t$. $S'(t) = 2t + 4$. $S''(t) = 2$.
а) $t = 1$. $v = 2 + 4 = 6$ (м/с); $a = 2$ (м/с^2); б) $t = 2{,}1$. $v = 4{,}2 + 4 = 8{,}2$ (м/с); $a = 2$ (м/с^2);
в) $t = 2$. $v = 8$ (м/с); $a = 2$ (м/с^2); г) $t = 3{,}5$. $v = 7 + 4 = 11$ (м/с); $a = 2$ (м/с^2).

§33. Вычисление производных

728. а) $y = 7$; б) $y = 2x$; в) $y = -6$; г) $y = -\frac{1}{x^2}$.

729. а) $y = \cos x$; б) $y = \frac{1}{2\sqrt{x}}$; в) $y = -\sin x$; г) $y = 0$.

730. *а)* $g'(x)=\frac{1}{2\sqrt{x}}$; $g'(x_0)=\frac{1}{4}$; **б)** $g'(x)=2x$. $g'(x_0)=-14$;

в) $g'(x)=-3$; $g'(x_0)=-3$; **г)** $g'(x)=-\frac{1}{x^2}$. $g'(x_0)=-4$.

731. *а)* $g'(x)=\cos x$; $g'(x_0)=0$; **б)** $g'(x)=-\sin x$; $g'(x_0)=-\frac{1}{2}$;

в) $g'(x)=-\sin x$; $g'(x_0)=0$; **г)** $g'(x)=\cos x$; $g'(x_0)=1$.

732. *а)* $h'(x_0)=7$; **б)** $h'(x)=\frac{1}{2\sqrt{x}}$; $h'(x_0)=\frac{1}{8}$; **в)** $h'(x)=-6$; **г)** $h'(x)=\frac{1}{2\sqrt{x}}$; $h'(x_0)=\frac{1}{6}$.

733. *а)* $h'(x)=\frac{1}{x^2}$; $h'(x_0)=\frac{1}{4}$; **б)** $h'(x)=\cos x$. $h'(x_0)=0$;

в) $h'(x)=2x$; $h'(x_0)=-\frac{1}{5}$; **г)** $h'(x)=-\sin x$; $h'(x_0)=0$.

734. *а)* $f'(x)=2x$; $f'(x_0)=-8$; **б)** $f'(x)=-\frac{1}{x^2}$; $f'(x_0)=-9$;

в) $f'(x)=-\frac{1}{x^2}$; $f'(x_0)=-4$; **г)** $f'(x)=2x$; $f'(x_0)=4$.

735. *а)* $f'(x)=\cos x$. $f(x_0)=\frac{1}{2}$; **б)** $f'(x)=-\sin x$. $f'(x_0)=\frac{\sqrt{2}}{2}$;

в) $f'(x)=-\sin x$. $f'(x)=-\frac{\sqrt{3}}{2}$; **г)** $f'(x)=\cos x$; $f'(x_0)=\frac{\sqrt{3}}{2}$.

736. *а)* $f(x)=x^2+c$; **б)** $f(x)=\sin x+c$; **в)** $f'(x)=3x+c$; **г)** $f(x)=\cos x+c$.

737. **а)** $y'=2x-7$; **б)** $y'=-6x-13$; **в)** $y'=14x+3\$$ **г)** $y'=-2x+8$.

738. **а)** $y'=12+\frac{1}{2\sqrt{x}}$; **б)** $y'=\frac{1}{2\sqrt{x}}-18x$; **в)** $y'=15+\frac{1}{2\sqrt{x}}$; **г)** $y'=\frac{1}{2\sqrt{x}}-10x$.

739. **а)** $y'=-\frac{1}{x^2}+4$; **б)** $y'=-4x+\frac{1}{x^2}$; **в)** $y'=-\frac{1}{x^2}-6$; **г)** $y'=20x-\frac{1}{x^2}$.

740. **а)** $y'=\frac{3}{\sqrt{x}}-\frac{3}{x^2}$; **б)** $y'=-\frac{1}{\sqrt{x}}+\frac{1}{x^2}$; **в)** $y'=\frac{5}{\sqrt{x}}-\frac{5}{x^2}$; **г)** $y'=-\frac{4}{\sqrt{x}}+\frac{1}{x^2}$.

741. **а)** $y'=\cos x$; **б)** $y'=-4\sin x$; **в)** $y'=-\sin x$; **г)** $y'=-2\cos x$.

742. **а)** $y'=-\sin x+2$; **б)** $y'=2\cos x-6$; **в)** $y'=\cos x-3$; **г)** $y'=-3\sin x+15$.

743. **а)** $y'=5\cos x-\sin x$; **б)** $y'=3\cos x-\sin x$; **в)** $y'=\cos x+\sin x$; **г)** $y'=-2\sin x+\cos x$.

744. **а)** $y'=5x^4$; **б)** $y'=10x^9$; **в)** $y'=4x^3$; **г)** $y'=201x^{200}$.

745. **а)** $y'=3x^2+10x^4$; **б)** $y'=4x^3-9x^8$; **в)** $y'=3x^2+400x^{99}$; **г)** $y'=4x^3-63x^8$.

746. **а)** $y'=5x^4+180x^{19}$; **б)** $y'=7x^6-64x^{15}$; **в)** $y'=6x^5-130x^9$; **г)** $y'=9x^8-126x^{20}$.

747. **а)** $y'=(x^4+2)(2x)+(x^2-1)(4x^3)$; **б)** $y'=(x^2+3)6x^5+(x^6-1)2x$.
в) $y'=2x(x^4-1)+(x^2+3)4x^3$; **г)** $y'=2x(x^7+4)+(x^2-2)(7x^6)$.

748. а) $y' = \frac{x-2}{\sqrt{x}} + 2\sqrt{x} = \frac{3x-2}{\sqrt{x}}$; б) $y' = \frac{x^3+1}{2\sqrt{x}} + 3x^2\sqrt{x} = \frac{7x^3+1}{2\sqrt{x}}$;

в) $y' = \frac{4x-5}{\sqrt{x}} + 8\sqrt{x} = \frac{12x-5}{\sqrt{x}}$; г) $y' = \frac{x^4+2}{2\sqrt{x}} + 4x^3\sqrt{x} = \frac{5x^4+2}{2\sqrt{x}}$.

749. а) $y' = \sin x + x\cos x$; б) $y' = \frac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x = \frac{\cos x - 2x\sin x}{2\sqrt{x}}$;

в) $y' = \cos x - x\sin x$; г) $y' = \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x - \frac{\sin x + 2x\cos x}{2\sqrt{x}}$.

750. а) $y' = -\frac{1}{x^2}(2x-3) + \frac{1}{x} + 2 = 2 + \frac{3}{x^2}$; б) $y' = \frac{1}{x^2}(6x+1) - \frac{6}{x} + 42 = 42 + \frac{1}{x^2}$;

в) $y' = -\frac{1}{x^2}(5x-2) + \frac{5}{x} + 40 = 40 + \frac{2}{x^2}$; г) $y' = \frac{1}{x^2}(3x+2) - \frac{3}{x} + 27 = 27 + \frac{2}{x^2}$.

751. а) $y' = \frac{3x^2(2x+4) - 2x^3}{4x^2+16x+16} = \frac{4x^3+12x^2}{4x^2+16x+16} = \frac{x^3+3x^2}{x^2+4x+4} = \frac{x^2(x+3)}{(x+2)^2}$;

б) $y' = \frac{2x\left(x^2-1\right) - 2x^3}{\left(x^2-1\right)^2} = \frac{-2x}{\left(x^2-1\right)^2}$;

в) $y' = \frac{2x(3-4x) + 4x^2}{(3-4x)^2} = \frac{6x-4x^2}{(3-4x)^2} = \frac{2x(3-2x)}{(3-4x)^2}$; г) $y' = \frac{x^2+1-2x^2}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)}$.

752. а) $y' = \frac{\frac{3}{2\sqrt{x}}(2x+9) - 6\sqrt{x}}{(2x+9)^2} = \frac{6x+27-12x}{2\sqrt{x}(2x+9)^2} = \frac{27-6x}{2\sqrt{x}(2x+9)^2}$;

б) $y' = \frac{x\cos x - \sin x}{x^2}$; в) $y' = \frac{-\frac{1}{\sqrt{x}}(8-3x) + 3\left(-2\sqrt{x}\right)}{(8-3x)^2} = \frac{3x-8-6x}{\sqrt{x}(8-3x)^2} = -\frac{8+3x}{\sqrt{x}(8-3x)^2}$;

г) $y' = \frac{-x\sin x - \cos x}{x^2} = \frac{\cos x + x\sin x}{x^2}$.

753. а) $y' = \frac{1}{\cos^2 x}$; б) $y' = -\frac{1}{\sin^2 x}$; в) $y' = \frac{1}{\cos^2 x}$; г) $y' = -\frac{1}{\sin^2 x}$.

754. а) $y' = 3\cos x - \frac{1}{\sin^2 x}$; б) $y' = \frac{1}{\cos^2 x} + \sin x$;

в) $y' = -\sin x + \frac{1}{\cos^2 x}$; г) $y' = \frac{6}{\cos^2 x} - \cos x$.

755. а) $y' = \operatorname{tg} x + \frac{x}{\cos^2 x}$; б) $y' = \sin x + \frac{\sin x}{\cos^2 x}$;

в) $y' = \operatorname{ctg} x - \frac{x}{\sin^2 x}$; г) $y' = -\cos x - \frac{\cos x}{\sin^2 x}$.

756. а) $y' = 6$; $y'(x_0) = 6$; б) $y' = -11$; $y'(x_0) = -11$;
в) $y' = 5$; $y'(x_0) = 5$; г) $y' = -20$; $y'(x_0) = -20$.

757. а) $y' = 2x + 2$; $y(x_0) = 2$; б) $y' = 3x^2 - 3$; $y'(x_0) = 3 - 3 = 0$;
в) $y' = 2x + 3$; $y'(x_0) = 5$; г) $y' = 3x^2 - 18x$; $y'(x_0) = 12 - 36 = -24$.

758. а) $y' = -\frac{2}{x^2}$; $y'(x_0) = -\frac{1}{8}$; б) $y' = \frac{1}{2\sqrt{x}}$; $y'(x_0) = \frac{1}{6}$;
в) $y' = -\frac{8}{x^2}$; $y'(x_0) = -8$; г) $y' = \frac{1}{2\sqrt{x}}$; $y'(x_0) = \frac{1}{4}$.

759. а) $y' = 2\cos x$; $y'(x_0) = 0$; б) $y' = 2\sin x$; $y(x_0) = \frac{\sqrt{3}}{2}$;
в) $y' = -\cos x$; $y(x_0) = -\frac{\sqrt{3}}{2}$; г) $y' = -4\sin x$; $y'(x_0) = -2\sqrt{2}$.

760. а) $y' = \frac{1}{\cos^2 x}$; $y'(x_0) = 2$; б) $y' = \frac{-2}{\sin^2 x}$; $y' = -\frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} = -\frac{8}{3} = -2\frac{2}{3}$;
в) $y' = -\frac{1}{\sin^2 x}$; $y'(x_0) = -4$; г) $y' = \frac{-4}{\cos^2 x}$; $y'(x_0) = -4$.

761. а) $y' = \frac{x\cos x - \sin x}{x^2}$; $y'(x_0) = -\frac{4}{\pi}$; б) $y' = \frac{x - 1 - x - 1}{(x-1)^2} = -\frac{2}{(x-1)^2}$; $y'(x_0) = -2$;
в) $y' = \frac{-x\sin x - \cos x}{x^2}$; $y'(x_0) = \frac{1}{\pi^2}$; г) $y' = \frac{2x + 2 - 2x}{(x+1)^2} = \frac{2}{(x+1)^2}$; $y'(x_0) = 2$.

762. а) $g'(x) = 3x^2 + 2$; $g'(x_0) = 14$; б) $g'(x) = \frac{\sqrt{x}}{2\sqrt{x}} + \frac{\sqrt{x} + 1}{2\sqrt{x}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2\sqrt{x}}$; $g'(x_0) = \frac{3}{2}$;
в) $g'(x) = 2x - 3$; $g'(x_0) = 5$;
г) $g'(x) = -\frac{1}{x^2}\left(\frac{4}{x} - 2\right) + \frac{1}{x}\left(-\frac{4}{x^2}\right) = -\frac{4}{x^3} + \frac{2}{x^2} - \frac{4}{x^3}$; $g'(x_0) = 4 \cdot 8 + 2 \cdot 4 + 4 \cdot 8 = 72$.

763. а) $g'(x) = -\frac{6}{x^2} - 2$; $g'(x_0) = -6 - 2 = -8$; б) $g'(x) = 6x^2 - 4$; $g'(x_0) = 20$;
в) $g'(x) = 8x + \frac{12}{x^2}$; $g'(x_0) = -16 + 3 = -13$; г) $g'(x) = -3x^2 + 4x$; $g'(x_0) = -4$.

764. а) $g'(x) = 2\cos x - 4$; $g'(x_0) = -4$; б) $g'(x) = \frac{1}{3\cos^2 x}$; $g'(x_0) = \frac{4}{3}$;
в) $g'(x) = 3\sin x + 1$; $g'\left(x_0 = -\frac{1}{2}\right)$; г) $g'(x) = \frac{1}{5\sin^2 x}$; $g'(x_0) = -\frac{4}{15}$.

765. а) $h'(x) = 6x^5 - 4$; $h'(x_0) = 2$; $tg\ \alpha = 2$; б) $h'(x) = \frac{1}{2\sqrt{x}}$; $h'(x_0) = 1$; $tg\ \alpha = 1$;
в) $h'(x) = -5x^4 - 4x$; $h'(x_0) = -5 + + = -1$; $tg\ \alpha = -1$;

г) $h'(x) = -\dfrac{25}{x^2}$; $h'(x_0) = -16$; $tg\,\alpha = -16$.

766. а) $h'(x) = \sin x$; $h'(x_0) = -1$; $tg\,\alpha = -1$; б) $h'(x) = \dfrac{2}{\cos^2 x}$; $h'(x_0) = 4$; $tg\,\alpha = 4$;

в) $h'(x) = -\cos x$; $h'(x_0) = -1$; $tg\,\alpha = -1$; г) $h'(x) = \dfrac{4}{\sin^2 x}$; $h'(x_0) = 8$; $tg\,\alpha = 8$.

767. а) $f'(x_0) = 2\sin x + x^2\cos x$; $f'\left(\dfrac{\pi}{2}\right) = \pi$;

б) $f'(x) = \sqrt{3}\cos x + \dfrac{2x}{\pi} + \dfrac{1}{2}$; $f'\left(\dfrac{\pi}{6}\right) = \dfrac{3}{2} + \dfrac{1}{3} + \dfrac{1}{2} = 2\dfrac{1}{3}$;

в) $f'(x) = 1 + \cos x + x(-\sin x)$; $f'(\pi) = 1 - 1 + 0 = 0$;

г) $f'(x) = -\sqrt{3}\sin x - \dfrac{\sqrt{3}}{2} + \dfrac{2x}{\pi}$; $f'\left(\dfrac{\pi}{3}\right) = -\dfrac{3}{2} - \dfrac{\sqrt{3}}{2} + \dfrac{2}{3} = \dfrac{4-9-3\sqrt{3}}{6} = -\dfrac{5+3\sqrt{3}}{6}$.

768. а) $f(x) = x^3 + x^2 + c$; б) $f(x) = \dfrac{7}{x} + c$; в) $f(x) = x^5 - x + c$; г) $f(x) = 9\sqrt{x} + c$.

769. а) $f'(x) = \dfrac{1}{\sqrt{x}} - 5 = 2 \Rightarrow x = \dfrac{1}{49}$; б) $f'(x) = 3 - \dfrac{1}{2\sqrt{x}} = 1 \Rightarrow x = \dfrac{1}{16}$.

770. а) $y' = 7(4x-9)^6 \cdot 4 = 28(4x-9)^6$; б) $y' = 12\left(\dfrac{x}{3}+2\right)^{11} \cdot \dfrac{1}{3} = 4\left(\dfrac{x}{3}+2\right)^{11}$;

в) $y' = 9(5x+1)^8 \cdot 5 = 45(5x+1)^8$; г) $y' = 14\left(\dfrac{x}{4}-3\right)^{13} \cdot \dfrac{1}{4} = \dfrac{7}{2}\left(\dfrac{x}{4}-3\right)^{13}$.

771. а) $y' = -5(3-x)^4$; б) $y' = -240(7-24x)^9$;

в) $y' = -\dfrac{6}{5}\left(12 - \dfrac{x}{5}\right)^5$; г) $y' = -117(15-9x)^{12}$.

772. а) $y' = 3\cos(3x-9)$; б) $y' = -2\cos(7-2x)$;

в) $y' = \dfrac{1}{2}\cos\left(\dfrac{x}{2}+1\right)$; г) $y' = -3\cos(5-3x)$.

773. а) $y' = -5\sin(5x+9)$; б) $y' = 4\sin\left(\dfrac{\pi}{3} - 4x\right)$;

в) $y' = -9\sin(9x-10)$; г) $y' = \dfrac{1}{2}\sin\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$.

774. а) $y' = \dfrac{5}{\cos^2\left(5x - \dfrac{\pi}{4}\right)}$; б) $y' = \dfrac{4}{\sin^2\left(\dfrac{\pi}{6} - 4x\right)}$;

в) $y' = \dfrac{2}{\cos^2\left(2x + \dfrac{\pi}{3}\right)}$; г) $y' = \dfrac{5}{\sin^2\left(\dfrac{\pi}{4} - 5x\right)}$.

775. а) $y' = \frac{-7}{2\sqrt{15-7x}}$; б) $y' = \frac{1}{4\sqrt{42+0{,}5x}}$; в) $y' = \frac{9}{2\sqrt{4+9x}}$; г) $y' = \frac{-1}{10\sqrt{50-0{,}2x}}$.

776. а) $y' = 21(3x-2)^6$; $y'(3) = 3\cdot 7^7$; б) $y' = -35(4-5x)^6$; $y'(-2) = -35\cdot 14^6$;
в) $y' = 10(2x+3)^4$; $y'(2) = 10\cdot 7^4$; г) $y' = -21(5-3x)^6$; $y'(-1) = -21\cdot 8^6$.

777. а) $y' = 2\cos\left(2x-\frac{\pi}{3}\right)$; $y'\left(\frac{\pi}{6}\right) = 2$; б) $y' = -2\cos\left(\frac{\pi}{6}-2x\right)$; $y'\left(\frac{\pi}{12}\right) = -2$;

в) $y' = 4\sin\left(\frac{\pi}{3}-4x\right)$; $y'\left(\frac{\pi}{8}\right) = -4\sin\frac{\pi}{6} = -2$;

г) $y' = -6\sin\left(6x-\frac{\pi}{4}\right)$; $y'\left(-\frac{\pi}{12}\right) = --6\sin\left(-\frac{3\pi}{4}\right) = 6\frac{\sqrt{2}}{2} = 3\sqrt{2}$.

778. а) $y' = \frac{2}{\cos^2\left(2x+\frac{\pi}{8}\right)}$; $y'(x_0) = \frac{2}{\cos^2\frac{\pi}{4}} = 4$;

б) $y' = \frac{1}{\sin^2\left(\frac{\pi}{6}-x\right)}$; $y'(x_0) = \frac{1}{\sin^2\left(-\frac{\pi}{6}\right)} = 4$;

в) $y' = \frac{2}{\cos^2\left(3x-\frac{\pi}{4}\right)}$; $y'(x_0) = \frac{3}{\cos^2 0} = 3$;

г) $y' = \frac{1}{\sin^2\left(\frac{\pi}{3}-x\right)}$; $y'(x_0) = \frac{1}{\sin^2\frac{\pi}{6}} = 4$.

779. а) $y' = \frac{3}{\sqrt{6x-1}}$; $y'(x_0) = \frac{3}{\sqrt{29}} = \frac{3\sqrt{29}}{29}$; б) $y' = -\frac{4}{\sqrt{4-8x}}$; $y'(x_0) = -2$;

в) $y' = \frac{7}{2\sqrt{7x+4}}$; $y'(x_0) = \frac{7}{10}$; г) $y' = -\frac{9}{2\sqrt{25-9x}}$; $y'(x_0) = -\frac{9}{8} = -1\frac{1}{8}$.

780. а) $y' = 10(2x+1)^4$; $y'(x_0) = 10$; б) $y' = \frac{7}{2\sqrt{7x-3}}$; $y'(x_0) = \frac{7}{4} = 1\frac{3}{4}$;

в) $y' = \frac{-12\cdot 4}{(12x-5)^2}$; $y'(x_0) = -\frac{48}{19^2} = -\frac{48}{361}$; г) $y' = \frac{-5}{2\sqrt{11-5x}}$; $y'(x_0) = -\frac{5}{8}$.

781. а) $y' = \cos\left(3x-\frac{\pi}{4}\right)$; $y'(x_0) = 0$; б) $y' = \frac{6}{\cos^2 6x}$; $y'(x_0) = \frac{6}{\cos^2\frac{\pi}{4}} = 12$;

в) $y' = 2\sin\left(\frac{\pi}{3}-2x\right)$; $y'(x_0) = 2\sin\left(-\frac{\pi}{3}\right) = -\sqrt{3}$;

г) $y' = \frac{-1}{6\sin^2\frac{x}{3}}$; $y'(x_0) = -\frac{1}{6\sin^2\frac{\pi}{3}} = -\frac{4}{9}$.

782. а) $h'(x)=\frac{7}{2}(0,5x+3)^6$; $h'(x_0)=\frac{7}{2}=3,5$; **б)** $h'(x)=\frac{8}{\sqrt{16x+21}}$; $h'(x_0)=\frac{8}{5}=1,6$;

в) $h'(x)=\frac{-18\cdot 4}{(4x+1)^2}$; $h'(x_0)=-\frac{72}{9}=-8$; **г)** $h'(x)=-\frac{1}{\sqrt{6-2x}}$; $h'(x_0)=-\frac{1}{2}$.

783. а) $y=(x-1)(x^2+x+1)=x^3-1$; $y'=3x^2$;
б) $y=(x^2+2x+4)(x-2)=x^3-8$; $y'=3x^2$;
в) $y=(x+1)(x^2-x+1)=x^3+1$; $y'=3x^2$;
г) $y=(x^2-9x+9)(x+3)=x^3+27$; $y'=3x^2$.

784. а) $y=\frac{x^9-3}{x^3}=x^6-\frac{3}{x^3}$; $y'=6x^5+\frac{9}{x^4}$;

б) $y=\frac{x^{15}}{x^{10}+1}$; $y'=\frac{15x^{14}(x^{10}+1)-10x^9\cdot x^{15}}{(x^{10}+1)}=\frac{5x^{24}+15x^{14}}{(x^{10}+1)^2}$;

в) $y=\frac{x^5+x}{x^5-1}$; $y'=\frac{(5x^4+1)(x^5-1)-5x^4(x^5+x)}{(x^5-1)^2}=\frac{5x^9+x^5-5x^4-1-5x^9+5x^5}{(x^5-1)^2}=\frac{-4x^5-5x^4-1}{(x^5-1)^2}$;

г) $y=\frac{x^{13}}{x^4-2}$; $y'=\frac{13x^{12}(x^4-2)-4x^3\cdot x^{13}}{(x^4-2)^2}=\frac{9x^{16}-26x^{12}}{(x^4-2)^2}$.

785. а) $y=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}=\cos x$; $y'=-\sin x$; **б)** $y=2\sin\frac{x}{2}\cos\frac{x}{2}=\sin x$; $y'=\cos x$;

в) $y=\cos^2 3x+\sin^2 3x=1$; $y'=0$; **г)** $y=-\sin\frac{x}{2}\cos\frac{x}{2}=-\frac{1}{2}\sin x$; $y'=-\frac{1}{2}\cos x$.

786. а) $y=\sin 2x\cos x-\cos 2x\sin x=\sin x$; $y'=\cos x$;

б) $y=\sin\frac{x}{3}\cos\frac{2x}{3}+\cos\frac{x}{3}\sin\frac{2x}{3}=\sin x$; $y'=\cos x$;

в) $y=\cos 3x\cos 2x-\sin 3x\sin 2x=\cos x$; $y'=-\sin x$;

г) $y=\cos\frac{x}{5}\cos\frac{4x}{6}-\sin\frac{x}{5}\sin\frac{4x}{5}=\cos x$; $y'=-\sin x$.

787. а) $f'(x)=2a\cos 2x-b\sin x$;

$$\begin{cases}2a\cos\frac{\pi}{3}-b\sin\frac{\pi}{6}=2\\ 2a\cos 9\pi-b\sin\frac{9\pi}{2}=-4\end{cases};\ \begin{cases}a-\frac{b}{2}=2\\ -2a-b=-4\end{cases};\ \begin{cases}2a-b=4\\ 2a+b=4\end{cases};\ \begin{cases}a=2,\\ b-0.\end{cases}$$

б) $f'(x)=-2a\sin 2x+4b\cos 4x$;

$$\begin{cases}-2a\sin\frac{7\pi}{6}+4b\cos\frac{7\pi}{3}=4\\ -2a\sin\frac{3\pi}{2}+4b\cos 3\pi=2\end{cases};\ \begin{cases}a+2b=4\\ 2a-4b=2\end{cases};\ \begin{cases}a+2b=4\\ a-2b=1\end{cases};\ \begin{cases}a=\frac{5}{2},\\ b=\frac{3}{4}.\end{cases}$$

788. а) $f'(x)=\frac{1}{2\sqrt{x}}-1=1$; $1-4\sqrt{x}=0$; $\sqrt{x}=\frac{1}{4}$; $x=\frac{1}{16}$;

б) $f'(x)=\frac{1}{2\sqrt{x}}+3=4; 1-2\sqrt{x}=0; \sqrt{x}=\frac{1}{2}; x=\frac{1}{4}$.

789. а) $f'(x)=\cos 2x=-\frac{\sqrt{2}}{2}; x=\pm\frac{3\pi}{8}+\pi n$;

б) $f'(x)=-2\cos x\sin x=-\sin 2x=\frac{1}{2}; x=(-1)^{k+1}\frac{\pi}{12}+\frac{\pi k}{2}$.

790. а) $f(x)$ $3x^2-4x^3<0; x^2(3-4x)<0;$ $x>\frac{3}{4}$;

б) $f'(x)=x^4-5x^2+6<0; (x^2-2)(x^2-3)<0; (x-\sqrt{2})(x+\sqrt{2})(x-\sqrt{3})(x+\sqrt{3})<0$;
$x\in(-\sqrt{3};-\sqrt{2})\cup(\sqrt{2};\sqrt{3})$.

791. а) $f(x)=2\cos 2x<0;$ $2x\in\left(\frac{\pi}{2}+2\pi n;\frac{3\pi}{2}+2\pi n\right); x\in\left(\frac{\pi}{4}+\pi n;\frac{3\pi}{4}+\pi n\right)$;

б) $f(x)=4\sin x+2<0;$ $\sin x<-\frac{1}{2}; x\in\left(-\frac{5\pi}{6}+2\pi n;-\frac{\pi}{6}+2\pi n\right)$.

792. а) $g'(x)=3x^2+4x^3>0; x^2(3+4x)>0;$ $x\in\left(-\frac{3}{4};0\right)\cup(0;+\infty)$;

б) $g'(x)=\frac{20}{(2-5x)^2}>0; x\in\Re$, но $x\neq\frac{2}{5}$.

793. а) $f(x)=-2\sin 2x>0; \sin 2x<0; 2x\in(-\pi+2\pi n; 2\pi n); x\in\left(-\frac{\pi}{2}+\pi n;\pi n\right)$;

б) $g'(x)=2\sin x\cos x=\sin 2x>0; 2x\in(2\pi n;\pi+2\pi n); x\in\left(+\pi n;\frac{\pi}{2}+\pi n\right)$.

794. а) $f(x)=-2\cos x\sin x+\cos x=0; \cos x(1-2\sin x)=0; \cos x=0;$
$x=\frac{\pi}{2}+\pi n; x=\frac{\pi}{2}; \sin x=\frac{1}{2}; x=(-1)^k\frac{\pi}{6}+\pi k; x=\frac{\pi}{6}$;

б) $f(x)=2\sin x\cos x+\sin x=0; \sin x(2\cos x+1)=0; \sin x=0; x=\pi n; x=\pi;$
$\cos x=-\frac{1}{2}; x=\pm\frac{2\pi}{3}+2\pi n; x=\frac{2\pi}{3}; x=4\pi$.

795. а) $h'(x)=3x^2-6x>0; x(3x-6)>0; x\in(-\infty;0)\cup(2;+\cup)$;

б) $h'(x)=\frac{2}{\sqrt{x}}-1>0; \frac{2}{\sqrt{x}}>1; \sqrt{x}<2; x\in(0;4)$;

в) $y'(x)=3x^2-4x^3=x^2(3-4x)>0; x\in(-\infty;0)\cup\left(0;\frac{3}{4}\right)$;

г) $h'(x)=\frac{1}{\cos^2 x}-4>0; \cos^2 x<\frac{1}{4}; \cos x\in\left(-\frac{1}{2};\frac{1}{2}\right); x\in\left(\frac{\pi}{3};\frac{2\pi}{3}\right)$.

796. а) $\varphi'(x)=\cos x<0;\ x\in\left(\frac{\pi}{2}+2\pi n;\ \frac{3\pi}{2}+2\pi n\right)$;

б) $\varphi'(x)=x^4-10x^2+9<0;\ (x^2-9)(x^2-1)<0;\ (x-3)(x+3)(x-1)(x+1)<0;$

в) $\varphi'(x)=-\frac{1}{\sin^2 x}+9<0;\ \sin^2 x<\frac{1}{9};\ \sin x\in\left(-\frac{1}{3};\frac{1}{3}\right);\ x\in\left(-\arcsin\frac{1}{3}+\pi n;\ \arcsin\frac{1}{3}+\pi n\right)$;

г) $\varphi'(x)=-\frac{1}{\sqrt{4-2x}}<0;\ x<2$.

797. а) $f'(x)=x^2-2x;\ g'(x)=15x-16;$

$x^2-2x=15x-16;\ x^2-17x+16=0;\ x=\frac{17+15}{2}=16;\ x=1;$

б) $f'(x)=\frac{1}{2\sqrt{x}};\ g'(x)=\frac{1}{x^2}$;

$\frac{1}{2\sqrt{x}}=\frac{1}{x^2};\ x\neq 0;\ x^2=2\sqrt{x};\ x^4=4x;\ x(x^3-4)=0;\ x=0$ не подходит; $x=\sqrt[3]{4}$.

798. а) $f'(x)=-2\sin 2x;\ g'(x)=\cos x;$

$-2\sin 2x=\cos x;\ \cos x\,(4\sin x+1)=0;\ \cos x=0;$

$x=\frac{\pi}{2}+\pi n;\ \sin x=-\frac{1}{4};\ x=(-1)^{k+1}\arcsin\frac{1}{4}+\pi k$;

б) $f'(x)=\frac{1}{\cos^2 x};\ g'(x)=-\frac{1}{\sin^2 x}$;

$\frac{1}{\cos^2 x}=-\frac{1}{\sin^2 x}$; $\cos^2 x+\sin^2=0$ – решений нет (т.к. $\cos^2 x+\sin^2=1$).

799. а) $g'(x)=3x^2-6x;\ h'(x)-3x;$

$3x^2-6x>3x;\ x^2-3x>0;\ x\in(-\infty;0)\cup(3;+\infty);$

б) $g'(x)=3\cos\left(3x-\frac{\pi}{6}\right);\ h'(x)=6$;

$3\cos\left(3x-\frac{\pi}{6}\right)>6;\ \cos\left(3x-\frac{\pi}{6}\right)>2$ – таких значений нет;

в) $g'(x)=\frac{1}{\cos^2 x};\ h'(x)=4$;

$\frac{1}{\cos^2 x}>4;\ \cos^2 x<\frac{1}{4};\ \cos^2 x\neq 0;\ \cos x\in\left(-\frac{1}{2};\frac{1}{2}\right);\ \cos\neq 0$;

$x\in\left(\frac{\pi}{3}+\pi n;\ \frac{\pi}{2}+\pi n\right)\cup\left(\frac{\pi}{2}+\pi n;\ \frac{2\pi}{3}+\pi n\right)$;

г) $g'(x)=2\sin\left(\frac{\pi}{4}-2x\right);\ h'(x)=-\sqrt{2}$;

$2\sin\left(\frac{\pi}{4}-2x\right)>-\sqrt{2};\ \sin\left(-\frac{\pi}{4}+2x\right)<\frac{\sqrt{2}}{2};\ 2x-\frac{\pi}{4}\in\left(-\frac{5\pi}{4}+2\pi n;\ \frac{\pi}{4}+2\pi n\right);\ x\in\left(-\frac{\pi}{2}+\pi n;\ \frac{\pi}{4}+\pi n\right)$.

800. а) $f'(x) = 2\cos(2x-3)$; $g'(x) = -2\sin(2x-3)$;

$\cos(2x-3) + \sin(2x-3) = 0$; $\sin\left(2x-3+\dfrac{\pi}{4}\right) = 0$; $x = \dfrac{3}{2} - \dfrac{\pi}{8} + \dfrac{\pi n}{2}$;

б) $g'(x) = \dfrac{15}{(7-5x)^2}$; $f'(x) = -\dfrac{30}{(5x-9)^2}$;

$\dfrac{2}{(5x-9)^2} = \dfrac{1}{(7-5x)^2}$; $(5x-9)^2 + 2(5x-7)^2 = 0$ – решений нет;

в) $f'(x) = \dfrac{3}{2\sqrt{3x-10}}$; $g'(x) = \dfrac{3}{\sqrt{6x+14}}$;

$\dfrac{1}{2\sqrt{3x-10}} = \dfrac{1}{\sqrt{6x+14}}$; $2\sqrt{3x-10} = \sqrt{6x+14}$; $12x - 40 = 6x + 14$;

$6x = 54$; $x = 9$; проверка: $2\sqrt{17} = \sqrt{68}$;

г) $f'(x) = -\dfrac{1}{\sin^2 x}$; $g'(x) = 2$; $\sin^2 = -\dfrac{1}{2}$ – решений нет.

801. а) $f(x) = \sin x \cos x = \dfrac{1}{2}\sin 2x$; $f'(x) = \cos 2x$; $g'(x) = \dfrac{1}{2}$;

$\cos 2x \le \dfrac{1}{2}$; $2x \in \left(\dfrac{\pi}{3} + 2\pi n; \dfrac{5\pi}{3} + 2\pi n\right)$; $x \in \left(\dfrac{\pi}{6} + \pi n; \dfrac{5\pi}{6} + \pi n\right)$;

б) $f(x) = \sin x \cos 2x + \sin 2x \cos x = \sin 3x$; $f'(x) = 3\cos 3x$; $g'(x) = -3$;

$3\cos 3x \le -3$; $\cos 3x \le -1$; $3x = \pi + 2\pi n$; $x = \dfrac{\pi}{3} + \dfrac{2\pi}{3}$;

в) $f(x) = \sin^2 x - \cos^2 x = -\cos 2x$; $f'(x) = 2\sin 2x$; $g'(x) = -2$;

$2\sin 2x \le -2$; $\sin 2x \le -1$; $2x = \dfrac{3\pi}{2} + 2\pi n$; $x = \dfrac{3\pi}{4} + \pi n$;

г) $f'(x) = \cos x - x\sin x$; $g'(x) = \cos x$;
$\cos x - x\sin x \le \cos x$; $x\sin x \ge 0$.

I: $\begin{cases} x \ge 0, \\ \sin x \ge 0; \end{cases}$ $x \in [2\pi n; \pi + 2\pi n]$; $n = 0, 1, 2...$

II: $\begin{cases} x \le 0, \\ \sin x \le 0; \end{cases}$ $x \in [-\pi + 2\pi n; 2\pi n]$; $n = 0, -1, -2...$

802. а) $h'(x) = 2x - 3 = \operatorname{tg} 45°$; $2x - 3 = 1$; $x = 2$;

б) $h'(x) = \dfrac{-4}{(x+2)^2} = \operatorname{tg} 135°$; $4 = (x+2)^2$; $x = 0$ или $x = 4$;

в) $h'(x) = \dfrac{2}{\sqrt{2x-4}} = \operatorname{tg} 60°$; $2 = \sqrt{3(2x-4)}$; $4 = 6x - 12$; $6x = 16$; $x = \dfrac{8}{3} = 2\dfrac{2}{3}$;

г) $h'(x) = \cos\left(4x - \dfrac{\pi}{3}\right) = 0$; $4x - \dfrac{\pi}{3} = \dfrac{\pi}{2} + \pi n$; $x = \dfrac{5\pi}{24} + \dfrac{\pi n}{4}$.

803. **а)** $y_1' = 7x^6$; $y_2' = 8x^7$; $7a^6 = 8a^7$; $a = \frac{7}{8}$;

б) $y' = \frac{1}{2\sqrt{x}}$; $y' = \frac{1}{\sqrt{x+8}}$; $2\sqrt{a} = a + 8$; $a = \frac{8}{3} = 2\frac{2}{3}$.

804. **а)** $f(x) = (2x - 1)^3 + c$; **б)** $f(x) = (4 - 5x)^4 + c$.

805. **а)** $f(x) = \frac{1}{2x+3} + c$; **б)** $f(x) = \sqrt{5x-7} + c$.

806. **а)** $f(x) = -\frac{1}{3}\cos\left(3x - \frac{\pi}{3}\right) + c$; **б)** $f(x) = \frac{4}{5}\mathrm{tg}(5x-1) + c$.

807. **а)** $y = ax^2 + bx + c$; по рисунку видно, что $b = 0$, $a < 0$. $y = ax^2 + c$; $y'(x) = 2ax$;

$A(-100; 0)$; $B(100; 0)$; $y'(100) = \mathrm{tg}\,(-15°)$; $200a = -\mathrm{tg}\,15°$; $a = \frac{\mathrm{tg}15°}{200}x^2 + c$.

Подставим точку $(100; 0)$; $0 = -50\mathrm{tg} + c$; $c = 50\,\mathrm{tg}\,15°$.

Уравнение профиля моста: $y = -\frac{\mathrm{tg}15°}{200}x^2 + 50\mathrm{tg}15°$.

808. **а)** $y = 4x^2 - |a|x$; $x(4x - |a|) = 0$; $x_1 = 0$; $x_2 = \frac{|a|}{4}$. Т.к. оси параболы направлены вверх и $x_2 > x_1$, то

1) $y'(x_1) = -\mathrm{tg}\,60°$ и $y'(x_2) = \mathrm{tg}\,30°$; 2) $y'(x_1) = -\mathrm{tg}\,30°$ и $y'(x_2) = \mathrm{tg}\,30°$.

1) $y'(0) = -|a| = -\mathrm{tg}60°$; $a = \pm\sqrt{3}$; $y'\left(\frac{\pi}{4}\right) = |a| = \mathrm{tg}60°$; $a = \pm\sqrt{3}$;

2) $y'(0) = -|a| = -\mathrm{tg}\,30°$; $a = \pm\frac{\sqrt{3}}{3}$.

б) $y = x^2 + |a|x$; $y' = 2x + |a|$; $x(4x - |a|) = 0$; $x_1 = 0$; $x_2 = -|a|$. Т.к. оси параболы направлены вверх и $x_1 > x_2$, то

1) $y'(x_2) = -\mathrm{tg}\frac{3\pi}{8}$; 2) $y'(x_2) = -\mathrm{tg}\frac{\pi}{8}$; $\mathrm{tg}\frac{3\pi}{4} = \frac{2\,\mathrm{tg}\frac{3\pi}{8}}{1 - \mathrm{tg}\frac{3\pi}{8}}$; $\mathrm{tg}^2\frac{3\pi}{8} - 2\mathrm{tg}\frac{3\pi}{8} - 1 = 0$;

$\mathrm{tg}\frac{3\pi}{8} = 1 \pm \sqrt{2}$, но т.к. $0 < \frac{3\pi}{8} < \frac{\pi}{2}$, то $\mathrm{tg}\frac{3\pi}{8} = 1 + \sqrt{2}$; $\mathrm{tg}\frac{\pi}{4} = \frac{2\mathrm{tg}\frac{\pi}{8}}{1 - \mathrm{tg}^2\frac{\pi}{8}}$; $\mathrm{tg}^2\frac{\pi}{8} + 2\mathrm{tg}\frac{\pi}{8} - 1 = 0$;

$\mathrm{tg}\frac{\pi}{8} = -1 \pm \sqrt{2}$, но т.к. $0 < \frac{\pi}{8} < \frac{\pi}{2}$, то $\mathrm{tg}\frac{\pi}{8} = -1 + \sqrt{2}$.

1) $y'(x_2) = -1 - \sqrt{2}$; $-|a| = -1 - \sqrt{2}$; $a = \pm(1 + \sqrt{2})$; 2) $y'(x_2) = 1 - \sqrt{2}$; $a = \pm(\sqrt{2} - 1)$.

§ 34. Уравнение касательной к графику функции

809. а) a: $\operatorname{tg}\alpha = 0$. b: $\operatorname{tg}\alpha < 0$. c: $\operatorname{tg}\alpha > 0$. б) a: $\operatorname{tg}\alpha < 0$. b: $\operatorname{tg}\alpha < 0$. c: $\operatorname{tg}\alpha > 0$.

810. а) $y' = 0$ при $x = 0$, $x = 3{,}5$; y' не существует при $x = -1$;
б) $y' = 0$ при $x = -4$, $x = -1{,}5$; y' не существует при $x = 4$;
в) $y' = 0$ при $x = -4$; y' не существует при $x = -2$;
г) $y' \neq 0$ при $x \in R$.

811. а) $f(x) = 4 + x^2$, $a = 2$. $f(x) = 2x$; $f'(a) = 4 \Rightarrow$ острый;
б) $f(x)$ $f(x)=1-\frac{1}{x}$; $a=3$; $f'(x)=\frac{1}{x^2}$; $f'(a)=\frac{1}{9}$ $\Rightarrow$ острый;
в) $f(x) = (1-x)^3$, $a = -3$. $f'(x) = -3(1-x)^2$; $f'(a) = -48 \Rightarrow$ тупой;
г) $f(x) = 2x - x^2$, $a = 1$. $f'(x) = 2 - 2x$; $f'(a) = 0 \Rightarrow$ острый.

812. $y = 1 - x^2$; $y'(x) = -2x$.
а) $A(0;\ 1)$; $y'(0) = 0 \Rightarrow \operatorname{tg}\alpha = 0$. б) $B(2;\ -3)$; $y'(2) = -4 \Rightarrow \operatorname{tg}\alpha = -4$.
в) $C\left(\frac{1}{2};\frac{3}{4}\right)$; $y'\left(\frac{1}{2}\right)=-1 \Rightarrow \operatorname{tg}\alpha = -1$; г) $D(-1;\ 0)$; $y'(-1) = 2 \Rightarrow \operatorname{tg}\alpha = 2$.

813. а) $f(x) = \frac{1}{2}x^2$, $a = 1$. $f'(x) = x$; $f'(a) = 1$, $\operatorname{tg}\alpha = 1$.
б) $f(x) = -2x^3$, $a = 2$. $f'(x) = -6x^2$; $f'(a) = -24$, $\operatorname{tg}\alpha = -24$.
в) $f(x) = 0{,}25x^4$, $a = -1$. $f'(x) = x^3$; $f(a) = -1$, $\operatorname{tg}\alpha = -1$.
г) $f(x) = -x^5$, $a = 1$. $f'(x) = -5x^4$; $f(a) = -5$. $\operatorname{tg}\alpha = -5$.

814. а) $f'(x) = 3x^2 - 4x$; $f'(a) = 7$; $\operatorname{tg}\alpha = 7$;
б) $f'(x)=\frac{x+3-x+1}{(x+3)^2}=\frac{4}{(x+3)^2}$; $f'(a)=\frac{1}{4}$; $\operatorname{tg}\alpha=\frac{1}{4}$;
в) $f'(x) = 4x^3 - 21x^3 + 12$; $f'(a) = 12$; $\operatorname{tg}\alpha = 12$;
г) $f'(x)=\frac{2x+2-2x+1}{(x+1)^2}=\frac{3}{(x+1)^2}$; $f'(a)=\frac{3}{4}$; $\operatorname{tg}\alpha=\frac{3}{4}$.

815. а) $f'(x)=\frac{1}{2\sqrt{x-7}}$; $f'(a)=\frac{1}{2}$; $\operatorname{tg}\alpha=\frac{1}{2}$;
б) $f'(x)=-\frac{5}{2\sqrt{4-5x}}$; $f'(a)=-\frac{5}{4}$; $\operatorname{tg}\alpha=-\frac{5}{4}$;
в) $f'(x)=\frac{1}{\sqrt{x+10}}$; $f'(a)=\frac{1}{\sqrt{5}}$; $\operatorname{tg}\alpha=\frac{\sqrt{5}}{5}$;
г) $f'(x)=-\frac{1}{4\sqrt{3{,}5-0{,}5x}}$; $f'(a)=-\frac{1}{8}$; $\operatorname{tg}\alpha=-\frac{1}{8}$.

816. а) $f'(x) = \cos x$; $f'(a) = 1$; $\operatorname{tg}\alpha = 1$; б) $f'(x)=\frac{2}{\cos^2 2x}$; $f'(a)=4$; $\operatorname{tg}\alpha=4$;
в) $f'(x) = -3\sin 3x$; $f'(a) = 3$; $\operatorname{tg}\alpha = 3$; г) $f'(x)=-\frac{1}{\sin^2 x}$; $f'(a)=-\frac{4}{3}$; $\operatorname{tg}\alpha=-\frac{4}{3}$.

817. а) $f'(x)=2x; f'(a)=1;\ \alpha=\frac{\pi}{4}$; б) $f'(x)=-9x^2; f'(a)=-1;\ \alpha=\frac{3\pi}{4}$;

в) $f'(x)=x^4; f'(a)=1;\ \alpha=\frac{\pi}{4}$; г) $f'(x)=-x^3; f'(a)=0; \alpha=0$.

818. а) $f'(x)=3x^2-6x+2; f'(a)=-1;\ \alpha=\frac{3\pi}{4}$; б) $f'(a)=-21x^2+20x+1; f'(a)=1;\ \alpha=\frac{\pi}{4}$.

819. а) $f'(x)=\frac{6-4x+4x-2}{(3-2x)^2}=\frac{4}{(3-2x)^2}; f'(a)=\frac{1}{4}; \alpha=\operatorname{arctg}\frac{1}{4}$;

б) $f'(x)=\frac{x-2-x+1}{(x-2)^2}=-\frac{1}{(x-2)^2}; f'(a)=-1; \alpha=\frac{3\pi}{4}$.

820. а) $f'(x)=\frac{3}{\sqrt{6x+7}}; f'=\frac{1}{\sqrt{3}}; \alpha=\frac{\pi}{4}$; б) $f'(x)=-\frac{1}{\sqrt{5-2x}}; f'(a)=-1; \alpha=\frac{3\pi}{4}$.

821. а) $f'(x)=-\frac{\sqrt{3}}{3}\sin\frac{x}{3}; f'(a)=-\frac{\sqrt{3}}{3}; \alpha=\frac{5\pi}{6}$; б) $f'(x)=\cos 2x;\ f'(a)=-1;\ \alpha=\frac{3\pi}{4}$.

822. а) $f'(x)=-\frac{2}{3\cos^2 x}+\frac{1}{3}\cos\frac{x}{3}; f'(a)=-\frac{2}{3}-\frac{1}{3}=-1; \alpha=\frac{3\pi}{4}$;

б) $f'(x)=-\sin x-\frac{\sqrt{3}}{8\sin^2\frac{x}{2}}; f'(a)=-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}=-\sqrt{3}; \alpha=\frac{2\pi}{3}$.

823. а) $f'(x)=2x; f'(a)=6; f(a)=9; y=9+6(x-3)=6x-9$;
б) $f'(x)=-1-3x^2; f'(a)=-1; f(a)=2; y=2-x$;
в) $f'(x)=3x^2; f'(a)=3; f(a)=1; y=1+3(x-1)=3x-2$;
г) $f'(x)=2x-3; f'(a)=-5; f(a)=1+3+5=9; y=9-5(x+1)=-5x+4$.

824. а) $f'(x)=\frac{9-3x+3x-2}{(3-x)^2}=\frac{7}{(x-3)^2}; f'(a)=7; f(a)=4; y=4+7(x-2)=7x-10$;

б) $f'(x)=\frac{10-2x+2x-5}{(5-x)^2}=\frac{5}{(5-x)^2}; f'(a)=5; f(a)=3; y=3+5(x-4)=5x-17$.

825. а) $f'(x)=-\frac{3}{(x+2)^4}; f'(a)=-3; f(a)=-1; y=-1-3(x+3)=-3x-10$;

б) $f'(x)=\frac{-4}{4(2x-1)^3}=-\frac{1}{(2x-1)^3}; f'(a)=-1; f(a)=\frac{1}{4}; y=\frac{1}{4}-x+1=-x+\frac{5}{4}$.

826. а) $f'(x)=-\frac{3}{\sqrt{3x-5}}; f'(a)=3; f(a)=2; y=2+3(x-2)=3x-4$;

б) $f'(x)=-\frac{1}{\sqrt{7-2x}}; f'(a)=-1; f(a); y=1-x+3=-x+4$.

827. а) $f'(x)=-\frac{1}{3}\sin\frac{x}{3}; f'(a)=0; f(a)=1; y=1$;

б) $f'(\ )=-\dfrac{2}{\sin^2 2x}$; $f'(a)=-2$; $f(a)=0$; $y=-2\left(x-\dfrac{\pi}{4}\right)=\dfrac{\pi}{2}-2x$;

в) $f'(x)=2\cos 2x$; $f'(a)=0$; $f(a)=1$; $y=1$;

г) $f'(x)=\dfrac{2}{3\cos^2\dfrac{x}{3}}$; $f'(a)=\dfrac{3}{2}$; $f(a)=0$; $y=\dfrac{2}{3}x$.

828. $y=9-x^2$; $9-x^2=0$; $x=\pm3$; $y'=-2x$; $y'(3)=-6$; $y'(-3)=6$;
$y=6(x+3)=6x+18$; $y=-6(x-3)=18-6x$.

829. $y=x^2-3x$; $x^2-3x-4=0$; $x=4$; $x=-1$; $y'=2x-3$; $y'(4)=8-3=5$; $y'(-1)=-5$;
$y=4+5(x-4)=5x-16$; $y=4-5(x+4)=-5x-1$.

830. $y=3x^3-4x^2+1$; $y'=9x^2-8x=1$; $9x^2-8x-1=0$; $x=\dfrac{4\pm5}{9}$; $x=1$; $x=\dfrac{1}{9}$

$y(1)=0$; $y\left(-\dfrac{1}{9}\right)=-\dfrac{1}{243}-\dfrac{4}{81}+1=\dfrac{230}{243}$; $y=x-1$; $y=\dfrac{230}{243}+\left(x+\dfrac{1}{9}\right)=x+\dfrac{257}{243}$.

831. $y=x^2$; $y'=2x$; $y=x_0^2+2x_0(x-x_0)=2x_0x-x_0^2$.

а) $y=2x+1$; $x_0=1$; $y_0=1$;

б) $y=-\dfrac{1}{2}x+5$; $x_0=-\dfrac{1}{4}$; $y_0=\dfrac{1}{16}$;

в) $\dfrac{3}{4}x-2=y$; $x_0=\dfrac{3}{8}$; $y_0=\dfrac{9}{64}$;

г) $y=-x+5$; $x_0=-\dfrac{1}{2}$; $y_0=\dfrac{1}{4}$.

832. а) $f'(x)=x^2-6x+10$; $y=\dfrac{x_0^3}{3}-3x_0^2+10x_0-4+\left(x_0^2-6x_0+10\right)(x-x_0)$;

$x_0^2-6x_0+10=1$; $x_0^2-6x_0+9=0$; $x_0=3$;

б) $f'(x)=x^3-2x$; $y=\dfrac{x_0^4}{4}-x_0^2+8+\left(x_0^3-2x_0\right)(x-x_0)$; $x_0^3-2x_0^2=0$; $x_0=0$; $x_0^2=2$; $x_0=\pm\sqrt{2}$;

в) $f'(x)=x^2-2x+2$; $y=\dfrac{x_0^3}{3}-x_0^2+2x_0-7+\left(x_0^2-2x_0+2\right)(x-x_0)$; $x_0^2-2x_0+2=1$; $x_0=1$;

г) $f'(x)=5x^3-3x^2$; $x_0=0$; $5x_0^3-3=0$; $x_0=\dfrac{3}{5}$.

833. а) $f'(x)=\cos x$; $\cos x_0=-1$; $x_0=\pi+2\pi n$; б) $f'(x)=-3\sin 3x$; $\sin 3x_0=0$; $x_0=\dfrac{\pi n}{3}$;

в) $f'(x)=\dfrac{1}{\cos^2 x}$; $\dfrac{1}{\cos^2 x}=1$; $\cos x=\pm1$; $x=\pi n$;

г) $f'(x)=\dfrac{1}{2}\cos\dfrac{x}{2}$; $\cos\dfrac{x}{2}=0$; $x_0=\pi+2\pi n$.

834. $y=\dfrac{x^3}{3}-2$; $y'=x^2$; $y=\dfrac{x_0^3}{3}-2+x_0^2(x-x_0)=x_0x-\dfrac{2}{3}x_0^3-2$;

а) $y=x-3$; $x_0^2=1$; $x_0=\pm1$; $y=x-\dfrac{2}{3}-2=x-\dfrac{8}{3}$; $x+\dfrac{2}{3}-2=x-1\dfrac{1}{3}$;

б) $y = 9x - 5;\ x_0^2 = \pm 3$; $y = 9x - 18 - 2 = 9x - 20$; $y = 9x + 18 - 2 = 9x + 16$.

835. а) $y' = x^2 + 5x; x_0^2 + 5x_0 = -1; x_0^2 + 5x_0 + 1 = 0; x_0 = \dfrac{-5 \pm \sqrt{21}}{2}$;

$$y = \left(\frac{-5 \pm \sqrt{21}}{2}\right)^3 \cdot \frac{1}{3} + \frac{5}{2}\left(\frac{-5 \pm \sqrt{21}}{2}\right)^2 + 8 + \left(\left(\frac{-5 \pm \sqrt{21}}{2}\right)^2 + 5\left(\frac{-5 \pm \sqrt{21}}{2}\right)\right)\left(x - \frac{\pm\sqrt{21} - 5}{2}\right);$$

б) $y' = x^2 + 2x - 1;\ x_0^2 + 2x_0 = 0; x_0 = 0, x_0 = -2; y(0) = 0, y(-2) = \dfrac{10}{3}$;

$y = x';\ y = -\dfrac{8}{3} + 4 + 2 - (x + 2) = -x + \dfrac{4}{3}$; $y = 0 - (x - 0) = -x$.

836. а) $y' = \dfrac{3x - 9 - 3x - 1}{(x-3)^2} = -\dfrac{10}{(x-3)^2}; -\dfrac{10}{(x_0 - 3)^2} = -1; (x_0 - 3)^2 = 10; x_0 = \pm\sqrt{10} + 3$;

$y(x_0) = \dfrac{3(\pm\sqrt{10} + 3) + 1}{\pm\sqrt{10} + 3 - 3} = \dfrac{\pm 3\sqrt{10} + 10}{\pm 10} = 3 \pm \sqrt{10}; y = 3 \pm \sqrt{10} - (x \mp \sqrt{10} - 3)$;

$y_1 = 3 + \sqrt{10} - x + \sqrt{10} + 3 = 6 + 2\sqrt{10} - x; y_2 = 3 - \sqrt{10} - x - \sqrt{10} + 3 = 6 - 2\sqrt{10} - x$;

б) $y' = -\dfrac{1}{(x+8)^2}; -\dfrac{1}{(x+8)^2} = -1$; $x_0 + 8 = \pm 1$; $x_0 = -9$, $x_0 = -7$; $y(-9) = 0$, $y(-7) = 2$;

$y = 2 - (x + 7) = -x - 5, y = -x - 9$.

837. а) $y' = -\dfrac{2}{\sqrt{x+7}}; -\dfrac{2}{\sqrt{x+7}} = -1$; $x_0 = -3$; $y(-3) = -8$; $y = -8 - (x + 3) = -x - 11$;

б) $y' = -\dfrac{1}{\sqrt{1-2x}}; -\dfrac{1}{\sqrt{1-2x}} = -1$; $x_0 = 0$; $y(0) = 1$; $y = 1 - x$.

838. а) $0{,}998^5$: $y = x^5 = f(x)$; $x = 0{,}998$; $a = 1$; $f(a) = 1$; $f'(x) = 5x^4$; $f'(a) = f'(1) = 5$;
$0{,}998^5 \approx 1 - 5 \cdot 0{,}002 = 0{,}99$;
б) $1{,}03^7 \approx 1 + 7 \cdot 0{,}03 = 1{,}21$.

839. а) $\sqrt{1{,}05} \approx 1 + \dfrac{1}{2} \cdot 0{,}05 = 1{,}025$; **б)** $\sqrt{3{,}99} \approx 2 - \dfrac{1}{2\sqrt{4}} \cdot 0{,}01 = 1{,}9975$.

840. а) $f'(x) = \sqrt{3}x^2 - 3\sqrt{3} = \sqrt{3}; x^2 = 4; x = \pm 2; f(-2) = \dfrac{8}{\sqrt{3}} - 6\sqrt{3} = -\dfrac{10}{\sqrt{3}}; f(-2) = \dfrac{10}{\sqrt{3}}$;

$y = \pm\dfrac{10}{\sqrt{3}} + \sqrt{3}(x \pm 2); y_1 = \sqrt{3}x - 2\sqrt{3} + \dfrac{10}{\sqrt{3}} = \sqrt{3}x + \dfrac{4}{\sqrt{3}}; y_2 = \sqrt{3}x + 2\sqrt{3} - \dfrac{10}{\sqrt{3}} = \sqrt{3}x - \dfrac{4}{\sqrt{3}}$.

б) $f'(x) = \dfrac{4}{\sqrt{3}} - \sqrt{3}x^2 = \dfrac{\sqrt{3}}{3}; x^2 = \dfrac{4}{3} - \dfrac{1}{3}; x = \pm 1; f(1) = \sqrt{3}; f(-1) = -\sqrt{3}$;

$y = \sqrt{3} + \dfrac{\sqrt{3}}{3}(x - 1) = \dfrac{\sqrt{3}}{3}x + \dfrac{2\sqrt{3}}{3}; y = -\sqrt{3} + \dfrac{\sqrt{3}}{3}(x + 1) = \dfrac{\sqrt{3}}{3}x - \dfrac{2\sqrt{3}}{3}$.

841. а) $y' = \dfrac{3x + 24 - 3x + 1}{(x+8)^2} = \dfrac{25}{(x+8)^2}$; $25 = (x + 8)^2$; $x + 8 = \pm 5$. $x = -3$. $x = -13$;

$y(-3)=\frac{-9-1}{5}=-2$; $y(-13)=\frac{-39-1}{-5}=8$; $y_1=-2+(x+3)=x+1$.

$y_2=8+(x+13)=x+21$. $(-1;0)$; $(-21;0)$.

б) $x-5=\pm3$; $x=8$; $x=2$; $y(8)=\frac{12}{3}=4$; $y(2)=\frac{6}{-3}=-2$.

$y_1=4-(x-8)=-x+12$. $y_2=-2-(x-2)=-x$; $(0;0)$; $(12;0)$.

842. Условие приведено не полностью.

843. $y=x_0^2+1+2x_0(x-x_0)=2x_0x-x_0^2+1$.

а) $A(-1;-2)$; $-2=-2x_0-x_0^2+1$; $x_0^2+2x_0-3=0$; $x_0=-3$, $x_0=1$; $y_1=-6x-8$; $y_2=2x$.

б) $A(0;0)$; $0=-x_0^2+1$; $x_0=\pm1$; $y=2x$; $y=-2x$.

в) $A(0;-3)$; $-3=-x_0^2+1$; $x_0=\pm2$; $y=4x-3$; $y=-4x-3$.

г) $A(-1;1)$; $1=2x_0-x_0^2+1$; $x_0=0$; $x_0=-2$; $y=1$; $y=-4x-3$.

844. а) $f'(x)=-2x-7$; $y=-x_0^2-7x_0+8+(-2x_0-7)(x-x_0)=(-2x_0-7)x+x_0^2+8$;
$x_0=0$, $x_0=2$; $f(0)=8$, $f(2)=-10$; $y=-10-11(x-2)=-11x+12$; $y=8-7x$;

б) $9=x_0^2+8$; $x=\pm1$; $f(1)=-1-7+8=0$; $f(-1)=-1+7+8=14$;
$y=9(x-1)=-9x+9$; $y=14-5(x+1)=-5x+9$.

845. а) $f'(x)=\frac{1}{2\sqrt{3-x}}$; $3=\sqrt{3-x_0}-\frac{1}{2\sqrt{3-x_0}}(-2-x_0)$; $3=\frac{6-2x_0+2+x_0}{2\sqrt{3-x_0}}$; $6\sqrt{3-x_0}=8-x_0$;

$108-36x_0=64-16x_0+x_0^2$; $x_0^2+20x_0^2-44=0$; $x_0=-22$; $x_0=2$; $f(-22)=5$; $f(2)=1$;

$y=5-\frac{1}{10}(x+22)=-\frac{x}{10}-\frac{11}{5}+5=-\frac{x}{10}+\frac{14}{5}$; $y=1-\frac{1}{2}(x-2)=-\frac{1}{2}x+2$;

б) $f'(x)=-\frac{1}{2\sqrt{3-x}}$; $0=\sqrt{3-x_0}-\frac{1}{2\sqrt{3-x_0}}(4-x_0)$; $6-2x_0-4+x_0=0$; $x_0=2$;

$f(2)=1$; $y=1-\frac{1}{2}(x-2)=-\frac{1}{2}x+2$.

846. а) $f'(x)=\frac{1}{2\sqrt{4x-3}}$; $y=\frac{2}{\sqrt{4x_0-3}}(x-x_0)+\sqrt{4x_0-3}$; $3\sqrt{4x_0-3}=2(2-x_0)+4x_0-3$;

$36x_0-27=4x_0^2+4x_0+1$; $4x_0-32x_0+28=0$; $x_0^2-8x_0+7=0$;

$x_0=7$; $x_0=1$; $y=\frac{2}{5}(x-7)+5=\frac{2}{5}x+\frac{11}{5}$; $y=2x-2+1=2x-1$.

б) $f'(x)=\frac{1}{\sqrt{2x+1}}$; $y=\frac{x-x_0}{\sqrt{2x_0+1}}+\sqrt{2x_0+1}$; $2\sqrt{2x_0+1}=1-x_0+2x_0+2x_0+1$;

$8x_0+4=x_0^2+4x_0+4$; $x_0^2-4x_0=0$; $x_0=0$, $x_0=4$; $y=x+1$; $y=\frac{x-4}{3}+3=\frac{x}{3}+\frac{5}{3}$.

847. а) $y'=-7\sin 7x-7\sin x$; $y\left(\frac{\pi}{6}\right)=-\frac{7\sqrt{3}}{2}+\frac{7\sqrt{3}}{2}=0$; $y'\left(\frac{\pi}{6}\right)=\frac{7}{2}-\frac{7}{2}=0$;

$y=(-7\sin 7a-7\sin a)(x-a)+\cos 7a+7\cos a$; $-7\sin 7a-7\sin a=0$; $\sin 4a\cos 3a=0$;

$\sin 4a=0$; $a=\frac{n}{4}$; $\cos 3a=0$; $3a=\frac{\pi}{2}+\pi n$; $a=\frac{\pi}{6}+\frac{\pi n}{3}$;

б) $y_1=2-14\sin 3x$; $y_2=6\sin 7x$; $y_1'=-42\cos 3x$; $y_2'=42\cos 7x$;

$y=2-14\sin 3a-42\cos 3a(x-a)$, $y=6\sin 7a+42\cos 7a(x-a)$; $42\cos 7a=-42\cos 3a$;

$\cos 7a+\cos 3a=0$; $\cos 5a\cos 2a=0$; $5a=\frac{\pi}{2}+\pi n$; $a=\frac{\pi}{10}+\frac{\pi n}{5}$; $a=\frac{\pi}{4}+\frac{\pi n}{2}$.

848. а) $y=\frac{1}{x^2}$; $\left|\frac{1}{2}xy\right|=2{,}25$; $x>0$; $y'=-\frac{2}{x^3}$; $y=\frac{2}{x_0^3}(x_0-x)+\frac{1}{x_0^2}=\frac{3}{x_0^2}-\frac{2x}{x_0^3}$;

$$\begin{cases} y=\frac{3}{x_0^2}-\frac{2x}{x_0^3}; & \text{при } x=0\ y=\frac{3}{x_0^2} \\ xy=\frac{9}{2} & \text{при } y=0\ x=\frac{3x_0}{2} \end{cases};\ \frac{3}{x_0^2}\cdot\frac{3x_0}{2}=\frac{9}{2};\ \frac{1}{x_0}=1;\ x_0=1;\ y=3-2x;$$

б) $y=\frac{1}{x^2}$, $x<0$, $-\frac{1}{2}xy=\frac{9}{8}$; $y=\frac{3}{x_0^2}-\frac{2x}{x_0^3}$; при $x=0$ $y=\frac{3}{x_0^2}$, при $y=0$ $x=\frac{3x_0}{2}$;

$\frac{3\cdot 3}{2}\cdot\frac{1}{x_0}=-\frac{9}{4}.\frac{1}{x_0}=-\frac{1}{2}$; $x_0=-2$; $y=\frac{3}{4}+\frac{2x}{8}=\frac{3+x}{4}$.

849. а) $y=x^3$, $x>0$, $\left|\frac{1}{2}xy\right|=\frac{2}{3}$; т.к. это кубическая парабола, то $xy=-\frac{4}{3}$.

$y=x_0^2+3x_0^2(x-x_0)=-2x_0^3+3x_0^2$; при $x=0$, $y=-2x_0^3$, при $y=0$, $x=\frac{2x_0^3}{3x_0^3}=\frac{2}{3}x_0$;

$xy=-\frac{4}{3}x_0^4=-\frac{4}{3}$; $x_0^4=1$, $x_0=\pm 1$, но $x_0>0$, значит $x_0=1$; $y=1+3(x-1)=3x-2$.

б) $y=x^3$, $x<0\Rightarrow xy=-\frac{27}{4}$; $y=-x_0^3+3x_0^2$; $xy=-\frac{4}{3}x_0^4=-\frac{27}{4}$;

$x_0^4=\left(\frac{3}{2}\right)^4$, $x_0=\pm\frac{3}{2}$, но $x_0<0\Rightarrow-\frac{3}{2}$, $y=\frac{4}{3}\cdot\frac{81}{16}+3x\cdot\frac{9}{4}=\frac{27}{4}+\frac{27}{4}x$.

850. а) $y=3-\frac{1}{2}x^2$; $B(0;z)$, $y'=-x$; $y=3-\frac{1}{2}x_0^2-x_0(x-x_0)=-x_0x+\frac{1}{2}x_0^2+3$; $z=\frac{1}{2}x_0^2+3$.

Т.к. график симметричен относительно Oy, тангенс угла наклона касательной ± 1.

$y'=-x_0$; $x_0=\pm 1$; $z=\frac{1}{2}x_0^2+3=\frac{1}{2}+3=3{,}5$; $B(0;3{,}5)$;

б) $y=0{,}5x^2-2{,}5$, $\alpha=90°$; $y'=x$; $y=\frac{1}{2}x_0^2-2{,}5+x_0(x-x_0)=x_0x-\frac{1}{2}x_0^2-\frac{5}{2}$.

Рассуждения аналогичны пункту **а**. $y'=x_0=\pm 1$; $y=-x-3$; $y=x-3$.

851. а) $y'=\sqrt{3}x$; $y=\frac{\sqrt{3}}{2}(x_0^2+1)+\sqrt{3}x_0(x-x_0)$. Рассуждения аналогичны № 850.

1. $y'=\sqrt{3}x_0=\pm\operatorname{tg}60°$; $\sqrt{3}x_0=\pm\sqrt{3}$; $x_0=\pm 1$; $y=\frac{\sqrt{3}}{2}\cdot 2-\sqrt{3}=0$; $B(0;0)$;

2. $y'=\sqrt{3}x_0=\pm\operatorname{tg}30°$; $\sqrt{3}x_0=\pm\frac{\sqrt{3}}{3}$; $x_0=\pm\frac{1}{3}$; $y=\frac{\sqrt{3}}{2}\left(1-\frac{1}{9}\right)=\frac{4\sqrt{3}}{9}$; $B\left(0;\frac{4\sqrt{3}}{9}\right)$;

б) $y' = -\frac{\sqrt{3}}{3}x$; $y = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{3}x_0^2 - \frac{\sqrt{3}x_0^2}{6} + \frac{\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{6}x_0^2 + \frac{\sqrt{3}}{6}$;

1. $y' = -\frac{\sqrt{3}}{3}x_0 = \pm\operatorname{tg}60°$; $\frac{\sqrt{3}}{3}x_0 = \pm\sqrt{3}$; $x_0 = \pm 3$;

$y = -\sqrt{3}x + \frac{3\sqrt{3}}{3} + \frac{\sqrt{3}}{6} = -\sqrt{3}x + \frac{5\sqrt{3}}{3}$; $y = \sqrt{3}x + \frac{5\sqrt{3}}{3}$;

2. $y' = -\frac{\sqrt{3}}{3}x_0 = \pm\operatorname{tg}30°$; $x_0 = \pm 1$; $y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$; $y = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$.

852. а) I: $y = x^2 - 2x + 6, x = 5$.
$y' = 2x - 2$; $y = 25 - 10 + 6 + (10 - 2)(x - 5) = 8x - 10$.
II: $y = x^2 + 2x - 6, x = -5$.
$y' = 2x + 2$; $y = 25 - 10 - 6 + (-10 + 2)(x + 5) = -8x - 31$; $8x - 10 = -8x - 31$;
$x = -\frac{3}{4}. \left(-\frac{3}{4}; -25\right)$.

б) I: $y = x^3 - (x - 1), x = 2$; $y' = 3x^2 + 1$.
II: $y = x^3 - x + 1, x = -2$. $y' = 3x^2 - 1$.
$y = -5 + 11(x + 2) = 11x + 17$. $13x - 17 = 11x + 17$; $x = 17$. $(17, 204)$.

853. а) $y' = 3x^2 - p$. $y(1) = 1 - p$; $y'(1) = 3 - p$; $y = 1 - p + (3 - p)(x - 1) =$
$= (3 - p)x + p - 3 - p + 1 = (3 - p)x - 2$; $3 = 2(3 - p) - 2$; $5 = -2p + 6$; $p = \frac{1}{2}$.

б) $y' = 3x^2 + 2px$; $y(1) = 1 + p$. $y'(1) = 3 + 2p$; $y = 1 + p + (3 + 2p)(x - 1)$; $2 =$
$= 1 + p + (3 + 2p)(3 - 1)$, $5p = -5$; $p = -1$.

§ 35. Применение производной для исследования функций на монотонность и экстремумы

854. а) $f'(a) > 0$; $f'(b) > 0$; $f'(c) < 0$; $f'(d) > 0$;
б) $f'(a) > 0$; $f'(b) = 0$; $f'(c) = 0$; $f'(d) > 0$.

855. а) функция возрастает: $\left(-\infty; \frac{c+b}{2}\right) \cup \left[d - \frac{c-b}{2}; +\infty\right)$, убывает:
$\left(\frac{b+c}{2}; d - \frac{c-d}{2}\right]$;

б) возрастает: $(-\infty; b] \cup [c; +\infty)$; убывает: $[b; c]$.

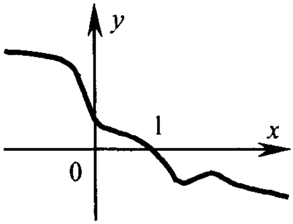
856. а) убывает: $[-2; 2]$; возрастает: $(+\infty; -2]$ *и* $[2; +\infty)$;
б) убывает: $[-4; 0] \cup [3; +\infty)$; возрастает: $(-\infty; -4] \cup [0; 3]$;
в) убывает: $[-4,5; +\infty)$; возрастает: $(-\infty; -4,5]$;
г) убывает: $(-\infty; -2,5] \cup [2,5; +\infty)$; возрастает: $[-2,5; 2,5]$.

857. Функция убывает на промежутке $(4, +\infty)$ (в).

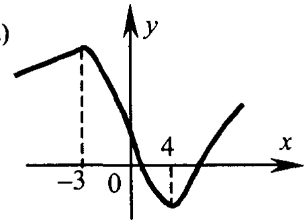
858. Для функции $g(x)$.

859. а) $f(x)$; б) $h(x)$.

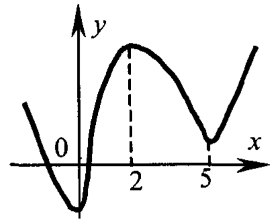
860.



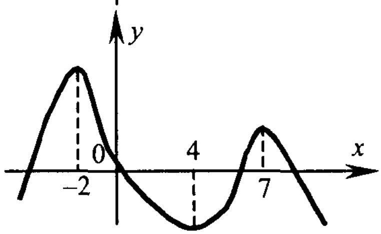
861. а)



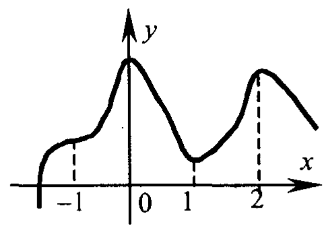
б)



в)



г)



862. **а)** $y' = -\sin x + 2$; $y' > 0$ при любых x.
б) $y' = 5x^4 + 3x^2 + 7$; $y' > 0$ при любых x.
в) $y' = \cos x + 3x^2 + 1$; $y' > 0$ при любых x.
г) $y' = 5x^4 + 12x^2 + 8$; $y' > 0$ при любых x.

863. **а)** $y = 2\cos 2x - 3$. $y < 0$ при любых x.
б) $y' = -3\cos 3x + 4$; $y > 0$ при $\forall\ x \Rightarrow$ утверждение неверно.

864. **а)** $y' = 5x^4 + 18x^2 \geq 0$, функция не убывает.
б) $y' = \cos x - 2 < 0$, монотонно убывает.
в) $y' = 1 + \sin x \geq 0$, функция не убывает.
г) $y' = -5 - 3x^2 \leq 0$, функция не возрастает.

865. **а)** $y' = 2x - 5$; при $x \geq \frac{5}{2}$ ф-ция возрастает, при $x \leq \frac{5}{2}$ ф-ция убывает.

б) $y' = 10x + 15$; при $x \geq -\frac{3}{2}$ ф-ция возрастаст; при $x \leq -\frac{3}{2}$ ф-ция убываст.

в) $y' = -2x + 8$; при $x \geq 4$ ф-ция возрастает, при $x \leq 4$ ф-ция убывает.

г) $y' = 2x - 1$; при $x \geq \frac{1}{2}$ ф-ция возрастает, при $x \leq \frac{1}{2}$ ф-ция убывает.

866. **а)** $y' = 3x^2 + 2$ возрастает при любых x;
б) $y' = 45 - 6x - 3x^2$; $-3(x^2 + 2x - 15) = 0$; $x \in [-5; 3]$ возрастает,
$x \in (-\infty; -5]$ и $[3; +\infty)$ убывает;
в) $y' = -6x^2 - 6x - 36 = 6(x^2 - x - 6) = 0$; $x \in [-2; 3]$ убывает, $x \in (-\infty; -2]$ и $[3; +\infty)$ возрастает;
г) $y' = 5x^4 + 5 = -5(x^4 - 1)$; $x \in [-1; 1]$ возрастает, $x \in (-\infty; -1]$ *и* $[1; +\infty)$ убывает.

867. а) $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$; $x \in (-\infty; -1] \cup [0; 1]$ убывает, $x \in [-1; 0] \cup [1; +\infty)$ возрастает.

б) $y' = -5x^4 - 1 = -(5x^4 + 1) < 0$; убывает при всех x.

в) $y' = -12x^3 + 12x^2 = -12x^2(x - 1)$; $x \in (-\infty; 1]$ возрастает, $x > 1$ убывает.

г) $y' = 25x^4$ возрастает при всех x.

868. а) $y' = -\dfrac{1}{(x+3)^2}$ – убывает на всей ОДЗ ($x \neq 3$);

б) $y' = \dfrac{9x+3-9x+3}{(3x+1)^2} = \dfrac{6}{(x+3)^2}$ – возрастает на всей ОДЗ $\left(x \neq \dfrac{1}{3}\right)$;

в) $y' = -\dfrac{2}{x^2}$ – убывает на всей ОДЗ ($x \neq 0$);

г) $y' = \dfrac{-6-4x-2+4x}{(3+2x)^2} = -\dfrac{8}{(3+2x)^2}$ – убывает на всей ОДЗ $\left(x \neq -\dfrac{3}{2}\right)$.

869. а) $y' = \dfrac{3}{2\sqrt{3x-1}}$ – возрастает а всей ОДЗ $\left(x \geq \dfrac{1}{3}\right)$;

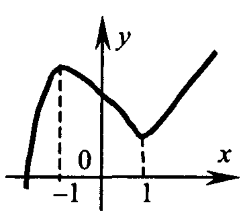
б) $y' = -\dfrac{1}{2\sqrt{1-x}} + 2 = 0$; $\dfrac{4\sqrt{1-x}-1}{2\sqrt{1-x}} = 0$; $x = \dfrac{15}{16}$; возрастает при $x \in \left(-\infty; \dfrac{15}{16}\right]$, убывает при $x \in \left[\dfrac{15}{16}; 1\right)$;

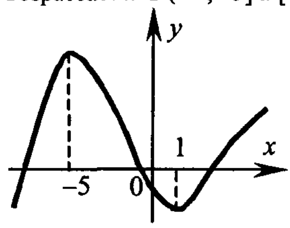
в) $y' = -\dfrac{1}{\sqrt{1-2x}}$; $x \in \left(-\infty; \dfrac{1}{2}\right]$ – убывает;

г) $y' = \dfrac{1}{\sqrt{2x-1}} - 1 = 0$; $\dfrac{1-2\sqrt{2x-1}}{\sqrt{2x-1}} = 0$; возрастает при $x \in \left[\dfrac{1}{2}; 1\right]$, убывает при $x \in [1; +\infty)$.

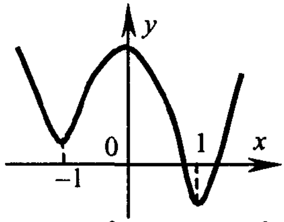
870. а) $y' = 3x^2 - 3$; убывает $x \in [-1; 1]$; возрастает $x \in (-\infty; -1] \cup [1; +\infty)$;

б) $y' = 3x^2 + 12x - 15$; убывает $x \in [-5; 1]$; возрастает $x \in (-\infty; -5]$ *и* $[1; +\infty)$.

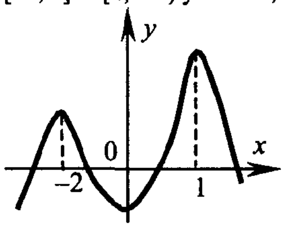




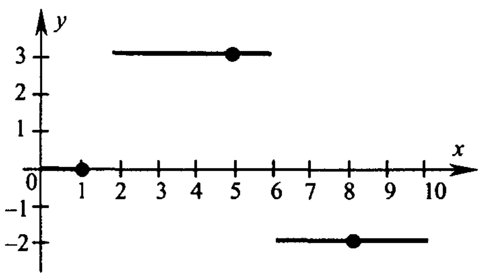
871. а) $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$;
$(-\infty; -1] \cup [0; 1]$ убывает; $[-1; 0]$ и $[1; +\infty)$ возрастает;



б) $y' = -4x^3 + 16x = -4x(x^2 - 4) - 4x(x - 2)(x + 2)$;
$[-2; 0] \cup [2; +\infty)$ убывает; $[-\infty; -2] \cup [0; 2]$ возрастает.



872.



873. а) $f'(b) = f'(d) = 0$; б) $f'(c) = 0$;
в) $f'(a) = f'(0) = 0$; г) нет точек, в которых производная равна нулю.

874. а) $f'(e)$; б) $f'(a), f'(b)$; в) $f'(b), f'(c)$; г) $f'(a), f'(b), f'(c), f'(d), f'(e)$.

875. а) 1; б) 2; в) 2; г) 2.

876. а) 2; б) 1; в) 2; г) 2.

877. а) $(-\infty; -5] \cup [-2; +\infty)$; б) $[-5; -2]$; в) -5; г) -2.

878. а) да; б) да; в) да; г) да.

879. а) да; б) нет; в) нет; г) да.

880. а) $y' = 12 - 3x^2$; $3x^2 = 12$; $x = \pm 2$. б) $y' = 9x^2 + 4x$; $x(9x + 4) = 0$; $x = 0$, $x = -\dfrac{4}{9}$.

в) $y' = 4x - 4x^3$; $4x(1 - x^2) = 0$; $x = 0$, $x = \pm 1$.
г) $y' = 4x^3 - 16x$; $4x(x^2 - 4) = 0$; $x = 0$, $x = \pm 2$.

881. а) $y' = 2 - \dfrac{8}{x^2} = 0$; $x = \pm 2$; б) $y' = \dfrac{1}{\sqrt{2x - 1}}$; $x = \dfrac{1}{2}$;

в) $y' = \dfrac{1}{5} - \dfrac{5}{x^2} = 0$; $x^2 - 25 = 0$; $x = \pm 5$; г) $y' = 4(x - 3)^3 = 0$; $x = 3$.

882. а) $y' = 4\cos 2x - 4\cos 4x = 0$; $\sin 3x \sin x = 0$; $x = \dfrac{\pi n}{3}$;

б) $y' = -2\sin 2x - 1 = 0$; $\sin 2x = -\frac{1}{2}$; $2x = (-1)^{k+1}\frac{\pi}{6} + \pi k$; $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$;

в) $y' = -3\sin 3x + 3\sin 5x$; $\sin 3x - \sin 5x = 0$; $\sin x \cos 4x = 0$; $x = \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{4}$;

г) $y' = \frac{1}{2}\cos\frac{x}{2} - \frac{1}{4}$; $\cos\frac{x}{2} = \frac{1}{2}$; $x = \pm\frac{2\pi}{3} + 4\pi n$.

В задачах 883-888,чтобы определить характер экстремума, необходимо проверить знак производной справа и слева от точки, где она равна нулю. Если слева $y' > 0$, а справа $y' < 0$, то это максимум, если слева $y' < 0$, а справа $y' > 0$, то это минимум.

883. а) $y' = 4x - 7 = 0$; $x = \frac{7}{4}$ – min; **б)** $y' = -5 - 2x = 0$; $x = -2{,}5$ max;

в) $y' = 8x - 6 = 0$; $x = \frac{3}{4}$ – min; **г)** $y' = -6x - 12 = 0$; $x = -2$ – max.

884. а) $y' = x^2 - 5x + 6 = 0$; $x = 2$, $x = 3$; $x = 2$ – max, $x = 3$ – min;
б) $y' = 3x^2 - 27 = 0$; $x = \pm 3$; $x = -3$ – max, $x = 3$ – min;
в) $y' = 3x^2 - 14x - 5 = 0$; $x = \frac{7+8}{3} = 5$; $x = -\frac{1}{3}$; $x = 5$ – min, $x = -\frac{1}{3}$ – max;
г) $y' = -6x^2 + 42x = 0$; $x^2 - 7x = 0$; $x(x-7) = 0$; $x = 0$, $x = 7$; $x = 7$ – max, $x = 0$ – min.

885. а) $y' = -25x^4 + 9x^2$; $x^2(9 - 25x^2) = 0$; $x = 0$, $x^2 = \frac{9}{25}$, $x = \pm\frac{3}{5}$; $x = \frac{3}{5}$ – max, $x = -\frac{3}{5}$ – min;
б) $y' = 4x^3 - 12x^2 - 16x$; $4x(x^2 - 3x - 4) = 0$; $x = 0$, $x = 4$, $x = -1$; $x = 4$, $x = -1$ – min, $x = 0$ – max;
в) $y' = 4x^3 - 100x$; $4x(x^2 - 25) = 0$; $x = 0$, $x = \pm 5$; $x = -5$, $x = 5$ – min, $x = 0$ – max;
г) $y' = 10x^4 + 20x^3 - 30x^2$; $10x^2(x^2 + 2x - 3) = 0$; $x = 0$, $x = -3$, $x = 1$, $x = -3$ – max, $x = 1$ – min.

886. а) $y' = 1 - \frac{4}{x^2} = 0$; $x = \pm 2$; $x = 2$ – min; $x = -2$ – max;

б) $y' = 1 - \frac{9}{x^2} = 0$; $x = \pm 3$; $x = 3$ – min; $x = -3$ – max.

887. а) $y' = 1 - \frac{1}{\sqrt{x-2}} = 0$; $\sqrt{x-2} = 1$; $x = 3$ – min;

б) $y' = \frac{4}{2x-1} - 1 = 0$; $\sqrt{2x-1} = 4$; $x = 8{,}5$ – max.

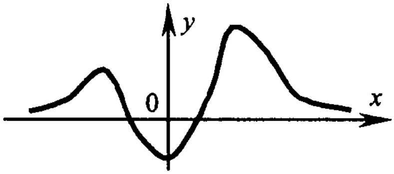
888. а) $y' = 1 + 2\sin x$; $\sin x = -\frac{1}{2}$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$; $x = -\frac{5\pi}{6}$ – max; $x = -\frac{\pi}{6}$ – min;

б) $y' = 2\cos x - 1$; $\cos x = \frac{1}{2}$; $x = \pm\frac{\pi}{3} + 2\pi n$; $x = \frac{5\pi}{3}$ – min; $x = \frac{7\pi}{3}$ – max.

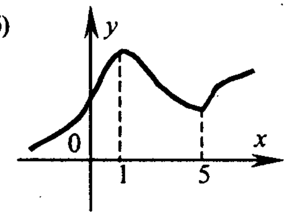
889. а) $y' = 3x^2 - 6ax + 27$; $x^2 - ax + 9 = 0$; $\frac{D}{4} = a^2 - 9 = 0$; $a = \pm 3$.

б) $y' = 3x^2 - 6ax + 75$; $x^2 - 2ax + 25 = 0$; $\frac{D}{4} = a^2 - 25 = 0$; $a = \pm 5$.

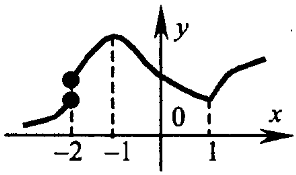
890. а)



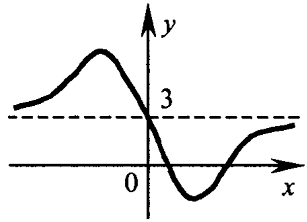
б)



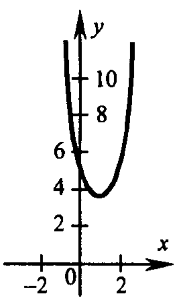
в)



г)

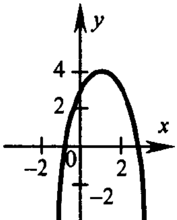


891. а)



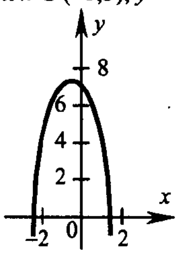
$y = 3x^2 - 4x + 5$; $y' = 6x - 4$; $x = \dfrac{2}{3}$ – min; при $x \geq \dfrac{2}{3}$ функция возрастает, при $x \leq \dfrac{2}{3}$ функция убывает; пересечение с Oy: (0;5), с Ox: нет; $y > 0$ при $x \in R$;

б)



$y = 3 + 2x - x^2$; $y' = 2 - 2x$; $x = 1$ – min; при $x \geq 1$ функция убывает при $x \leq 1$ функция возрастает пересечение с Oy: (0;3), с Ox: (3;0), (–1;0); $y > 0$ при $x \in (-1;3)$; $y < 0$ при $x < -1$, $x > 3$;

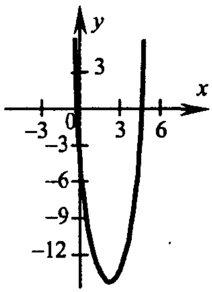
в)



$y = 7 - x - 2x^2$; $y' = -1 - 4x$; $x = -\frac{1}{4}$ – max; при $x \geq \frac{1}{4}$ функция убывает; при $x \leq \frac{1}{4}$ функция возрастает; пересечение с Oy: $(0;7)$, с Ox:

$\left(\frac{-1-\sqrt{57}}{4};0\right)$, $\left(\frac{-1+\sqrt{57}}{4};0\right)$; $y < 0$ при $x \in \left(\frac{-1-\sqrt{57}}{4};\frac{-1+\sqrt{57}}{4}\right)$, $y > 0$ при $x \in \left(-\infty;\frac{-1-\sqrt{57}}{4}\right) \cup \left(\frac{-1+\sqrt{57}}{4};+\infty\right)$;

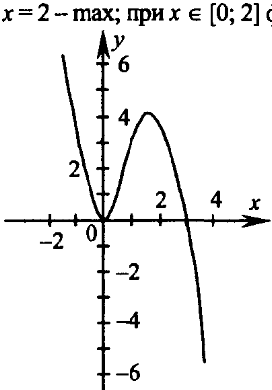
г)



$y = 5x^2 - 15x - 4$; $y' = 10x - 15$; $x = \frac{3}{2}$ – min; при $x \geq \frac{3}{2}$ функция возрастает, при $x \leq \frac{3}{2}$ функция убывает; пересечение с Oy: $(0;-4)$, с Ox:

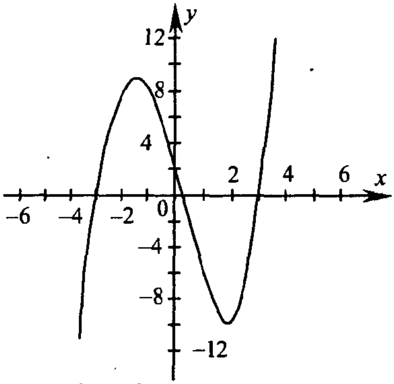
$\left(\frac{15-\sqrt{305}}{10};0\right)$, $\left(\frac{15+\sqrt{305}}{10};0\right)$; $y > 0$ при $x \in \left(-\infty;\frac{15-\sqrt{305}}{10}\right) \cup \left(\frac{15+\sqrt{305}}{10};+\infty\right)$, $y > 0$ при $x \in \left(\frac{15-\sqrt{305}}{10};\frac{15+\sqrt{305}}{10}\right)$.

892. **а)** $y = 3x^2 - x^3$; $y' = 6x - 3x^2$; $x = 0$, $x = 2$; $y(0) = 0$, $y(2) = 12 - 8 = 4$, $x = 0$ – min, $x = 2$ – max; при $x \in [0;\ 2]$ функция возрастает , при $x \in (-\infty;\ 0] \cup [2;\ +\infty)$ убывает.

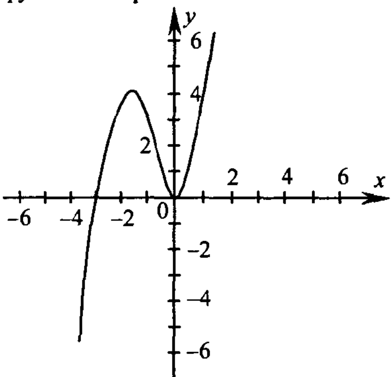


б) $y' = -9x + x^3$; $y' = -9 + 3x^2$; $x^2 = 3$; $x = \pm\sqrt{3}$;

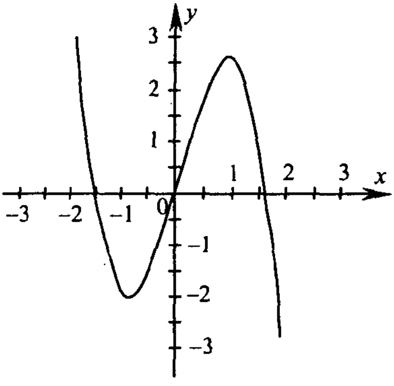
$y(\sqrt{3}) = -9\sqrt{3} + 3\sqrt{3}$; $y(-\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3}$; $x = \sqrt{3}$ – min;, $x = -\sqrt{3}$ – max ; при $x \in (-\infty; -\sqrt{3}] \cup [\sqrt{3}; +\infty)$ функция возрастает, при $x \in [-\sqrt{3}; \sqrt{3}]$ функция убывает.



в) $y = x^3 + 3x^2$; $y' = 3x^2 + 6x$; $x = 0$, $x = -2$, $y(0) = 0$, $y(-2) = -8 + 12 = 4$; $x = 0$ – min, $x = -2$ – max; при $x \in [-2; 0]$ функция убывает, при $x \in (-\infty; -2] \cup [0; +\infty)$ функция возрастает.

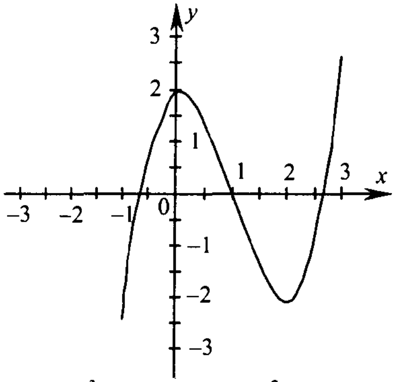


г) $y = 3x - x^3$; $y' = 3 - 3x^2$; $x = \pm 1$, $y(1) = 3 - 1 = 2$, $y(-1) = -3 + 1 = -2$; $x = 1$ – max, $x = -1$ – min; при $x \in [-1; 1]$ функция возрастает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция убывает.

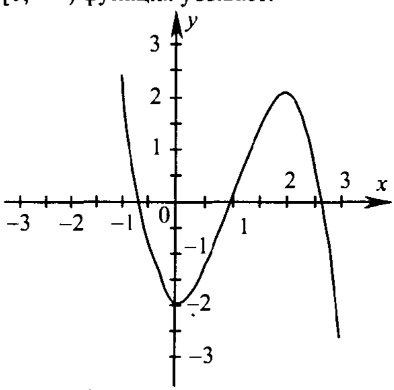


893. а) $y = x^3 + 3x^2$; $y' = 3x^2 + x$; $x = 0$, $x = 2$, $y(0) = 0$, $y(-2) = -8 + 12 = 4$; $x = 0$ – min,

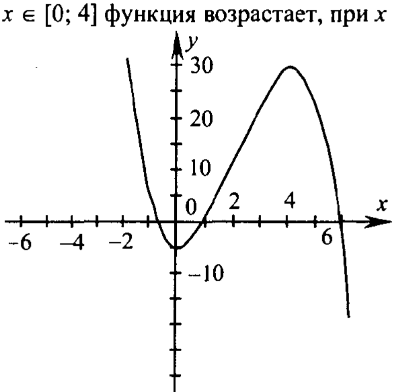
$x = -2$ – max; при $x \in [0;2]$ функция убывает, при $x \in (-\infty; 0] \cup [2; +\infty)$ функция возрастает.



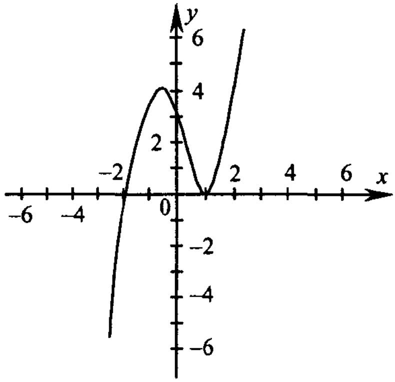
б) $y = -x^3 + 3x - 2$; $y' = -3x^2 + 3$; $x = \pm 1$, $y(1) = -1 + 3 - 2 = 0$, $y(-1) = 1 - 3 - 2 = -4$; $x = 1$ – max, $x = -1$ – min; при $x \in [-1; 1]$ функция возрастает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция убывает.



в) $y = -x^3 + 6x^2 - 5$; $y' = -3x^2 + 12x = -x^2 + 4x$, $x = 0$, $x = 4$, $x = 0$ – min, $x = 4$ – max, при $x \in [0; 4]$ функция возрастает, при $x \in (-\infty; 0] \cup [4; +\infty)$ функция убывает.

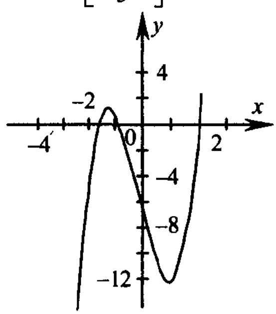


г) $y = x^3 - 3x + 2$; $y' = 3x^2 - 3$; $x = \pm 1$; $x = 1$ – min, $x = -l$ – max; при $x \in [-1; 1]$ функция убывает, при $x \in (-\infty; 0] \cup [4; +\infty)$ функция возрастает.

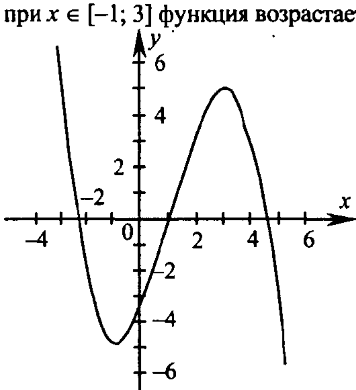


894. а) $y = 2x^3 + x^2 - 8x - 7$; $y' = 6x^2 + 2x - 8$; $x = \frac{-1-7}{6} = -\frac{4}{3}$; $x = 1 - \min, x = -\frac{4}{3} - \max$;

при $x \in \left[-\frac{4}{3}; 1\right]$ функция убывает, при $x \in \left(-\infty; -\frac{4}{3}\right] \cup [1; +\infty)$ функция возрастает.



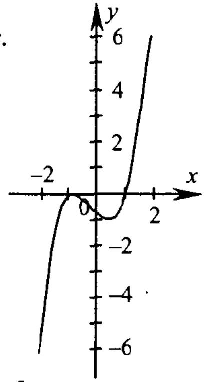
б) $y = -\frac{x^3}{3} + x^2 + 3x - \frac{11}{3}$; $y' = -x^2 + 2x + 3$; $x = 3$, $x = -1$; $x = -1 - \min$, $x = 3 - \max$;
при $x \in [-1; 3]$ функция возрастает, при $x \in (-\infty; -1] \cup [3; +\infty)$ функция убывает.



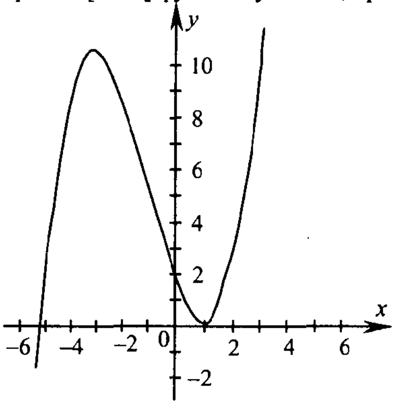
в) $y = x^3 + x^2 - x - 1$; $y' = 3x^2 + 2x - 1$; $x = \frac{-1-2}{3} = -1, x = \frac{1}{3}$, $x = \frac{1}{3} - \min$,

$x = -1 - \max$; при $x \in \left[-1; \frac{1}{3}\right]$ функция убывает, при $x \in (-\infty; -1] \cup \left[\frac{1}{3}; +\infty\right)$

функция возрастает.



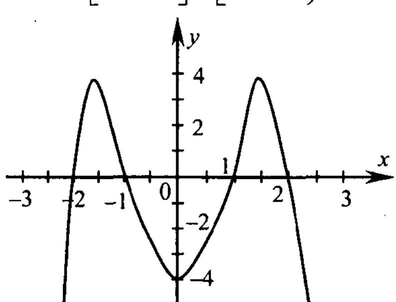
г) $y=\frac{x^3}{3}+x^2-3x+\frac{5}{3}$; $y'=x^2+2x-3$, $x=-3$, $x=1$; $x=1$ – min, $x=-3$ – max; при $x\in[-3;\ 1]$ функция убывает, при $x\in(-\infty;\ -3]\cup[1;\ +\infty)$ функция возрастает.



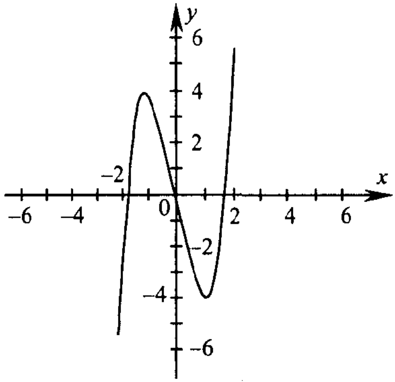
895. а) $y=-x^4+5x^2-4$; $y'=-4x^3+10x$, $2x(-2x^2+5)=0$, $x=0$, $x=\sqrt{\frac{5}{2}}$, $x=-\sqrt{\frac{5}{2}}$;

$x=0$ –min, $x=\pm\sqrt{\frac{5}{2}}$ max; при $x\in\left(-\infty;-\frac{\sqrt{5}}{\sqrt{2}}\right]\cup\left[0;\frac{\sqrt{5}}{\sqrt{2}}\right]$ функция возрастает,

при $x\in\left[-\frac{\sqrt{5}}{\sqrt{2}};0\right]\cup\left[\frac{\sqrt{5}}{\sqrt{2}};+\infty\right)$ функция убывает;

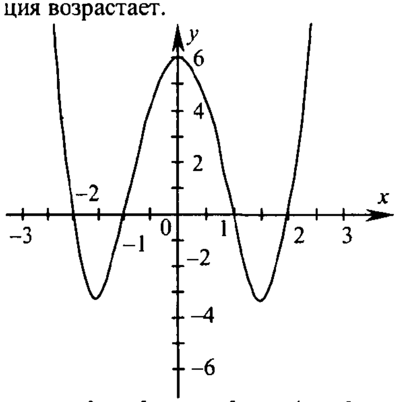


б) $y = x^5 - 5x$; $y' = 5x^4 - 5$; $x^4 - 1 = 0$; $x = \pm 1$; $x = 1$ – min, $x = -l$ – max;
при $x \in [-1;\ 1]$ функция убывает, при $x \in (-\infty;\ -1] \cup [1;\ +\infty)$ функция возрастает.

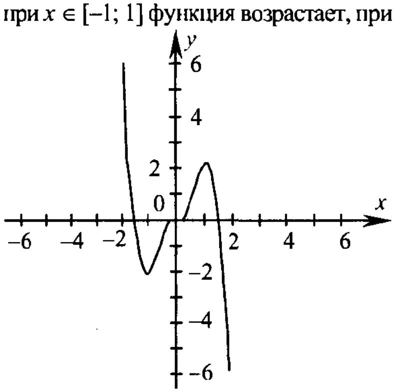


в) $y = 2x^4 - 9x^2 + 7$; $y' = 8x^3 - 18x$; $2x(4x^2 - 9) = 0$; $x = 0$ – max, $x = \pm\frac{3}{2}$ – min;

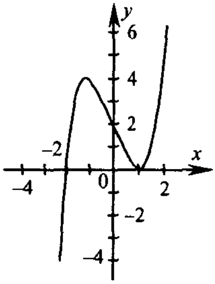
при $x \in \left(-\infty; -\frac{3}{2}\right] \cup \left[0; \frac{3}{2}\right]$ функция убывает, при $x \in \left[-\frac{3}{2}; 0\right] \cup \left[\frac{3}{2}; +\infty\right)$ функция возрастает.



г) $y = 5x^3 - 3x^5$, $y' = 15x^2 - 15x^4$, $15x^2(1 - x^2) = 0$, $x = 0$, $x = \pm 1$, $x = l$ – max, $x = -1$ – min,
при $x \in [-1;\ 1]$ функция возрастает, при $x \in (-\infty;\ -1] \cup [l;\ +\infty)$ функция убывает.

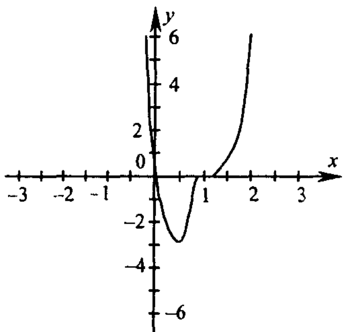


896. **а)** $y = (x-1)^2(x+2)$; $y' = 2(x-1)(x+2) + (x-1)^2 = (x-1)(2x+4+x-1) =$
$= (x-1)(3x+3) = 0$; $(x-1)(3x+1) = 0$, $x = 1$, $x = -1$, $x = -1$ – max, $x = 1$ – min,
при $x \in [-1; 1]$ функция убывает, при $x \in (-\infty; -1] \cup [l; +\infty)$ функция возрастает.



б) $y = \frac{256}{9}x(x-1)^3$; $y' = \frac{256}{9}\left((x-1)^3 + 3(x-1)^2x\right) = \frac{256}{9}(x-1)^2(4x-1)$;

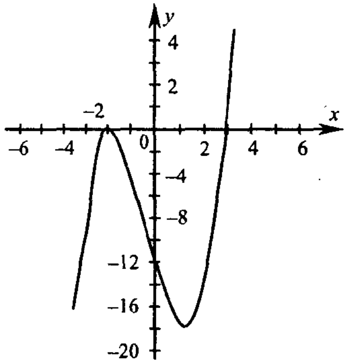
$\frac{256}{9}(x-1)^2(4x-1) = 0$; $x = \frac{1}{4}$ – min, $x = 1$; при $x \geq \frac{1}{4}$ функция возрастает, при $x \leq \frac{1}{4}$ функция убывает.



в) $y = (x+2)^2(x-3)$, $y' = 2(x+2)(x-3) + (x+2)^2 = (x+2)(3x-4)$, $(x+2)(3x-4) = 0$,

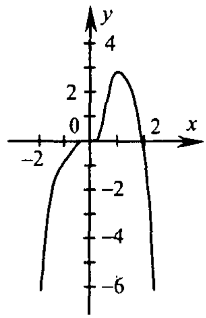
$x = -2$ – max, $x = \frac{4}{3}$ – min, при $x \in \left[-2; \frac{4}{3}\right]$ функция убывает, при

$x \in (-\infty; -2] \cup \left[\frac{4}{3}; +\infty\right)$ функция возрастает.

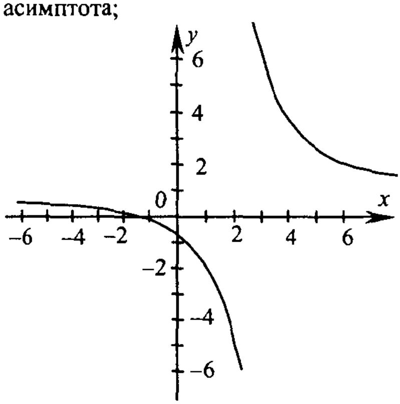


г) $y = x^3(2-x)$; $y' = (3x^2(2-x) - x^3) = x^2(6-4x)$, $x^2(6-4x) = 0$, $x = 0$, $x = \frac{3}{2}$ – max;

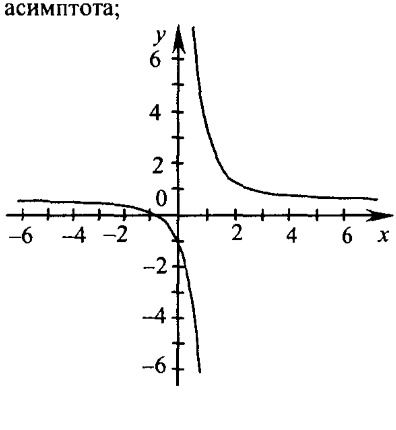
при $x \geq \frac{3}{2}$ функция убывает, при $x \leq \frac{3}{2}$ функция возрастает.



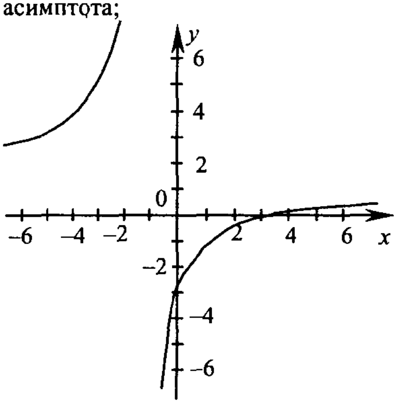
897. а) $y' = \frac{x-3-x-2}{(x-3)^2} = -\frac{5}{(x-3)^2}$; $-\frac{5}{(x-3)^2} = 0$, везде убывает, $x \neq 3$; $x = 3$ – асимптота;



б) $y' = \frac{6x-2-6x-3}{(3x-1)^2} = -\frac{5}{(3x-1)^2}$; $-\frac{5}{(3x-1)^2} = 0$, везде убывает, $x \neq \frac{1}{3}$; $x = \frac{1}{3}$ – асимптота;

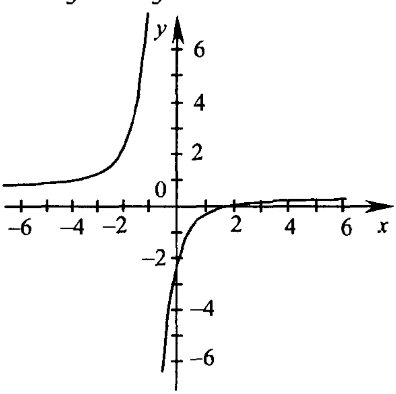


в) $y' = \frac{x+1-x+3}{(x+1)^2} = \frac{4}{(x+1)^2}; \frac{4}{(x+1)^2} = 0$ – везде возрастает, $x \neq 1$; $x = 1$ – асимптота;

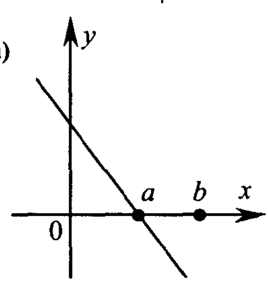


г) $y' = \frac{6x+4-6x+9}{(3x+2)^2} = \frac{13}{(3x+2)^2}; \frac{13}{(3x+2)^2} = 0$ – везде возрастает,

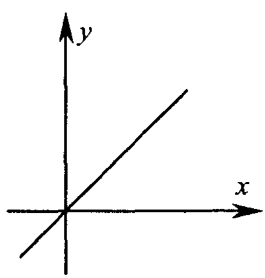
$x \neq -\frac{2}{3}$; $x = -\frac{2}{3}$ – асимптота.



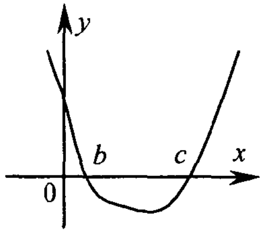
898. а)



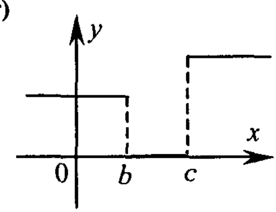
б)



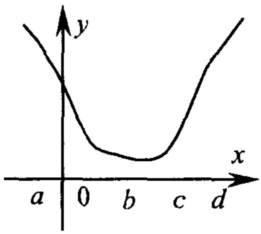
в)



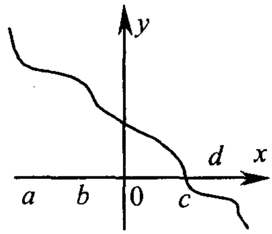
г)



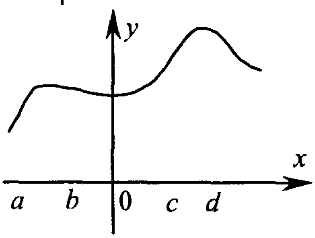
899. а)



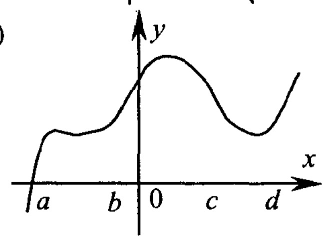
б)



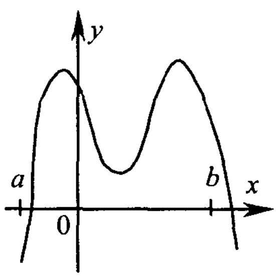
в)



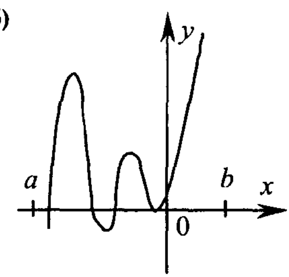
г)



900. а)



б)



901. а) да; **б)** да; **в)** нет; **г)** нст.

902. а) $y' = \cos x - \frac{1}{2}$; $\cos x = \frac{1}{2}$; $x = \pm\frac{\pi}{3} + 2\pi n$; $x = -\frac{\pi}{3} + 2\pi n - \min$; $x = \frac{\pi}{3} + 2\pi n - \max$;

возрастает при $x \in \left[-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right]$, убывает при $x \in \left[\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right]$;

б) $y' = \frac{1}{2} + \sin x$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$; $x = -\frac{\pi}{6} + 2\pi n - \max$; $x = \frac{7\pi}{6} + 2\pi n - \min$;

возрастает при $x \in \left[-\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n\right]$, убывает при

$x \in \left[\frac{7\pi}{6} + 2\pi n; \frac{11\pi}{6} + 2\pi n\right]$;

в) $y'=\frac{\sqrt{2}}{2}-\sin x;\ x=(-1)^k\frac{\pi}{4}+\pi k;\ x=\frac{\pi}{4}+2\pi n-\max;\ x=\frac{3\pi}{4}+2\pi n-\min$;

возрастает при $x\in\left[\frac{3\pi}{4}+2\pi n;\frac{9\pi}{4}+2\pi n\right]$, убывает при $x\in\left[\frac{\pi}{4}+2\pi n;\frac{3\pi}{4}+2\pi n\right]$;

г) $y=x-\sin x;\ y'=1-\cos x;\ x=2\pi n$, возрастает на R.

903. а) $y'=-\frac{1}{2}\sin\frac{x}{2};\ \sin\frac{x}{2}=0;\ x=2\pi n;\ x=4\pi n-\max,\ x=2\pi+4\pi n-\min$; возрастает при $x\in[-2\pi+4\pi n;4\pi n]$, убывает при $x\in[4\pi n;2\pi+4\pi n]$,

б) $y'=-\frac{1}{3}\cos\frac{x}{3};\ x=\frac{3\pi}{2}+6\pi n-\min;\ x=-\frac{3\pi}{2}+6\pi n-\max$; убывает при

$x\in\left[6\pi n-\frac{3\pi}{2};\frac{3\pi}{2}+6\pi n\right]$, возрастает при $x\in\left[\frac{3\pi}{2}+6\pi n;\frac{9\pi}{2}+6\pi n\right]$.

904. а) $y'=1-2\cos 2x;\ \cos 2x=-\frac{1}{2};\ 2x=\pm\frac{\pi}{3}+2\pi n;\ x=\pm\frac{\pi}{6}+\pi n;\ x=\frac{\pi}{6}+\pi n-\min$;

$x=-\frac{\pi}{6}+\pi n-\max$; убывает при $x\in\left[\pi n-\frac{\pi}{6};\frac{\pi}{6}+\pi n\right]$, возрастает при $x\in\left[\frac{\pi}{6}+\pi n;\frac{5\pi}{6}+\pi n\right]$;

б) $y'=1-2\sin\frac{x}{2};\ \sin\frac{x}{2}=\frac{1}{2};\ x=(-1)^k\frac{\pi}{3}+2\pi k;\ x=\frac{\pi}{3}+4\pi n-\max;\ x=\frac{5\pi}{3}+4\pi n-\min$;

убывает при $x\in\left[\frac{\pi}{3}+4\pi n;\frac{5\pi}{3}+4\pi n\right]$, возрастает при $x\in\left[-\frac{7\pi}{3}+4\pi n;\frac{\pi}{3}+4\pi n\right]$.

905. а) I: $x\ge 3;\ y=x-5;\ y'=1$; возрастает при $x\ge 3$;
II: $x\le 3;\ y=1-x;\ y'=-1$; убывает при $x\le 3$;
Ответ: $x\in(-\infty;3]$ – убывает, $x\in[3;+\infty)$ – возрастает; $x=3-\min$.

б) I: $x\ge 1,\ x<0;\ y=-\frac{1}{x}+1;\ y'=\frac{1}{x^2}$ – везде возрастает;

II: $x\in(0;1];\ y=-1+\frac{1}{x};\ y'=-\frac{1}{x^2}$ – везде убывает;

Ответ: $x\in(0;1]$ – убывает, $x\in(-\infty;0)\cup[1;+\infty)$ – возрастает; $x=1-\min$.

в) I: $x\ge 2,\ x\le -3;\ y=x^2+x-6;\ y'=2x+1;\ x=-\frac{1}{2}$; возрастает при $x\ge-\frac{1}{2}$,

убывает при $x\le-\frac{1}{2}$;

II: $x\in[-3;2];\ y=-x^2-x+6;\ y'=-2x-1;\ x=-\frac{1}{2}$; возрастает при $x\le-\frac{1}{2}$,

убывает при $x\ge-\frac{1}{2}$;

Ответ: $x\in(-\infty;-3]\cup\left[-\frac{1}{2};2\right]$ – убывает; $x\in\left[-3;\frac{1}{2}\right]\cup[2;+\infty)$ – возрастает;

$x=-3,\ x=2-\min;\ x=-\frac{1}{2}-\max$.

г) I: $x\ge 0;\ y=x^2-2x;\ y'=2x-2;\ x=1$; возрастает при $x\ge 1$, убывает при $x\le 1$;

II: $x \le 0$; $y = x^2 + 2x$; $y' = 2x + 2$; $x = -1$; возрастает при $x \ge -1$, убывает при $x \le -1$;

Ответ: $x \in (-\infty;-1] \cup [0;1]$ – убывает; $x \in [-1;0] \cup [1;+\infty)$ – возрастает;

$x = \pm 1$ – min; $x = 0$ – max.

906. а) $y = \left|x^3 - 3x\right|$; $y = \left|x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)\right|$;

I: $x \in \left[-\sqrt{3};0\right] \cup \left[\sqrt{3};+\infty\right)$; $y' = 3x^2 - 3$; $x = \pm 1$; возрастает при $x \in (-\infty;-1] \cup [1;+\infty)$, убывает при $x \in [-1;-1]$;

II: $x \in \left(-\infty;-\sqrt{3}\right] \cup \left[0;\sqrt{3}\right]$ $y' = 3 - 3x^2$, $x = \pm 1$; возрастает при $x \in [-1;-1]$, убывает при $x \in (-\infty;-1] \cup [1;+\infty)$;

Ответ: $x \in \left(-\infty;-\sqrt{3}\right] \cup [-1;0] \cup \left[1;\sqrt{3}\right]$ – убывает,

$x \in \left[-\sqrt{3};-1\right] \cup [0;1] \cup \left[\sqrt{3};+\infty\right)$, $x = \pm\sqrt{3}$ – возрастает, $x = 0$ – min, $x = \pm 1$ – max.

б) $y = \left|x - x^3\right|$; $y = \left|x\left(\sqrt{3} - x\right)\left(\sqrt{3} + x\right)\right|$;

I: $x \in \left(-\infty;-\sqrt{3}\right] \cup \left[0;\sqrt{3}\right]$ $y' = 1 - 3x^2$, $x = \pm\dfrac{1}{\sqrt{3}}$; возрастает при

$x \in \left[-\dfrac{1}{\sqrt{3}};\dfrac{1}{\sqrt{3}}\right]$, убывает при $x \in \left(-\infty;-\dfrac{1}{\sqrt{3}}\right] \cup \left[\dfrac{1}{\sqrt{3}};+\infty\right)$;

II: $x \in \left[-\sqrt{3};0\right] \cup \left[\sqrt{3};+\infty\right)$; $y' = 3x^2 - 1$; $x = \pm\dfrac{\sqrt{3}}{3}$; возрастает при

$x \in \left(-\infty;-\dfrac{1}{\sqrt{3}}\right] \cup \left[\dfrac{1}{\sqrt{3}};+\infty\right)$, убывает при $x \in \left[-\dfrac{1}{\sqrt{3}};\dfrac{1}{\sqrt{3}}\right]$;

Ответ: $x \in (-\infty;-1] \cup \left[-\dfrac{1}{\sqrt{3}};0\right] \cup \left[\dfrac{1}{\sqrt{3}};1\right]$ – убывает,

$x \in \left[-1;-\dfrac{1}{\sqrt{3}}\right] \cup \left[0;\dfrac{1}{\sqrt{3}}\right] \cup [1;+\infty)$, $x = \pm 1$, $x = 0$ – min, $x = \pm\dfrac{1}{\sqrt{3}}$ – max.

907. а) $y' = 5x^4 + 3 > 0\ \forall\ x \in (-\infty;+\infty)$;

б) $y' = \dfrac{2}{x^2} + \dfrac{3}{x^4} > 0\ \forall\ x \in (-\infty;+\infty)$;

в) $y' = x^6 + 21x^2 + 2$;

г) $y' = 21 + \dfrac{5}{x^6} > 0\ \ \forall\ x > 0$.

908. а) $y' = 7 + 2\sin 2x > 0\ \forall\ x \in (-\infty;+\infty)$;

б) $y' = \dfrac{3}{\cos^2 x} > 0\ \forall x \in \left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right)$;

в) $y' = \dfrac{1}{\sin^2 x} > 0\ \forall x \in (0;\pi)$;

г) $y' = 10 + 3\cos 3x > 0\ \forall\ x \in (-\infty;+\infty)$.

909. а) $y = 2x^3 + 2x^2 + 11x - 35$, $y' = 6x^2 + 4x + 11$, $D = 16 - 4 \cdot 6 \cdot 11 < 0$, следовательно, $6x^2 + 4x + 11 > 0\ \forall x \in (-\infty;+\infty)$;

б) $y = 3x^3 - 6x^2 + 41x - 137$; $y' = 9x^2 - 12x + 41$, $\dfrac{D}{4} = 36 - 41 \cdot 9 < 0$, следовательно, $9x^2 - 12x + 41 > 0\ \forall x \in (-\infty;+\infty)$.

910. а) $y' = \dfrac{16x+4-16x}{(4x+1)^2} = \dfrac{4}{(4x+1)^2} > 0\ \forall x \in \left(-\dfrac{1}{4}; +\infty\right)$;

б) $y' = \dfrac{2x-10-2x+13}{(x-5)^2} = \dfrac{3}{(x-5)^2} > 0\ \forall x \in (-\infty; 5)$.

911. а) $y = -x^3 - 5x + 3$, $y' = -3x^2 - 5 < 0\ \forall x \in (-\infty; +\infty)$;
б) $y = -2x^5 - 7x^3 - x + 8$, $y' = -10x^4 - 21x^2 - 1 < 0\ \forall x \in (-\infty; +\infty)$;
в) $y = -x^3 + 3x^2 - 6x + l$, $y = -3x^2 + 6x - 6 = -3(x^3 - 2x + 1) = -3(x-1)^2 < 0\ \forall x \in (-\infty; +\infty)$;
г) $y = -4x^3 + 4x^2 - 2x + 9$, $y' = -12x^2 + 8x - 2$, $\dfrac{D}{4} = 16 - 24 < 0 \Rightarrow 12x^2 + 8x - 2 < 0$
$\forall x \in (-\infty; +\infty)$.

912. а) $y' = \dfrac{3x+6-3x-7}{(x+2)^2} = -\dfrac{1}{(x+2)^2} < 0\ \forall x \in (-\infty; +\infty)$;

б) $y' = \dfrac{-8x-4+8x-2}{(2x+1)^2} = -\dfrac{6}{(2x+1)^2} < 0\ \forall x \in \left(-\infty; -\dfrac{1}{2}\right)$.

913. а) $y' = -7\sin x - 15\cos 3x - 22 < 0\ \forall x \in (-\infty; +\infty)$, убывает на R;
б) $y' = -21\sin 7x - 4\cos\dfrac{x}{2} - 25 < 0\ \forall x \in (-\infty; +\infty)$, убывает на R.

914. а) $y' = 3x^2 + a$, при $a \geq 0$ возрастает на R;
б) $y' = x^2 - 2ax + 5$, $\dfrac{D}{4} = a^2 - 5$, при $a \in \left[-\sqrt{5}; \sqrt{5}\right]$ возрастает на R.

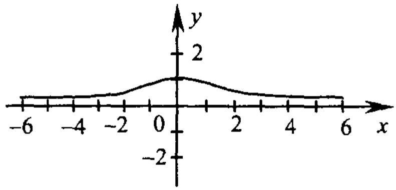
915. а) $y' = a + \sin x$, при $a > 1$ возрастает на R;
б) $y' = 4\cos 2x - a$, при $a < -4$ возрастает на R.

916. а) $y' = b - 2x - 3x^2$, $D = 4 - 4 \cdot (-3) \cdot b = 4 + 12b < 0$, при $b < -\dfrac{1}{3}$ убывает на R;

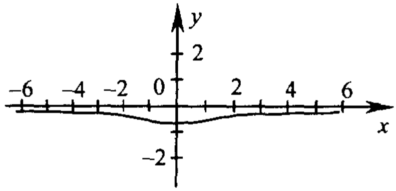
б) $y' = -\dfrac{1}{\sqrt{x+3}} + b < 0$, $b < \dfrac{1}{\sqrt{x+3}}$, при $b < 0$ убывает при $\forall x \in (-3; +\infty)$;
в) $y' = 3x^2 + 2bx + 3$. При $x = 0$ $y' = 3$, следовательно, ни при каких b функция не может убывать на всей области определения.
г) $y' = -2b - \dfrac{1}{2\sqrt{1-x}} < 0$, при $b \geq 0$.

917. а) $y' = 6x^2 - 6x = 6x(x-1)$, возрастает $x \leq 0$, $x \geq 1$ при $a \leq -1$, $a \geq 2$, следовательно, $a - 1 \geq 1$, $a + 1 \leq 0$; $a \geq 2$, $a \leq -1$.
б) $y' = -3x^2 + 3 = -3(x^2 - 1)$, возрастает $x \in [-1; 1]$, убывает $x \leq -1$, $x \geq 1$ при $a \in \left(-\infty; -\dfrac{3}{2}\right] \cup [1; +\infty)$, следовательно $a > 1$, $a + \dfrac{1}{2} < -1$; $a \geq 2$, $a \leq -\dfrac{3}{2}$ при
$a \in \left(-\infty; -\dfrac{3}{2}\right] \cup [1; +\infty)$.

918. а) $y' = -\dfrac{1}{(x^2+1)^2}$, $y' < 0$ при $x > 0$, $y' > 0$ при $x < 0$, асимптота $y = 0$, $x = 0$ – max;

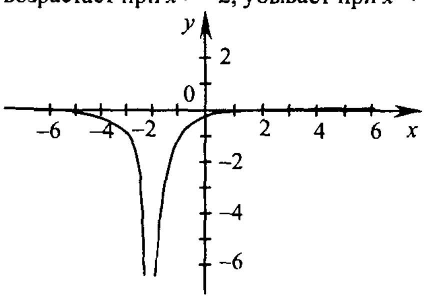


б) $y' = -\dfrac{2}{\left(x^2+4\right)^2}$, $y' < 0$ при $x < 0$, $y' > 0$ при $x > 0$, асимптота $y = 0$, $x = 0$ – min.



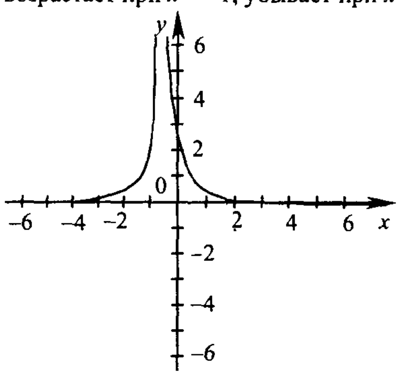
919. а) $y = -\dfrac{1}{x^2+4x+4} = -\dfrac{1}{(x+2)^2}$; $y' = \dfrac{2}{(x+2)^3}$,

возрастает при $x > -2$, убывает при $x < -2$, асимптота $x = -2$;



б) $y = \dfrac{1}{x^2+4x+1} = \dfrac{1}{(x+1)^2}$; $y' = -\dfrac{2}{(x+1)^3}$

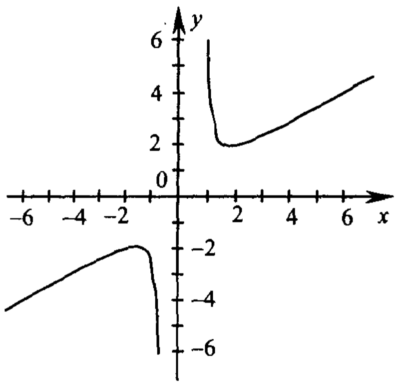
возрастает при $x < -1$, убывает при $x > -1$, асимптота $x = -1$.



920. а) $y' = \frac{1}{2} - \frac{2}{x^2}$; $y' > 0: \frac{1}{2} - \frac{2}{x^2} > 0, x^2 > 4$,

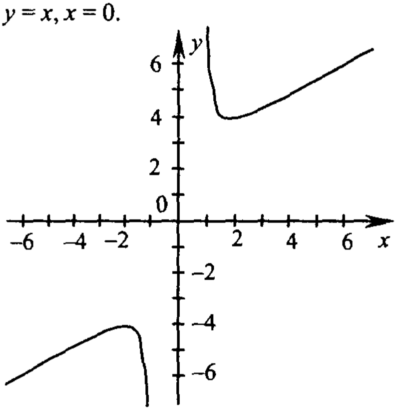
$y' > 0$ при $x \in (-\infty; -2) \cup (2; +\infty)$, $y' < 0$ при $x \in (-2; 0) \cup (0; 2)$,
при $x \in (-\infty; -2] \cup [2; +\infty)$ возрастает, при $x \in [-2; 0) \cup (0; 2]$ убывает;

$x = 2$ – min, $x = -2$ – max, асимптоты: $y = \frac{x}{2}$, $x = 0$;



б) $y = \frac{x^2 + 4}{x} = x + \frac{4}{x}$; $y' = 1 - \frac{4}{x^2}$; $x = \pm 2$; $1 - \frac{4}{x^2} > 0$; $\frac{x^2}{4} > 1$; $x^2 > 4$; $|x| > 2$;

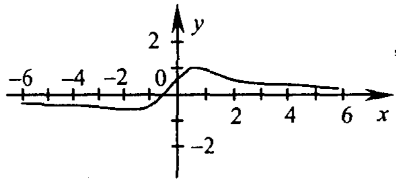
при $|x| \geq 2$ возрастает, при $|x| \leq 2$ убывает; $x = -2$ – min, $x = 2$ – max; асимпт
$y = x$, $x = 0$.



921. а) $y' = \frac{4x^2 + 4x - 4x^2 - 2}{\left(x^2 + 2\right)^2} = \frac{2x}{\left(x^2 + 2\right)}$, $y' > 0$ при $x^2 + x - 4 < 0$;

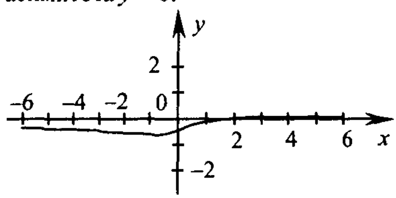
при $x \in \left(\frac{-1 - \sqrt{17}}{2}; \frac{-1 + \sqrt{17}}{2}\right)$ возрастает, при

$x \in \left(-\infty; \frac{-1 - \sqrt{17}}{2}\right) \cup \left(\frac{-1 + \sqrt{17}}{2}; +\infty\right)$ убывает, асимптота $y = 0$;

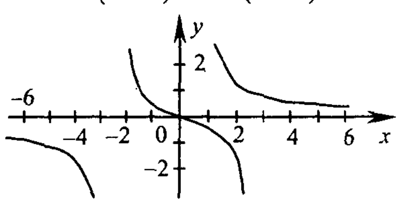


б) $y' = \dfrac{x^2 + 5 - 2x^2 + 4x}{\left(x^2 + 5\right)^2} = \dfrac{-x^2 + 4x + 5}{\left(x^2 + 5\right)^2}$; $y' > 0$ при $x^2 - 4x - 5 < 0$; $x \le -1$; $x = 5$;

при $x \in [-1;\ 5]$ возрастает, при $x \le 1$, $x \ge 5$ убывает; $x = -1$ – min, $x = 5$ – max, асимптота $y = 0$.



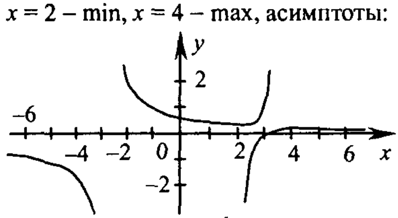
922. а) $y' = \dfrac{x^2 - 4 - 2x^2}{\left(x^2 - 4\right)^2} = -\dfrac{x^2 + 4}{\left(x^2 - 4\right)^2}$, убывает при всех $x \ne 2$, асимптоты $x = \pm 2, y = 0$;



б) $y' = \dfrac{x^2 - 8 - 2x^2 + 6x}{\left(x^2 - 8\right)^2} = \dfrac{-x^2 + 6x - 8}{\left(x^2 - 8\right)^2}$, $y' > 0$ при $x^2 - 6x + 8 < 0$; $x = 4$, $x = 2$;

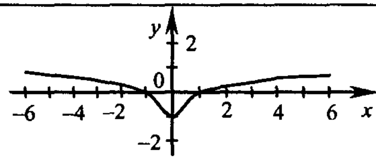
при $x \in \left[2; 2\sqrt{2}\right) \cup \left(2\sqrt{2}; 4\right]$ возрастает, при $x \le 2$, $x \ge 4$ убывает;

$x = 2$ – min, $x = 4$ – max, асимптоты: $x = \pm 2\sqrt{2}$, $y = 0$.

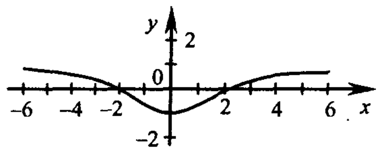


923. а) $y' = \dfrac{2x^3 + 2x - 2x^3 + 2x}{\left(x^2 + 1\right)^2} = \dfrac{4x}{\left(x^2 + 1\right)^2}$;

при $x \ge 0$ возрастает, при $x \le 0$ убывает, $x = 0$ – min;

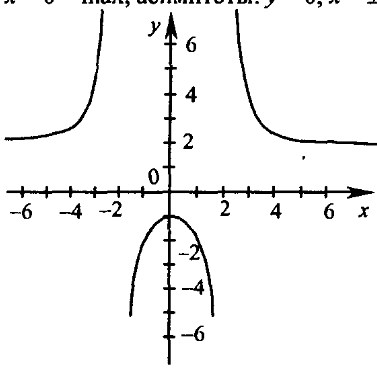


б) $y' = \dfrac{2x^3 + 2x - 2x^3 + 2x}{\left(x^2+1\right)^2} = \dfrac{4x}{\left(x^2+1\right)^2}$; при $x \geq 0$ возрастает, при $x \leq 0$ убывает, $x = 0$ -



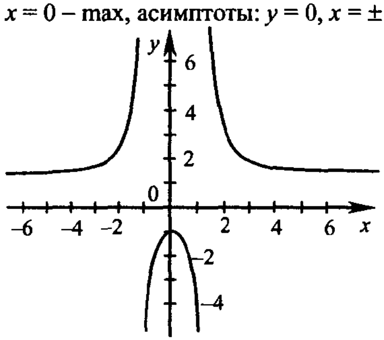
924. а) $y' = \dfrac{2x^3 - 8x - 2x^3 - 8x}{\left(x^2-4\right)^2} = \dfrac{-16x}{\left(x^2-4\right)^2}$;

при $x \in [0; 2) \cup (2; +\infty)$ убывает, при $x \in (-\infty; -2) \cup (-2; 0]$ возрастает; $x = 0$ – max, асимптоты: $y = 0$, $x = \pm 2$;

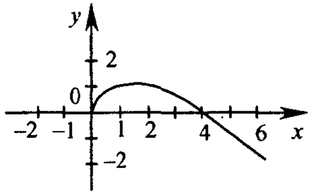


б) $y' = \dfrac{2x^3 - 2x - 2x^3 - 2x}{\left(x^2-1\right)^2} = \dfrac{-4x}{\left(x^2-1\right)^2}$;

при $x \in [0; 1) \cup (1; +\infty)$ убывает, при $x \in (-\infty; -1) \cup (-1; 0]$ возрастаст; $x = 0$ – max, асимптоты: $y = 0$, $x = \pm 1$.

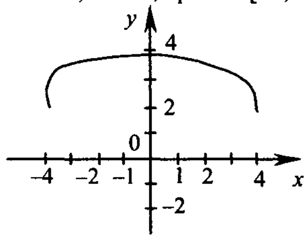


925. а) $y' = \frac{1}{\sqrt{x}} - 1$, при $x \in [1; +\infty)$ убывает, при $x \in [0; 1]$ возрастает, $x = 1$ – max;

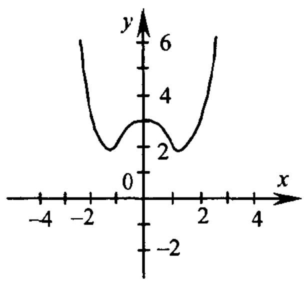


б) $y' = \frac{1}{2\sqrt{x+4}} - \frac{1}{\sqrt{9-3x}} = \frac{\sqrt{9-3x} - 2\sqrt{x+4}}{2\sqrt{x+4}\sqrt{9-3x}}$; $\sqrt{9-3x} > 2\sqrt{x+4}$; $9 \; 3x > 4x + 16$;

$7x < -7$, $x < -1$; при $x \in [-4; -1]$ возрастает, при $x \in [-1; 3]$ убывает; $x = -1$ –max.



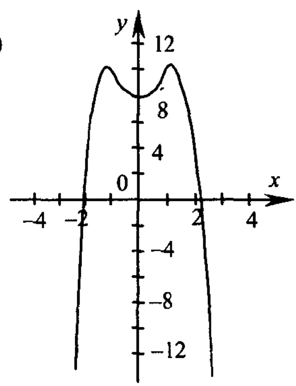
926. а)



б) Количество корней в данном уравнении – это количество пересечения графиков $y = x^4 - 2x^2 + 3$ *и* $y = a$.

Из рисунка видно, что такой случай имеет место, когда прямая $y = a$ касается графика функции в точке $(0; y(0))$, $y(0) = 3$, следовательно, $a = 3$.

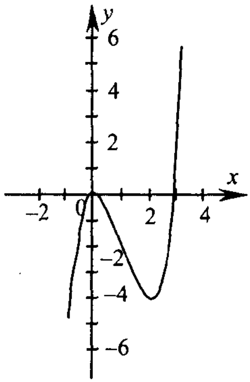
927. а)



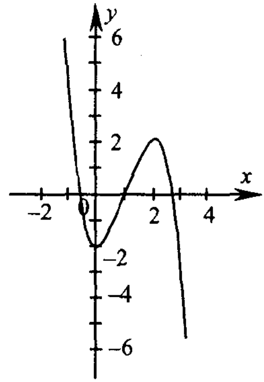
б) Чтобы уравнение не имело корней, необходимо, чтобы прямая лежала выше графика функции $y = -x^4 + 2x^2 + 8$.

Найдем точки максимума: $y' = -4x^3 + 4x = 4x(1 - x^2) = 0$, $x = 0$ – точка минимума, $x = \pm 1$ – точки максимума, $y(1) = y(-1) = -1 + 2 + 8 = 9$. Следовательно, $a > 9$.

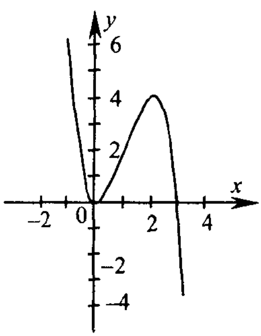
928. а) 3 корня



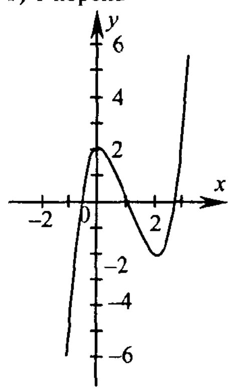
б) 1 корень



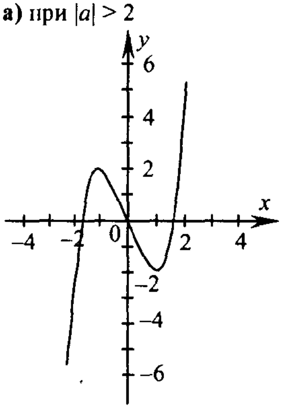
в) 3 корня



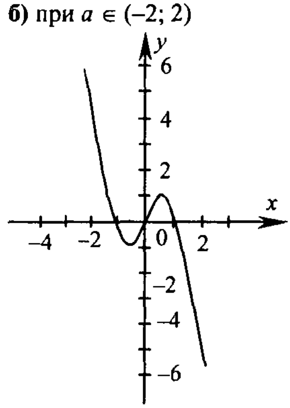
г) 1 корень



929. а) при $|a| > 2$



б) при $a \in (-2; 2)$



930. а) $y_1 = x^3 + 5$, $y_2 = 15 - x$,

$y_1' = 3x^2 + 5$, возрастает на R; $y_2' = -1$, убывает на R, один корень $x = 2$;

б) $y_1' = 5x^4 + 9x^2 + 7$, возрастает на R, один корень $x = -1$;

в) $2x^5 + 3x^3 + 12x - 17 = 0$; $y' = 10x^4 + 9x^2 + 12$, возрастает на R, один корень $x = -1$;

г) $x^5 + 4x^3 + 8x - 13 = 0$, $y' = 5x^4 + 12x^2 + 8$, возрастает на R, один корень $x = -1$.

931. а) $\sin 5x - 2\cos x - 8x - x^5 + 2 = 0, y_1 = \sin 5x - 2\cos x - 8x, y_2 = x^5 - 2, y_1 = 5\cos 5x +$ $+ 2\sin x - 8$ – убывает на R; $y_2' = 5x^4 + 8$ – возрастает на R, только одно решение $x = 0$.

б) $4\cos 3x + 5\sin\dfrac{x}{2} + 15x = 4 - x^3, y_1 = 4\cos 3x + 5\sin\dfrac{x}{2} + 15x, y_2 = 4 - x^3$,

$y_1' = -12\sin 3x + \dfrac{5}{2}\sin\dfrac{x}{2} + 15$ – возрастает на R; $y_2' = 4 - 3x^2$ – убывает на R,

только одно решение $x = 0$.

932. а) $3\cos\dfrac{\pi x}{2} + 5\sin\dfrac{\pi x}{2} + 18x = 46 - x^5 - 22x^3$; $y_1 = 3\cos\dfrac{\pi x}{2} + 5\sin\dfrac{\pi x}{2} + 18x$;

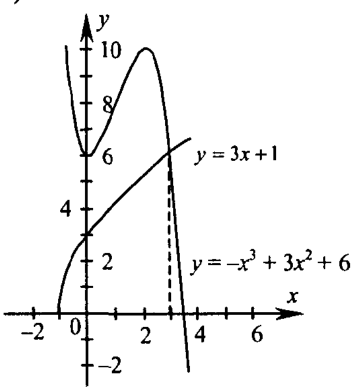
$y_2 = 46 - x^5 - 22x^3$, $y_1' = -\dfrac{3\pi}{2}\sin\dfrac{\pi x}{2} + \dfrac{5\pi}{2}\cos\dfrac{\pi x}{2} + 18$, возрастает на R;

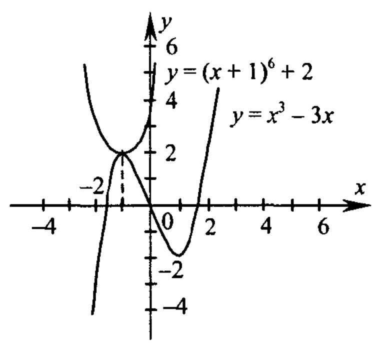
$y_3' = -5x^4 - 66x^2$, убывает на R, один корень $x = 1$;

б) $2\sin\dfrac{\pi}{2}x - 2\cos \pi x - 8x = x^5 - 50, y_1 = 2\sin\dfrac{\pi}{2}x - 2\cos \pi x - 8x, y_2 = x^5 - 50$,

$y_2' = \pi\cos\dfrac{\pi x}{2} + 2\pi\sin \pi x - 8$, убывает на R; $y_2' = 5x^4$, возрастает на R, один корень $x = 2$.

933. а) $x = 3$ б) $x = -1$.





§36. Применение производной для отыскания наибольших и наименьших значений величин

934. а) $y' = 3 \Rightarrow$ функция растет на R; $x_{min} = -1, y(-1) = -9, y_{min} = -9$; $x_{max} = 4$, $y(-1) = 6, y_{max} = 6$.

б) $y' = \dfrac{8}{x^2} > 0 \Rightarrow$ функция растет при $x \in \left[\dfrac{1}{4};8\right]$; $x_{min} = \dfrac{1}{4}, y\left(\dfrac{1}{4}\right) = -32, y_{min} = -32$;

$x_{max} = 8, y(8) = -1, Y_{max} = -1$.

в) $y' = -\dfrac{1}{2} \Rightarrow$ функция убывает на R;

$x_{min} = 6, y(6) = 1, y_{min} = 1$; $x_{max} = -2, y(-2) = 5, y_{max} = 5$.

г) $y' = -\frac{3}{x^2} < 0 \Rightarrow$ функция убывает при $[0{,}3;\ 2]$; $x_{\max} = 0{,}3$, $y(0{,}3) = 10$,

$y_{\max} = 10$; $x_{\min} = 2$, $y(2) = \frac{3}{2}$, $y_{\min} = \frac{3}{2}$.

935. а) $y' = 2x - 8$, $x = 4$ – точка минимума;
$y(4) = 16 - 32 + 19 = 3$; $y(-1) = 1 + 8 + 19 = 28$; $y(5) = 25 - 40 + 19 = 4$; $y_{\max} = 28$, $y_{\min} = 3$.
б) $y' = 2x + 4$, $x = -2$, $-2 \notin [0;2]$; $y(0) = -3$, $y(2) = 4 + 8 - 3 = 9$; $y_{\max} = 9$, $y_{\min} = -3$.
в) $y' = 4x - 8$, $x = 2$; $y(7) = 8 - 16 + 6 = -2$; $y(-1) = 2 + 8 + 6 = 16$; $y(4) = 32 - 32 + 6 = 6$; $y_{\max} = 16$, $y_{\min} = -2$.
г) $y' = -6x + 6$, $x = 1$; $y(1) = -3 + 6 - 10 = -7$; $(-2) = -12 - 12 - 10 = -34$;
$y(9) = -243 + 54 - 10 = -199$; $y_{\max} = -7$, $y_{\min} = -199$.

936. а) $y' = 2\cos x$; $x = \frac{\pi}{2}$; $y = -\frac{\pi}{2}$; $y_{\max} = 2$; $y_{\min} = -2$; б) $y_{\max} = 2$; $y_{\min} = -2$;
в) $y_{\max} = 6$; $y_{\min} = 0$; г) $y_{\max} = \frac{1}{2}$; $y_{\min} = -\frac{1}{2}$.

937. а) $y_{\max} = -\frac{\sqrt{3}}{3}$; $y_{\min} = -\sqrt{3}$; б) $y_{\min} = -3\sqrt{3}$; $y_{\max} = 0$;
в) $y_{\max} = 0$; $y_{\min} = -\frac{2\sqrt{3}}{3}$; г) $y_{\max} = \frac{1}{2}$; $y_{\min} = 0$.

938. а) $y_{\max} = 3$; $y_{\min} = 0$; б) $y_{\max} = 2$; $y_{\min} = 0$;
в) $y_{\max} = -2$; $y_{\min} = -4$; г) $y_{\max} = -2$; $y_{\min} = -3$.

939. а) $y_{\max} = 192$; $y_{\min} = 0$; б) $y_{\max} = -\frac{6}{100\,000} = -\frac{3}{50\,000}$; $y_{\min} = -192$;
в) $y_{\max} = 0$; $y_{\min} = -3$; г) $y_{\max} = 9$; $y_{\min} = 0$.

940. а) $y_{\max} = 1$; $y_{\min} = 0$; б) $y_{\max} = 1$; $y_{\min} = 0$; в) $y_{\max} = 1$; $y_{\min} = 0$; г) $y_{\max} = 1$; $y_{\min} = -1$.

941. $y' = 3x^2 - 18x + 24 = 0$; $x^2 - 6x + 8 = 0$; $x = 4$, $x = 2$;
$y_{\max} = y(2) = 8 - 36 + 48 - 1 = 19$; $y_{\min} = y(4) = 64 - 144 + 96 - 1 = 15$;
а) $y(-1) = -1 - 9 - 24 - 1 = -35$; $y(3) = 17$; $y_{\min} = y(-1) = -35$; $y_{\max} = y(2) = 19$;
б) $y(3) = 27 - 81 + 72 - 1 = 17$; $y(6) = 35$; $y_{\min} = y(4) = 15$; $y_{\max} = y(6) = 15$;
в) $y_{\max} = y(2) = 19$; $y_{\min} = y(-2) = -8 - 36 - 48 - 1 = -93$;
г) $y_{\max} = y(5) = 125 - 225 + 120 - 1 = 19$; $y_{\min} = y(4) = 15$.

942. $y' = 3x^2 + 6x - 45 = 0$; $x^2 + 2x - 15 = 0$; $x = -5$, $x = 3$;
$y_{\max} = y(-5) = -125 + 75 + 225 - 2 = 173$; $y_{\min} = y(3) = 27 + 27 - 135 - 2 = -83$;
а) $y_{\max} = 173$; $y_{\min} = -2$; б) $y_{\max} = -43$; $y_{\min} = -72$;
в) $y_{\max} = 173$; $y_{\min} = 45$; г) $y_{\max} = -2$; $y_{\min} = -72$.

943. $y' = 3x^2 - 18x + 15 = 0$; $x^2 - 6x + 5 = 0$; $x = 5$, $x = 1$;
$y_{\max} = y(1) = 1 - 9 + 15 - 3 = 4$; $y_{\min} = y(5) = 125 - 225 + 75 - 3 = -28$;
а) $y_{\max} = 4$; $y_{\min} = -3$; б) $y_{\max} = -12$; $y_{\min} = -28$;
в) $y_{\max} = 4$; $y_{\min} = -28$; г) $y_{\max} = 4$; $y_{\min} = -28$.

944. $y' = 4x^3 - 24x^2 + 20x = 0$; $4x(x^2 - 6x + 5) = 0$; $x = 0, x = 5, x = 1$; $y(0) = 1$;
$y_{max} = y(1) = 1 - 8 + 10 + 1 = 4$; $y_{min} = y(5) = 625 - 1000 + 250 + 1 = -124$;
а) $y_{max} = 10$; $y_{min} = -7$; **б)** $y_{max} = 4$; $y_{min} = -124$;
в) $y_{max} = 121$; $y_{min} = -44$; **г)** $y_{max} = 148$; $y_{min} = -124$.

945. $y' = 1 - \dfrac{4}{(x-1)^2} = 0$; $(x - 1)^2 = 4$; $x = -1, x = 3$;

$y_{max} = y(3) = 3 + 2 = 5$, $y_{min} = y(-1) = -1 + \dfrac{4}{-2} = -1 - 2 = -3$;

а) $y_{max} = 6$; $y_{min} = 5$; **б)** $y_{max} = -3$; $y_{min} = -4$.

946. а) $y' = -\dfrac{1}{\sin^2 x} + 1 \le 0$; $y_{max} = 1 + \dfrac{\pi}{4}$; $y_{min} = -1 + \dfrac{3\pi}{4}$;

б) $y' = 2\cos x - 1$; $x = \pm\dfrac{\pi}{3} + \pi n, x = \dfrac{\pi}{3}$; $y_{max} = y\left(\dfrac{\pi}{3}\right) = \sqrt{3} - \dfrac{\pi}{3}$; $y_{min} = -\pi$;

в) $y' = -2\sin x + 1$; $\sin x = \dfrac{1}{2}$; $x = \dfrac{\pi}{6}$; $y_{max} = y\left(\dfrac{\pi}{6}\right) = \sqrt{3} + \dfrac{\pi}{6}$; $y_{min} = -\dfrac{\pi}{2}$;

г) $y' = \dfrac{1}{\cos^2 x} - 1$; $x = \pi n, x = 0$; $y_{max} = y\left(\dfrac{\pi}{3}\right) = \sqrt{3} - \dfrac{\pi}{3}$; $y_{min} = y(0) = 0$.

947. а) $y' = 3x^2 - 4x = 0$; $x(3x - 4) = 0$; $x = 0,\ x = \dfrac{4}{3}$;

y_{max} не существует; $y_{min} = y\left(\dfrac{4}{3}\right) = \dfrac{64}{27} - \dfrac{32}{9} + 1 = -\dfrac{5}{27}$;

б) $y' = 1 - \dfrac{1}{\sqrt{x}}$; $x = 1$; y_{max} не существует; $y_{min} = y(1) = 1 - 2 = -1$;

в) $y' = x^4 - 2x = 0$; $x(x^3 - 2) = 0$; $x = 0,\ x = \sqrt[3]{2}$; $y_{max} = y(0) = 0$; y_{min} не существует;

г) $y' = \dfrac{4x^3}{(x^4+1)^2} = 0$; y_{max} не существует; $y_{min} = y(0) = 0$.

948. а) $y' = 1 - \dfrac{1}{x^2}$; $x = \pm 1$; $y_{max} = y(-1) = -1 - 1 = -2$; y_{min} не существует;

б) $y' = \dfrac{3x^2 + 9 - 6x^2}{(x^2+3)^2} = -3 \cdot \dfrac{x^2 - 3}{x^2 + 3}$; $x = \pm\sqrt{3}$; $y_{max} = y\left(\sqrt{3}\right) = \dfrac{3\sqrt{3}}{3+3} = \dfrac{\sqrt{3}}{2}$; $y_{min} = y(0) = 0$;

в) $y' = -2 + \dfrac{1}{2x^2} = 0$; $4x^2 = 1$; $x = \pm\dfrac{1}{2}$; $y_{max} = y\left(\dfrac{1}{2}\right) = -1 - 1 = -2$; y_{min} не существует;

г) $y' = \dfrac{1}{\sqrt{2x+6}} - 1 = 0$; $\sqrt{2x+6} = 1$; $x = -\dfrac{5}{2}$; $y_{max} = y\left(-\dfrac{5}{2}\right) = 1 + \dfrac{5}{2} = 3{,}5$, y_{min} не существует.

949. а) $\begin{cases} a+b=24 \\ ab=\max \end{cases}$; $\begin{cases} a=24-b \\ 24b-b^2=y \end{cases}$; $y'=24-2b$; $b=12$, $y(12)=144$; $b=12$, $a=12$.

б) $ab=484$; $a=\dfrac{484}{b}$; $\dfrac{484}{b}+b=y$; $y'=1-\dfrac{484}{b^2}$; $b=22$, $a=22$.

950. а) $a-b=10$; $a=10+b$; $10b+b^2=y$; $y'=2b+10$; $b=-5$, $a=5$.

б) $a-b=98$; $a=98+b$; $b^2+98b=y$; $y'=2b+98$; $b=-49$, $a=49$.

951. а) $a(a+36)=y$; $a^2+36a=y$; $y'=2a+36$; $a=-18$, $b=-18+36=18$.

б) $a(a-28)=y$; $a^2-28a=y$; $y'=2a-28$; $a=14$, $b=-14$.

952.а) $\begin{cases} a+b=3 \\ y=3a+b^3 \end{cases}$; $\begin{cases} a=3-b \\ y=3a+b^3 \end{cases}$; $y=9-3b+b^3$; $y'=3b^2-3$; $b=\pm1$, но т.к. по условию $b>0$, то $b=1$, $a=2$.

б) $\begin{cases} a+b=5 \\ y=ab^3 \end{cases}$; $\begin{cases} a=5-b \\ y=ab^3 \end{cases}$; $e=5b^3-b^4$; $y'=15b^2-4b^3=b^2(15-4b)$; $b=0$, $b=\dfrac{15}{4}$

но т.к. по условию $b>0$, то $b=\dfrac{15}{4}$; $a=\dfrac{5}{4}$.

953. а) $\begin{cases} 2a+2b=56 \\ ab=y \end{cases}$; $\begin{cases} a=28-b \\ 28b-b^2=y \end{cases}$; $y'=28-2b$; $2b=28$; $b=14$, $a=14$;

б) $\begin{cases} a+b=36 \\ ab=y \end{cases}$; $\begin{cases} a=36-b \\ 36b-b^2=y \end{cases}$; $y'=36-2b$; $2b=36$; $b=18$, $a=18$.

954. а) $\begin{cases} a+b=100 \\ ab=y \end{cases}$; $\begin{cases} a=100-b \\ 100b-b^2=y \end{cases}$; $y'=100-2b$; $2b=100$; $b=50$, $a=50$;

б) $\begin{cases} a+b=120 \\ ab=y \end{cases}$; $\begin{cases} a=120-b \\ 120b-b^2=y \end{cases}$; $y'=120-2b$; $2b=120$; $b=60$, $a=60$.

955. а) $\begin{cases} ab=16 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\dfrac{16}{b} \\ 2a+2b=y \end{cases}$; $y=\dfrac{32}{b}+2b$; $y'=2-\dfrac{32}{b^2}$; $b=4$, $a=4$;

б) $\begin{cases} ab=64 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\dfrac{164}{b} \\ 2a+2b=y \end{cases}$; $y=\dfrac{128}{b}+2b$; $y'=2-\dfrac{128}{b^2}$; $b=8$, $a=8$.

956. $\begin{cases} ab=2500 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\dfrac{2500}{b} \\ 2a+2b=y \end{cases}$; $y=\dfrac{5000}{b}+2b$; $y'=2-\dfrac{5000}{b^2}$; $b=50$, $a=50$.

957. $KD=DM=x$; $BH_1=\dfrac{3\sqrt{2}}{2}$; $DH_2=\dfrac{x\sqrt{2}}{2}$ (H_1 и H_2 – точки пересечения BD с PE и KM соответственно);

$$S=\frac{1}{2}(PE+KM)\cdot H_1H_2=\frac{1}{2}\left(x\sqrt{2}+3\sqrt{2}\right)\left(8\sqrt{2}-\frac{3\sqrt{2}}{2}-\frac{x\sqrt{2}}{2}\right)=\frac{1}{2}(39+10x-x_2);$$

$$S'=\frac{1}{2}(10-2x)=0;\ x=5;\ S=\frac{1}{2}(39+50-25)=32.$$

958. **а)** $y_{\max}=\sqrt{2};\ y_{\min}=0$; **б)** $y_{\max}=\sqrt{2};\ y_{\min}=1$;

в) $y_{\max}=\sqrt{2};\ y_{\min}=0$; **г)** $y_{\max}=\sqrt{2};\ y_{\min}=0$.

959. **а)** $y=2-5\sin(x+a);\ y_{\max}=7;\ y_{\min}=-3;$

б) $y=3\sin x-4\cos x+l;\ y=5\sin(x-\alpha)+l;\ y_{\max}=6,\ y_{\min}=-4.$

960. **а)** $y'=4x^3+24x^2+48x+32=0;\ x^3+6x^2+12x+8=0;\ (x+2)(x^2-2x+4+6x)=0;$
$(x+2)(x^2+4x+4)=(x+2)^3=0;\ x=-2;$
$y(-2)=16-64+96-64+21=133;\ y(-3)=81-216+216-96+21=6;$
$y(0)=21;\ y_{\min}=y(-2)=5,\ y_{\max}=y(6)=21.$
б) $y'=4x^3-12x^2+12x-4=0;\ x^3-3x^2+3x-1=0;$
$(x-1)^3=0;\ x=1;\ y_{\max}=y(4)=71,\ y_{\min}=y(1)=1-4+6-4-9=-10.$

961. **а)** $y=x^2-5|x|+6$, $[0;4]$; на этом промежутке $x\geq 0\Rightarrow y=x^2-5x+6;\ y'=2x-5;$
$x=\frac{5}{2};\ y_{\max}=y(0)=6;\ y_{\min}=y\left(\frac{5}{2}\right)=\frac{25}{4}-\frac{25}{2}+6=-\frac{25}{4}+6=-\frac{1}{4}.$
б) $y=x^2-5|x|+6$, $[-5;0]$; на этом промежутке $x\leq 0\Rightarrow y=x^2+5x+6;\ y'=2x+5;$
$x=-\frac{5}{2};\ y_{\max}=y(0)=6;\ y_{\min}=y\left(-\frac{5}{2}\right)=\frac{25}{4}-\frac{25}{2}+6=-\frac{25}{4}+6=-\frac{1}{4}.$
в) $y=x^2+8|x|+7$, $[1;5]$; на этом промежутке $x\geq 0\Rightarrow y=x^2+8x+7;\ y'=2x+8;$
$x=-4$ – не подходит;
$y_{\max}=y(5)=25+40+7=72;\ y_{\min}=y(1)=16.$
г) $y=x^2+8|x|+7$, $[-8;-2]$; на этом промежутке $x\leq 0\Rightarrow y=x^2-8x+7;\ y'=2x-8-$
$-\ x=4$ – не подходит; $y_{\max}=y(-8)=64+64+7=135;\ y_{\min}=y(-2)=4+32+7=43.$

962. **а)** $y=x^3-3x,\ (-\infty;0];\ y'=3x^2-3;\ x=-1;\ y_{\max}=y(-1)=-1+3=2;\ y_{\min}$ – не существует.
б) $y=x^3-3x,\ [0;+\infty);\ y_{\min}=y(1)=-2;\ y_{\max}$ – не существует.

963. **а)** $y'=\dfrac{x^4+3-4x^2}{\left(x^4+3\right)^2}=\dfrac{3-3x^4}{\left(x^4+3\right)^2};\ x=\pm 1;\ y_{\max}=y(1)=\frac{1}{4};\ y_{\min}(0)=0;$

б) $y_{\min}=y(-1)=-\frac{1}{4};\ y_{\max}=y(0)=0.$

964. **а)** $y=\sin^2\frac{x}{2}\sin x=\frac{1}{2}\sin x-\frac{1}{2}\cos x\sin x=\frac{1}{2}\sin x-\frac{1}{4}\sin 2x;$

$$y'=\frac{1}{2}\cos x-\frac{1}{2}\cos 2x=0;\ \sin\frac{3x}{2}\sin\frac{x}{2}=0;\ x=2\pi n,\ x=\frac{2\pi n}{3};\ x=-\frac{2\pi}{3};$$

$$y\left(-\frac{2\pi}{3}\right)=-\frac{3}{4}\cdot\frac{\sqrt{3}}{2}=-\frac{3\sqrt{3}}{8}-y_{\min};\ y_{\max}=0.$$

б) $y' = -\cos\frac{x}{2}\cdot\sin\frac{x}{2}\cos x - \cos^2\frac{x}{2}\sin x = 0; \cos\frac{x}{2}\left(\sin\frac{x}{2}\cos x + \cos\frac{x}{2}\sin x\right) = 0$;

$\cos\frac{x}{2}\cdot\sin\frac{3x}{2} = 0;\ x = \pi + 2\pi n,\ x = \frac{2\pi k}{3}$;

$y(0) = 1;\ y(\pi) = 0;\ y\left(\frac{2\pi}{3}\right) = \frac{1}{4}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{8};\ y_{\min} = -\frac{1}{8},\ y_{\max} = 1$.

965. а) I: $x \ge 1;\ y = x^2 - 3x + 4;\ y' = 2x - 3 = 0;\ x = \frac{3}{2}$;

$y(1) = 1 - 4 + 5 + (1 - 1) = 2;\ y\left(\frac{3}{2}\right) = \frac{9}{4} - 6 + 5 + \left(1 - \frac{3}{2}\right) = \frac{7}{4};\ y(4) = 8$;

II: $x \le 1;\ y = x^2 - 5x + 6;\ y' = 2x - 5 = 0;\ x = \frac{5}{2}$ – не подходит; $y(0) = 6$,

$y_{\min} = \frac{7}{4};\ y_{\max} = 8$;

б) I: $x \ge 1;\ y = x^3 - 3x - 1;\ y' = 3x^2 - 3;\ x = 1;\ y(1) = -3;\ y(3) = -17$;

II: $x \le 1;\ y = 1 - 3x - x^3;\ y' = -3 - 3x^2;\ y(1) = -3 = y_{\max};\ y(-1) = 5;\ y_{\min} = -3;\ y_{\max} = 17$.

966. а) $y' = 2 - \frac{8}{\sqrt{16x-4}};\ x = \frac{5}{4};\ y\left(\frac{5}{4}\right) = \frac{5}{2} - 4 = -\frac{3}{2};\ y\left(\frac{1}{4}\right) = \frac{1}{2};\ y\left(\frac{17}{4}\right) = \frac{17}{2} - 8 = \frac{1}{2};\ y \in \left[-\frac{3}{2};\frac{1}{2}\right]$.

б) $y' = \frac{1}{\sqrt{x-1}} - \frac{1}{2};\ x = 5;\ y(5) = 4 - \frac{5}{2} = \frac{3}{2};\ y(1) = -\frac{1}{2};\ y(10) = 6 - 5 = 1;\ y \in \left[-\frac{1}{2};\frac{3}{2}\right]$.

967. а) $y' = \sqrt{x+2} + \frac{x}{2\sqrt{x+2}} = \frac{2x+4+x}{2\sqrt{x+2}} = \frac{3x+4}{2\sqrt{x+2}}$;

$x = -\frac{4}{3};\ y\left(-\frac{4}{3}\right) = -\frac{4}{3}\sqrt{-\frac{4}{3}+2} = -\frac{4\sqrt{6}}{9};\ y \in \left[-\frac{4}{9}\sqrt{6};+\infty\right)$;

б) $y' = \sqrt{1-2x} - \frac{x}{\sqrt{1-2x}} = \frac{1-2x-x}{\sqrt{1-2x}} = \frac{1-3x}{\sqrt{1-2x}};\ x = \frac{1}{3}$;

$y\left(\frac{1}{3}\right) = \frac{1}{3}\sqrt{1-\frac{2}{3}} = \frac{1}{3\sqrt{3}};\ y \in \left(-\infty;\frac{\sqrt{3}}{9}\right]$.

968. а) $y' = \sqrt{x+a} + \frac{x}{2\sqrt{x+a}} = \frac{3x+2a}{2\sqrt{x+a}};\ x = -\frac{2a}{3};\ y\left(-\frac{2a}{3}\right) = -\frac{2a}{3}\sqrt{\frac{a}{3}} = -6\sqrt{3}$;

$a\sqrt{a} = 27;\ a = 27^{\frac{2}{3}} = 9$.

б) $y' = -\sqrt{x} + \frac{a-x}{2\sqrt{x}} = \frac{a-x-2x}{2\sqrt{x}} = \frac{a-3x}{2\sqrt{x}};\ x = \frac{a}{3};\ y\left(\frac{a}{3}\right) = \frac{2a}{3}\sqrt{\frac{a}{3}} = 10\sqrt{5}$;

$a\sqrt{a} = 15\sqrt{15};\ a = 15$.

969. а) $a_9 = 1$; $\begin{cases} a_1 + 8d = 1 \\ (a_1 + 3d)(a_1 + 6d)(a_1 + 7d) = y \end{cases}$; $\begin{cases} a_1 = 1 - 8d \\ (1 - 5d)(1 - 2d)(1 - d) = y \end{cases}$;

$y = (1 + 10d^2 - 7d)(1 - d) = 1 - d + 10d^2 - 10d^3 - 7d + 7d^2 = 1 - 8d + 17d^2 - 10d^3$;

$y' = -8 + 34d - 30d^2 = 0$; $15d^2 - 17d + 4 = 0$; $D = 289 - 240 = 49$;

$d_1 = \frac{17+7}{30} = \frac{4}{5}$; $d_2 = \frac{1}{3}$; $y\left(\frac{4}{5}\right) = (1-4)\left(1 - \frac{8}{5}\right)\left(1 - \frac{4}{5}\right) = -3 \cdot \left(-\frac{3}{5}\right)\frac{1}{5} = \frac{9}{25}$;

$y\left(\frac{1}{3}\right) = \left(1 - \frac{5}{3}\right)\left(1 - \frac{2}{3}\right)\left(1 - \frac{1}{3}\right) = -\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = -\frac{4}{27}$; $d = \frac{4}{5}$.

б) $\begin{cases} a_1 + d = 6 \\ (a_1 + 2d)a_1(a_1 + 5d) = y \end{cases}$; $\begin{cases} a_1 = 6 - d \\ (6 + d)(6 - d)(6 + 4d) = y \end{cases}$; $y = (36 - d^2)(6 + 4d) =$

$= 216 + 144d - 6d^2 - 4d^3$; $y' = 144 - 12d - 12d^2 = 0$; $d^2 + d - 12 = 0$; $d_1 = 3$; $d_2 = -4$;

$y(3) = 9 \cdot 3 \cdot 18 = 486$; $y(-4) = 2 \cdot 10 \cdot (-10) = -200$; $d = -4$.

970. а) $y_1 = 2x^2$; $y_2 = 4x$. Длина отрезка равна $4x - 2x^2 = f(x)$, $x \in [0; 2]$; $f'(x) = 4 - 4x$; $x_0 = 1$; $f(1) = 4 - 2 = 2$;

б) $y_1 = x^2$; $y_2 = -2x$. Длина отрезка равна $-2x - x^2 = f(x)$, $x \in [-2; 0]$; $f'(x) = -2 - 2x$; $x_0 = -1$; $f(-1) = 2 - 1 = 1$.

971. а) $\sqrt{(0-x)^2 + (1{,}5 - y)^2} = f(x)$; $f(x) = \sqrt{x^2 + \left(1{,}5 - x^2\right)^2} = \sqrt{x^2 + \frac{9}{4} - 3x^2 + x^4} =$

$= \sqrt{x^4 - 2x^2 + \frac{9}{4}}$; $f'(x) = \frac{4x^3 - 4x}{2\sqrt{x^4 - 2x^2 + \frac{9}{4}}} = 2 \cdot \frac{x^3 - x}{\sqrt{x^4 - 2x^2 + \frac{9}{4}}}$;

$x = 0, x = \pm 1$; $f(0) = \frac{3}{2}$; $f(1) = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2} = f(-1)$; $(1;1), (-1;1)$;

б) $f(x) = \sqrt{(4{,}5 - x)^2 + y^2} = \sqrt{\frac{81}{4} - 9x + x^2 + x} = \sqrt{\frac{81}{4} - 8x + x^2}$;

$f'(x) = \frac{2x - 8}{2\sqrt{\frac{81}{4} - 8x + x^2}}$; $x = 4$; $y(4) = 2$; $(4; 2)$.

972. Пусть h – высота трапеции;

$S = \frac{1}{2}\left(15 + 2\sqrt{225 - h^2} + 15\right)h = \left(15 + \sqrt{225 - h^2}\right)h$;

$S' = 15 + \sqrt{225 - h^2} - \frac{h^2}{\sqrt{225 - h^2}}$; $15\sqrt{225 - h^2} + 225 - 2h^2 = 0$;

$50\,625 - 225h^2 = 50\,625 - 900h^2 + 4h^2$; $4h^2 - 675h^2 = 0$; $h^2(4h^2 - 675) = 0$;

$h^2 = \frac{675}{4}$; $a = 15 + 2\sqrt{225 - \frac{675}{4}} = 15 + 2 \cdot \frac{15}{2} = 30$.

973. а) Пусть α – угол между основанием и боковой стороной x – сторона прямоугольника, которая совпадает с высотой, y – его другая стороной.

Тогда $\operatorname{tg}\alpha = 5 = \dfrac{x}{80-y}$; $80 - y = \dfrac{x}{5}$; $y = 80 - \dfrac{x}{5}$; $S = 80x - \dfrac{x^2}{5}$; $S' = 80 - \dfrac{2x}{5}$;

$x = 200$, но $x \in (0;\ 100] \Rightarrow x = 100$, $y = 60$; $S = 6000$.

б) $a = 24$, $b = 8$, $h = 12$.

Пусть α – угол между большим основанием трапеции и ее боковой стороной, x – сторона прямоугольника, которая совпадает с высотой, y – его другая сторона. $\operatorname{tg}\alpha = \dfrac{3}{4} = \dfrac{x}{24-y}$; $24 - y = \dfrac{4x}{3}$; $y = 24 - \dfrac{4x}{3}$; $S = 24x - \dfrac{4x^2}{3}$; $S' = 24 - \dfrac{8x}{3}$;

$x = 9$, $y = 12$; $S = 108$.

974. **а)** Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

Тогда $\operatorname{tg}\alpha = \dfrac{7-5}{9-3} = \dfrac{2}{6} = \dfrac{1}{3} = \dfrac{x}{9-y}$; $x = 3 - \dfrac{1}{3}y$; $y = -3x + 9$; $S = -3x^2 + 9x$;

$S' = -6x + 9$; $x = \dfrac{3}{2}$; $y = -\dfrac{9}{2} + 9 = \dfrac{9}{2}$, но $AE \cdot AB = 21 \Rightarrow S_{\max} = 21$.

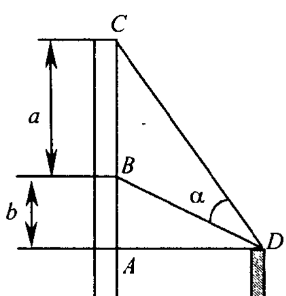
б) $a = 7$, $b = 18$, $c = 3$, $m = 1$;

Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

Тогда $\operatorname{tg}\alpha = \dfrac{7-1}{18-3} = \dfrac{2}{5} = \dfrac{x}{18-y}$; $18 - y = \dfrac{5}{2}x$; $y = 18 - \dfrac{5}{2}x$;

$S = 18x - \dfrac{5}{2}x^2$; $S' = 18 - 5x$; $x = \dfrac{18}{5}$; $y = 9$; $S = 32{,}4$; $AE \cdot AB = 21 \Rightarrow S_{\max} = 32{,}4$.

975. $AC = a$; $AB = b$; $CB = AC - AB = a - b$; $AD = x$; $BD = \sqrt{x^2 + b^2}$; $CD = \sqrt{x^2 + a^2}$.



По теореме косинусов $(a-b)^2 = x^2 + b^2 + x^2 + a^2 - 2\sqrt{(x^2+b^2)(x^2+a^2)}\cos\alpha$;

$2x^2 + 2ab = 2\sqrt{(x^2+b^2)(x^2+a^2)}\cos\alpha$; $\cos\alpha = \dfrac{x^2+ab}{\sqrt{(x^2+b^2)(x^2+a^2)}}$;

$$f(x) = \frac{x^2+ab}{\sqrt{(x^2+b^2)(x^2+a^2)}} = \frac{x^2+ab}{\sqrt{x^4 + x^2(a^2+b^2) + b^2a^2}};$$

$$f'(x)=\frac{2x\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}-\frac{\left(x^2+ab\right)\left(4x^3+2x\left(a^2+b^2\right)\right)}{2\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}}}{\left(x^2+b^2\right)\left(x^2+a^2\right)}=0;$$

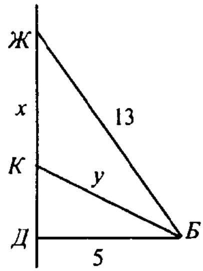
$$2x\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}=x\left(x^2+ab\right)\frac{2x^2+a^2+b^2}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}};$$

$2(a^2b^2+x^2a^2+x^2b^2+x^4)=2x^4+2x^2ab+x^2a^2+x^2b^2+a^3b+ab^3;$

$2ab(ab-x^2)+a^2(x^2-ab)+b^2(x^2-ab)=0;\ (x^2-ab)(a-b)^2=0;\ x=\sqrt{ab}$;

$\alpha_{\max}$ при $x=\sqrt{ab}$.

976. ЖД = 12; x – путь пешехода по дороге, y – путь пешехода по лесу.



Суммарное время $t=\frac{x}{5}+\frac{y}{3}$; $ДК=12-x$;

$$y=\sqrt{25+(12-x)^2};\ t'=\frac{1}{5}+\frac{2x-24}{2\cdot 3\sqrt{x^2-24x+169}}=\frac{1}{5}+\frac{x-12}{3\sqrt{x^2-24+169}}=0;$$

$\frac{3}{5}\sqrt{x^2-24x+169}=12-x;\ 9x^2-9\cdot 24x+9\cdot 169=25(144-24x+x^2);$

$16x^2-384x+2079=0;\ x_1=8{,}25;\ x_2=15{,}75$ – не подходит;

$x=\frac{33}{4};\ y=\sqrt{25+3{,}75^2}=\frac{25}{4};\ t=\frac{x}{5}+\frac{4}{5}=\frac{33}{20}+\frac{25}{12}=\frac{56}{15}\approx$ 3 часа 44 минуты.

977. Пусть x – длина стороны основания бака, y – его высота.

$V=x^2y=32;\ S=x^2+4xy=x^2+\frac{128}{x};\ S'(x)=2x-\frac{128}{x^2}=0;\ x^3=64;\ x=4,\ y=2.$

Ответ: 4 дм, 4 дм, 2 дм.

978. $V=x^2y=343$ (x – длина стороны основания бака, y – его высота);

$S=2x^2+4xy=2x^2+\frac{1372}{x};\ S'=4x-\frac{1372}{x^2}=0;\ x=7,\ y=7.$

Ответ: 7 м, 7 м, 7 м.

979. Пусть $2x$ и $3x$ – длины сторон основания короба, y – его высота.

$V=6x^2y=576;\ S=12x^2+6xy+4xy=12x^2+\frac{960}{x};\ S'=24x-\frac{960}{x^2}=0;$

$x^3 = 40$; $x_0 = 2\sqrt[3]{5}$, $y_0 = \dfrac{576}{6} \cdot \dfrac{1}{4\sqrt[3]{25}} = 24\dfrac{\sqrt[3]{5}}{5}$.

Ответ: $4\sqrt[3]{5}$ м; $6\sqrt[3]{5}$ м; $\dfrac{24\sqrt[3]{5}}{5}$ м.

980. $V = \sqrt{\left(d^2 - x^2\right)x}$ (x – длина бокового ребра призмы);

$V' = d^2 - 3x^2 = 0$; $x = \dfrac{d}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}d$.

981. $a = 2\sqrt{p^2 - h^2}$; $S_{\text{осн}} = 4\left(p^2 - h^2\right)$; $V = \dfrac{1}{3} \cdot 4\left(p^2 - h^2\right)h$;

$V'(h) = \dfrac{4}{3}\left(p^2 - 3h^2\right) = 0$; $h = \dfrac{f}{\sqrt{3}} = \dfrac{p\sqrt{3}}{3}$.

982. Пусть x – диаметр цилиндра, h – его высота; $2x + 2h = p$;

$V = \pi\left(\dfrac{x}{2}\right)^2 h = \pi\left(\dfrac{p-2h}{2}\right)^2 h$; $V' = \left(\dfrac{\pi}{4}\left(p^2 - 4ph + 4h^2\right)h\right) = \dfrac{\pi}{4}\left(p^2 - 8ph + 12h^2\right) = 0$;

$12h^2 - 8ph + p^2 = 0$; $h_{1,2} = \dfrac{8p \pm 4p}{24}$; $h_1 = \dfrac{p}{6}$; $h_2 = \dfrac{p}{2}$;

h_2 – не подходит, т.к. $2h_2 = p$, тогда $x = 0$, чего быть не может; значит $h = \dfrac{p}{6}$.

983. $S = 2\pi Rh + 2\pi R^2$ – площадь боковой поверхности; $V = \pi R^2 h$;

$h = \dfrac{V}{\pi R^2}$; $S = \dfrac{2V}{R} + 2\pi R^2$; $S'(R) = -2\dfrac{V}{R^2} + 4\pi R = 0$; $R^3 = \dfrac{V}{2\pi}$; $R = \sqrt[3]{\dfrac{V}{2\pi}}$.

Содержание

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